AIRPLANE EQUILIBRIUM AND STABILITY: A SIMPLIFIED TEACHING APPROACH

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Abstract

This work is concerned with a reformulation of the study of basic equilibrium and stability of airplanes, within a course in flight mechanics at the undergraduate and/or graduate levels. The typical learning path involves a step-by-step process that basically traces back to the traditional approach in preliminary aircraft design. The student goes through a substantial number of formulæ which are mathematically simple, but considerably unattractive and uneasy to remember, being 'cluttered' by a remarkable amount of quantities that describe specific details of the assumed airplane model. Some possible drawbacks of this approach include: (a) the different appearance of formulæ derived within the same general hypotheses, but with different levels of refinement in modeling the airplane and its aerodynamic field; and (b) the need of reconsider the derivation of a formula from the very start when looking at an aircraft different from the model assumed (*e.g.* canard instead of traditionally tailed, tailless, etc.). Also important is the fact that the student may develop the impression that the validity of a general result in trim and stability analysis is related to the way it was derived.

Motivated by this considerations, we propose a simplified formulation where the stress is placed on the fundamental qualitative features of the governing equations without *a priori* assumptions on a particular airplane model. Therefore, design details on the surface arrangement, layout of the controls, and so on, are left aside while deriving, in a general way, all the fundamental results concerning trim and stability. In this way, we are able to formulate the problem of airplane equilibrium and stability, under the customary assumptions of linear, low subsonic, steady-state aerodynamics, by first stating the *constitutive laws* pertaining to the flight conditions at hand: stick-fixed steady level flight, stick-free steady level flight, steady maneuvering flight, etc. From the investigation of the constitutive laws, appropriate characteristic points are introduced: the first is the well known neutral point, while the second is termed the control point. The latter, for a traditionally tailed design, is located close to the aerodynamic center of the tailplane. The introduction of the control point allows for a reduction of the complex system of aerodynamic forces acting on the airplane to an equivalent one consisting in only two applied forces: the first, termed the *attitude lift*, depends on the angle of attack only, while the second, termed the *control lift*, depends on the elevator deflection only. Fig. 1 shows the equilibrium for level flight conditions and should be contrasted with fig. 2, which represents

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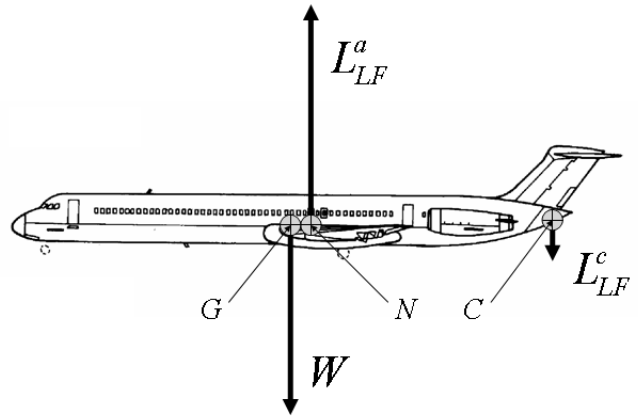

FIGURE 1. The level flight simplified force diagram resulting from the decomposition of the aerodynamic forces into *attitude lift* component L^a_{LF} applied at the neutral point N, and *control lift* component L^c_{LF} applied at the control point C. W represents the weight applied at the center of mass G.

the ‘classical’ situation adopted in flight mechanics textbooks and courses. Basic considerations of trim and stability are easily carried out within this framework by simple, intuitive reasoning upon this force diagram. In order to help the reader to familiarize with this approach, applications to level flight in both the stick-fixed and stick-free settings, as well as to maneuvering flight, are briefly addressed.

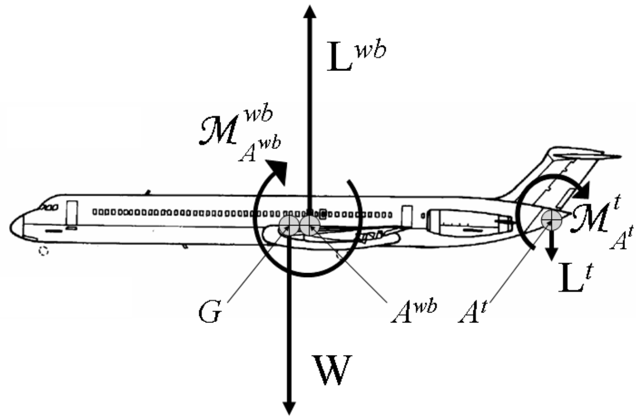

FIGURE 2. The level flight ‘classical’ force diagram resulting from the decomposition of the aerodynamic forces into wing-body and tail contribution: L^{wb} is the wing-body lift, A^{wb} the wing-body aerodynamic center and $\mathcal{M}^{wb}_{A^{wb}}$ the corresponding wing-body pitching moment, L^t is the tail lift, A^t the tail aerodynamic center and $\mathcal{M}^t_{A^t}$ the corresponding tail pitching moment. W represents the weight applied at the center of mass G.