RECENT ADVANCES IN INVARIANT ALGORITHMS FOR STRUCTURAL AND MULTIBODY DYNAMICS

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Abstract

In this work we report our latest results on the problem of constructing numerical integration schemes that, by design, preserve fundamental properties of the mechanical system being simulated. In fact, many systems in mathematical physics are characterized by a number of qualitative characteristics that are typically lost when conventional numerical discretization processes are used. Among these characteristics we find purely geometrical notions, such as the preservation of nonlinear configuration manifolds, or the preservation of constraint manifolds, or frame-indifference, and also conservation of important dynamic invariants such as momenta and energy. In recent years it has become widely recognized that the 'gentler' the numerical treatment of a problem is with respect to its qualitative characteristics, the better the numerical solution typically becomes in terms of accuracy, stability, and overall performance [7, 8].

Control on the total mechanical energy has a crucial role in this context, since the proof of a discrete energy bound in a typical time step guarantees unconditional stability of the underlying integrator. To this end, dedicated energy-preserving (EP) schemes for non-linear problems have been developed, given that classical unconditionally stable algorithms loose their properties in the non-linear regime.

However, when dealing with stiff problems in structural and flexible multibody dynamics, it has become increasingly evident that energy preservation does not really imply robustness of the scheme, due to the inability of controlling the possible excitation of high frequency components in the response. Energy decaying (ED) schemes represent the most recent attempt at trying to develop practically robust algorithms [1, 3, 4, 6]. ED schemes are carefully constructed so that the underlying time discretization implies the existence of a discrete law of energy decay in a typical time step, insuring unconditional stability in the non-linear regime together with the appropriate damping of the unresolved and spurious high frequencies. In fact, high frequency dissipation is a well understood and appreciated property that represents a customary requirement for many practical engineering applications, even in the linear regime. Fig. 1 clearly shows the improvement offered by an ED scheme in contrast to the typical output of an EP scheme for a stiff flexible multibody problem.

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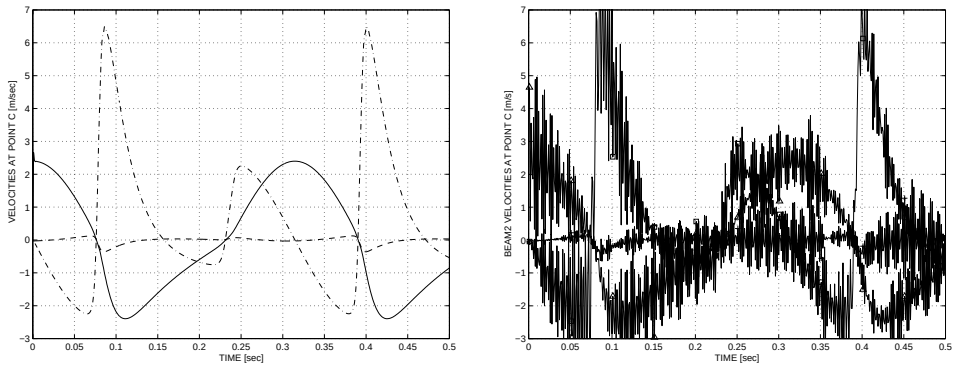

FIGURE 1. Time history of velocities at a hinge in a flexible four bar mechanism computed with the α-MP-ED scheme of Ref. [5] at $\alpha = 1$ (left) and with the EP scheme of Ref. [4] (right).

An innovative family of 2nd to 4th order accurate, non-linear ED methods was recently proposed in refs. [2, 5]. The method is based on a simple discretization stencil characterized by an excellent low-pass filtering behavior, as shown by the spectral radius and the relative period error, two common performance indices that refer to linear problems. These are contrasted with those of the well known *generalized-α* methods [9] in fig. 2, at equal values of the asymptotic spectral radius ρ_∞. The improvement is quite clear, and should be added to the fact that *generalized-α* methods, as well as other common multistep methods that are indeed unconditionally stable and high-frequency dissipative in the linear case, do not guarantee the same properties in the non-linear case.

The drawback of EP and ED methods is that the basic discretization scheme has to be specialized to each model implemented, since they must 'understand' the equations being solved. Thus, details for a beam or a shell will be slightly different, reflecting the diverstity of the governing equations

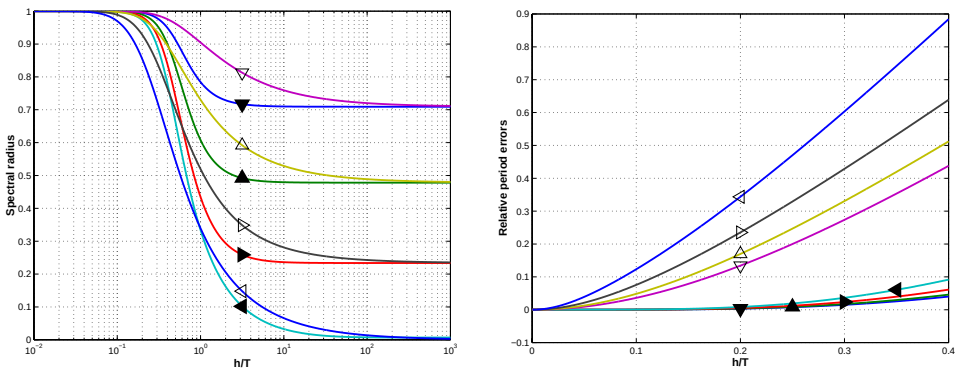

FIGURE 2. Spectral radius (left) and relative period error (right) vs. non-dimensional time-step size for various α-MP-ED schemes (black triangles) compared with the generalized-α scheme (white triangles) with similar asymptotic behavior.

for these two models. However, what is gained by this approach seems to amply justify this minor limitation.

The derivation of EP and ED schemes can be performed in view of additional invariant preservations. A remarkable example is the conservation of the system total linear and angular momenta, which are commonly assumed as important observables of mechanical systems. As in the case of energy preservation/decay, we show how the discretization schemes can be designed so as to imply momentum preservation, first for unconstrained bodies under suitable loading conditions, and eventually to multibody systems subjected to ideal, time-independent constraints. The latter case amounts to the exact preservation of Newton's *Third Law of Action and Reaction*, a result that, to the authors' knowledge, has not been achieved to date in the present context.

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