Accurate and Efficient Direct Numerical Simulation of drag reduction via riblets

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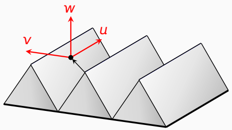
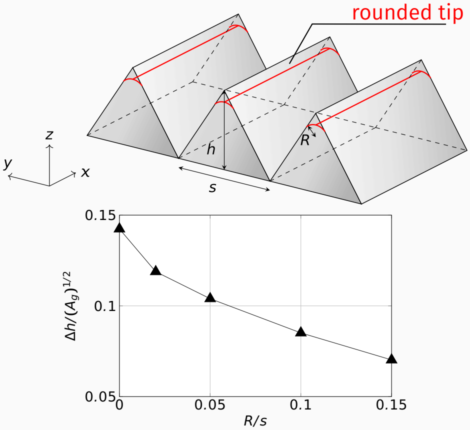


The drag reduction performance of the riblets depends on the sharpness of their tip.

Consequences for DNS:
An extremely fine grid is required near the tip.



Adapted from Garcia–Mayoral & Jimenez, Phil. Trans. R. Soc. A (2011)

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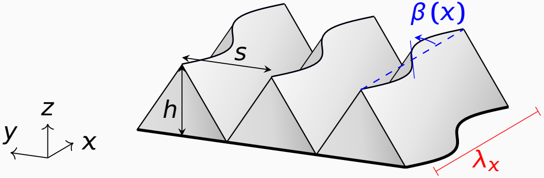
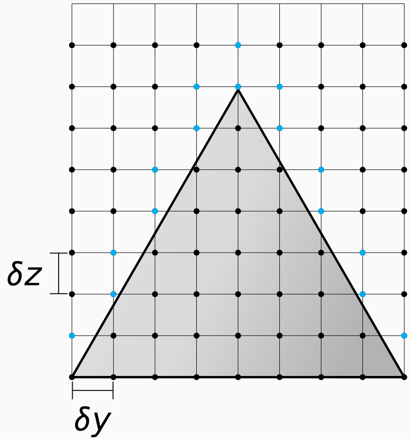
Simulation of three dimensional riblets particularly expensive



Riblets resolved in immersed boundary solver
(Luchini, Eur. J. Mech. B Fluids (2016))

- second-order finite differences on a staggered grid
- implicit deferred correction of $\nabla^2 \mathbf{u}$
  - solution behaves linearly at the wall



Riblets resolved in immersed boundary solver
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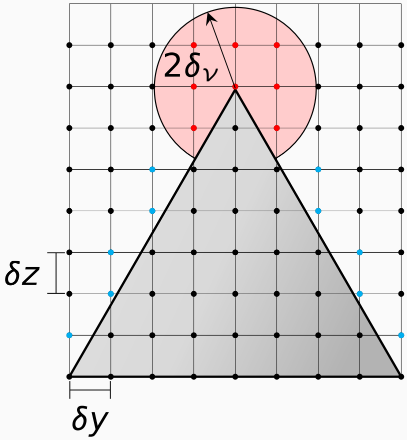
- second-order finite differences on a staggered grid
- implicit deferred correction of $\nabla^2 \mathbf{u}$ and ∇p
  - solution behaves linearly at the wall
  - solution behaves as Stokes eigensolution at the riblet tip

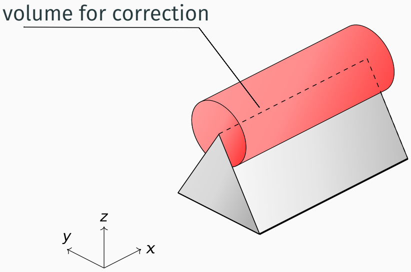


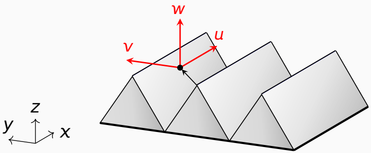
Analytical solution
of the Stokes problem
around infinite corner:

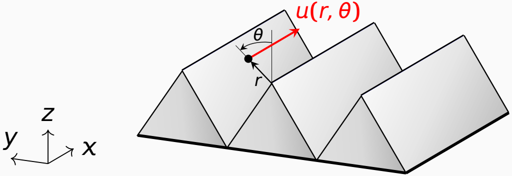
$$\nabla \cdot \mathbf{u} = 0$$

$$\cancel{\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u}} + \frac{1}{\rho}\nabla p = \nu \nabla^2 \mathbf{u}$$

- linear equations
- two (|| and ⊥) uncoupled problems!



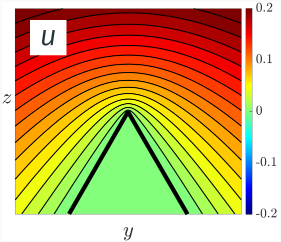




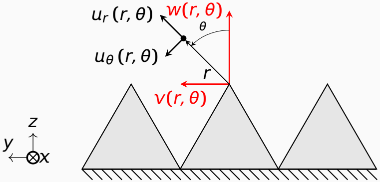
1D Laplace problem

$$0 = \nu\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$\downarrow$$



The problem ⊥ to the crest

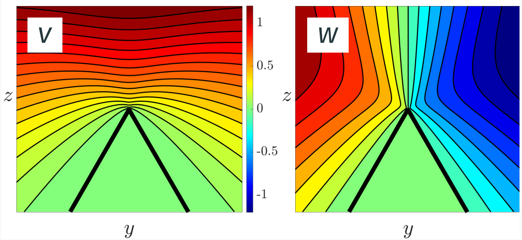


2D Stokes problem

$$\frac{\partial P}{\partial y} = \nu\left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

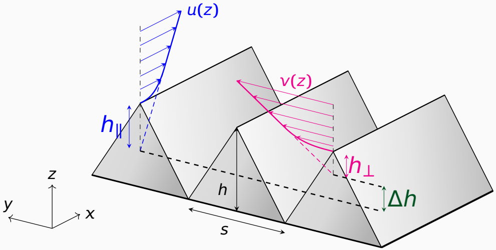
$$\frac{\partial P}{\partial z} = \nu\left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$

$\downarrow$



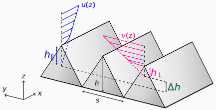
Validation: protrusion heights

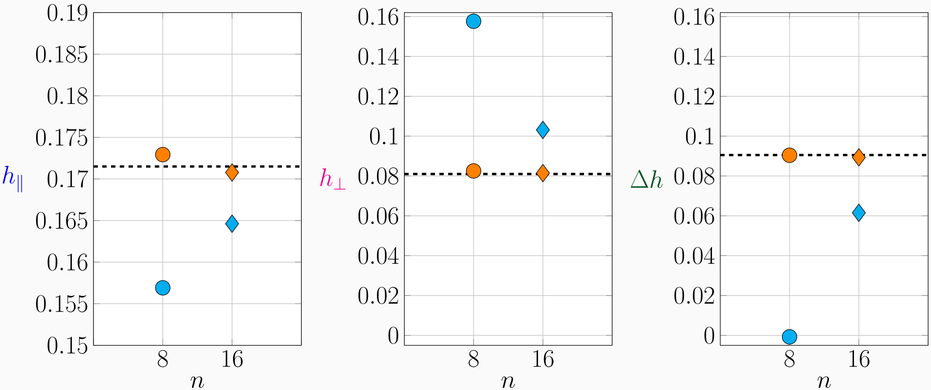
- For Stokes flows, the protrusion height $\Delta h = h_{\parallel} - h_{\perp}$ can be computed exactly
- For turbulent flows, drag reduction performance is related to Δh



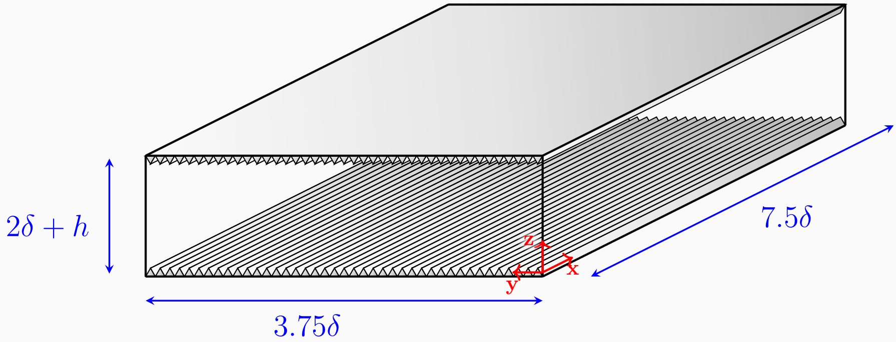
Validation: protrusion heights (2)

Protrusion heights without and with corner correction with 8 (●) and 16(◆) points per riblet (n):





Turbulent simulations: computational domain



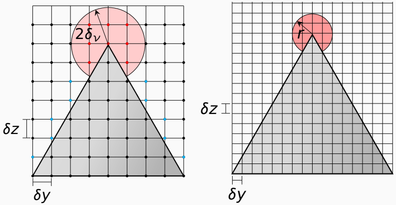
Turbulent simulations: simulation parameters

We performed DNS of turbulent channel flows with both walls covred by riblets:

- Constant Pressure Gradient (CPG) $Re_\tau = 200$
- s^+ is changed by changing the phsical riblet size s/δ

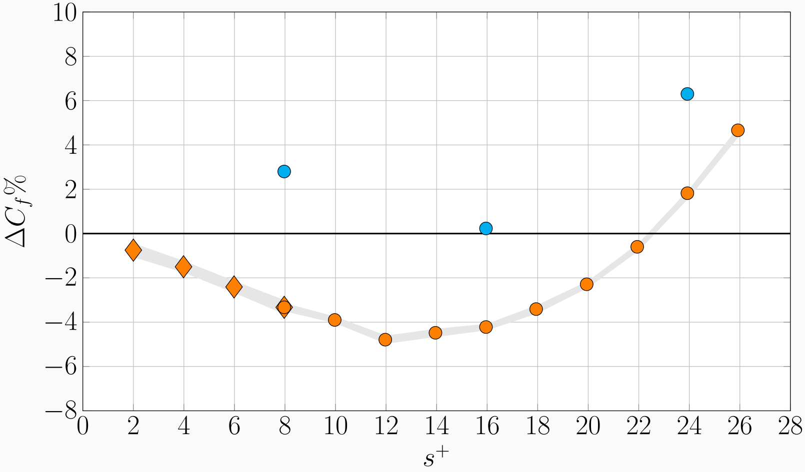
	s^+	Re_τ	L_x^+	L_y^+	δx^+	δy^+	δz^+
Grid 1	$\mathbf{8 \div 26}$	200	1500	750	2.0	$\mathbf{1.0}$	$0.8 \div 2.4$
Grid 2	$\mathbf{2 \div 8}$	200	1500	750	2.0	$\mathbf{0.5}$	$0.8 \div 2.4$

Example of riblet discretization at $s^+ = 8$:



Straight riblets: Friction Coefficient

without and with corner correction with Grid 1 (●) and Grid 2 (♦):



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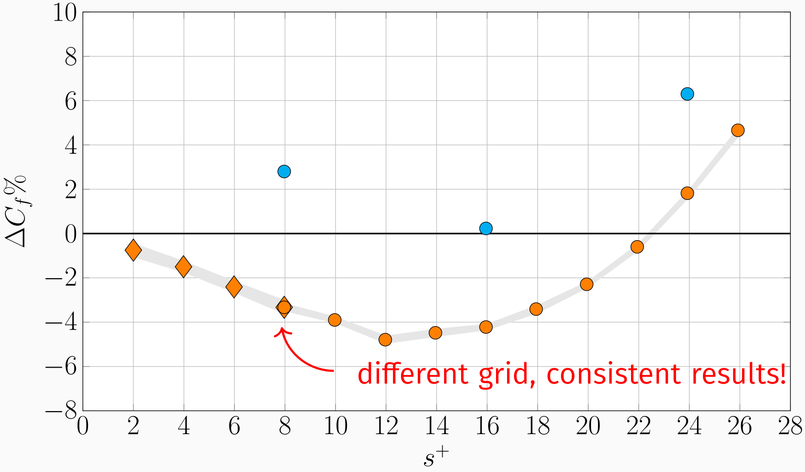

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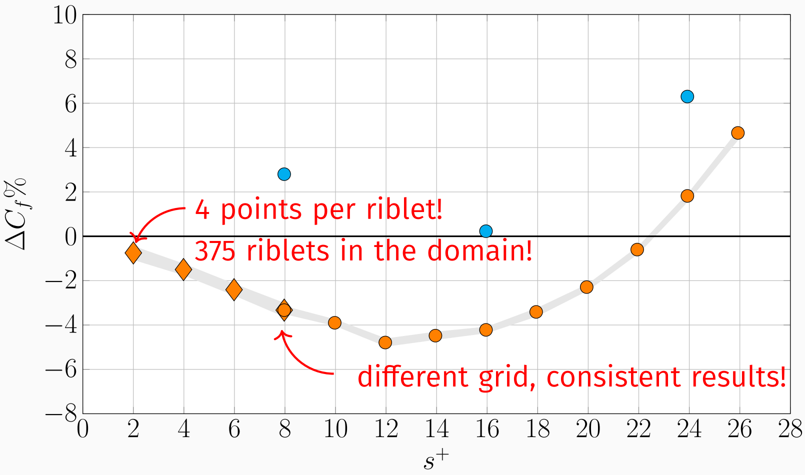
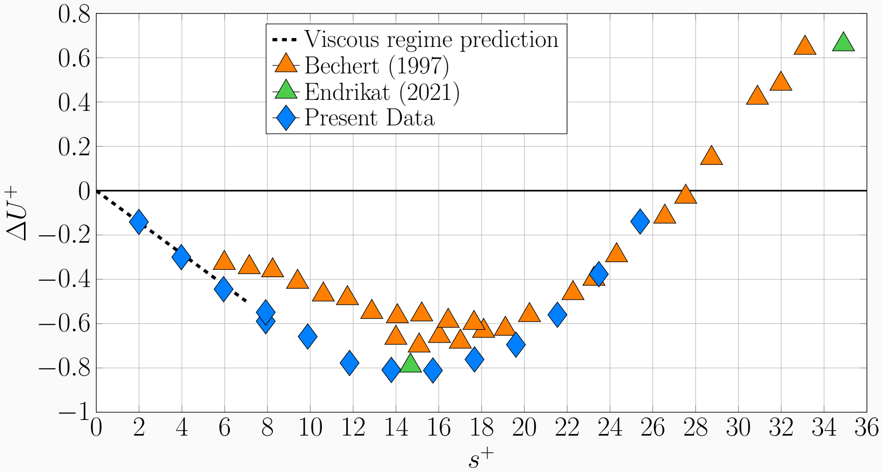
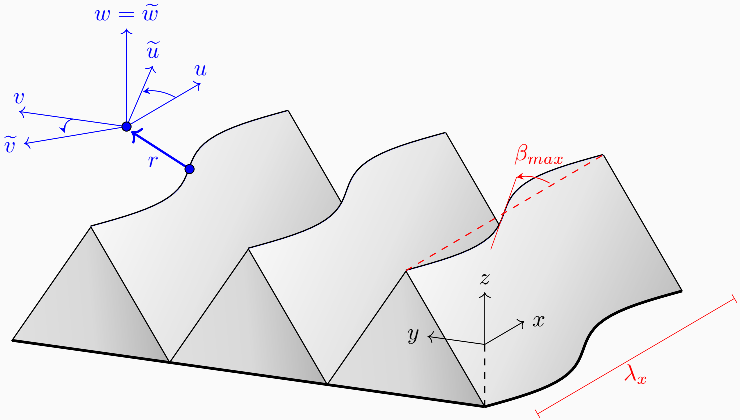


Straight riblets: ΔU^+

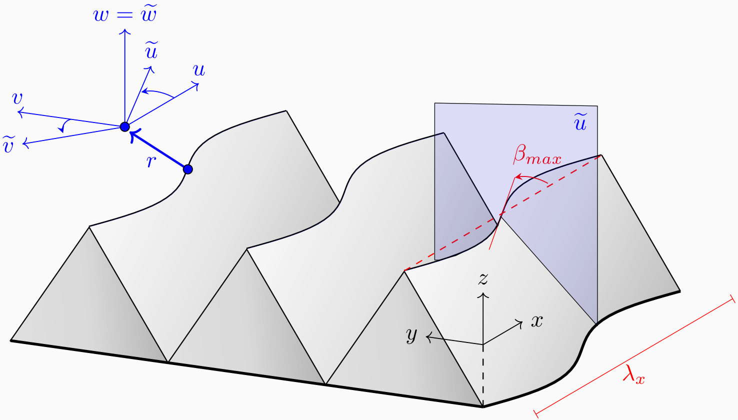
Comparison with literature data



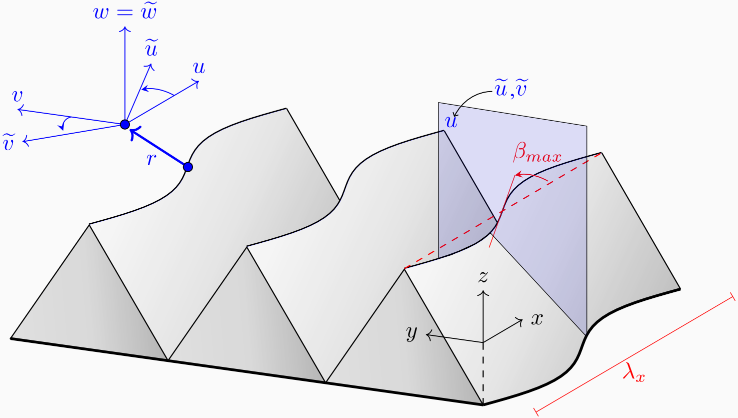


$w = \widetilde{w}$
$\widetilde{u}$
u
v
$\widetilde{v}$
r
β_{max}
z
x
y
λ_x

Sinusoidal Riblets



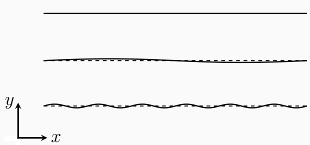
Sinusoidal Riblets

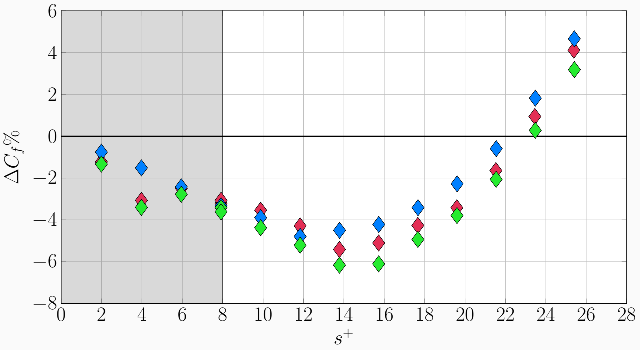


Straight

Long $(\lambda_x = 1500, \beta_{max} = 2°)$

Short $(\lambda_x = 250, \beta_{max} = 12°)$





Conclusions

An analytical correction for the tip singularity has been developed

- accurate: increased accuracy in computing Δh
- efficient: accurate results with as low as 4 points per riblet

Conclusions

An analytical correction for the tip singularity has been developed

- accurate: increased accuracy in computing Δh
- efficient: accurate results with as low as 4 points per riblet

and Outlook:

- spanwise inhomogeneous grid, still some work to do...
- we are interested in 3d riblets, but the result is general

Thank you

for your kind attention!

complaints, comments, suggestions:

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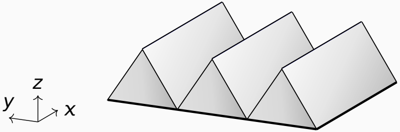
For comparison, Endrikat et al. (2021)

- at least 32 points per riblet
- resolution for riblets at $s^+ = 10$: $\delta y^+ = 0.057 \div 1.52$, $\delta z^+ = 0.0334 \div 7.02$
- domain size $2.6\delta \times 0.64\delta \times 2\delta$

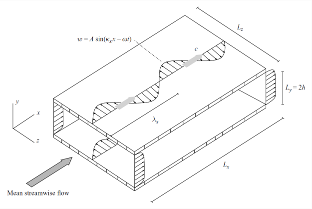
Our computational cost (finer grid)

- 91k core hours on bwUniCluster 2.0

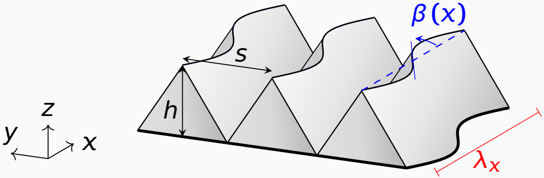
Preliminary extension to 3D sinusoidal riblets



$+$

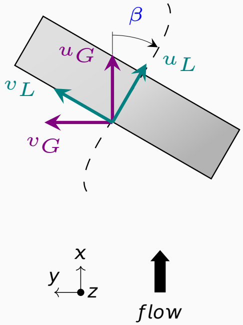


$=$

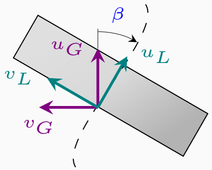


3D RIBLETS: ISSUES

- Global reference frame:
  decoupling into 1D Laplace and 2D Stokes problems fails
- Local reference frame: decoupling is possible, but velocity components are intermixed
    - discretization becomes explicit
    - discretization becomes challenging due to staggered grid



3D RIBLETS: PROVISIONAL SOLUTION



Assumption: local misalignment of the riblets section is small ($\beta(x)_{max} = 2°$, $\lambda_x^+ = 1500$)

$$\begin{Bmatrix} u_G \\ v_G \end{Bmatrix} = \begin{bmatrix} f(\beta, c_{lap}, c_{st}) & \cancel{g(\beta, c_{lap}, c_{st})} \\ \cancel{p(\beta, c_{lap}, c_{st})} & q(\beta, c_{lap}, c_{st}) \end{bmatrix} \begin{Bmatrix} u_L \\ v_L \end{Bmatrix}$$

Solution: limitation to the diagonal components of the correction matrix

3D riblets: preliminary results

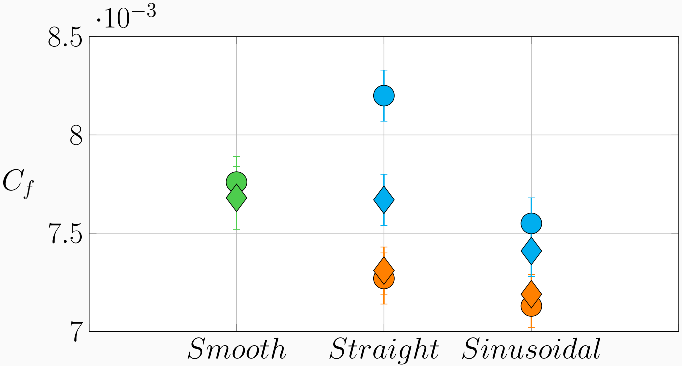
Friction coefficient for the cases

- smooth
- with riblets
  - without corner correction
  - with corner correction

with

- 8 (●)
- 16(◆)

points per riblet



Analytical Corner Correction: Stokes problem with streamfunction-vorticity formulation

$$\begin{cases} \nabla \cdot \mathbf{u} = 0 \\ \nabla^2 \mathbf{u} - \nu^{-1} \nabla p = 0 \end{cases} \implies \begin{cases} \nabla^2 \boldsymbol{\psi} = \boldsymbol{\omega} \\ \nabla^2 \boldsymbol{\omega} = 0. \end{cases}$$

The steady $\psi - \omega$ Stokes system in polar coordinates is

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r}\frac{\partial \psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \theta^2} = \omega$$
$$\frac{\partial^2 \omega}{\partial r^2} + \frac{1}{r}\frac{\partial \omega}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \omega}{\partial \theta^2} = 0.$$

Analytical Corner Correction: polar coordinates

By imposing a variable separation for $\psi(r,\theta) = \mathrm{P}(r)\mathrm{F}(\theta)$ and $\omega(r,\theta) = \mathrm{R}(r)\mathrm{G}(\theta)$, calling $\chi = \mathrm{G}''/\mathrm{G}$ and $k = -\sqrt{\chi} < 0$:

$$\begin{aligned} & r^2\mathrm{R}'' + r\mathrm{R}' - \chi\mathrm{R} = 0 \\ & \mathrm{G}'' + \chi\mathrm{G} = 0 \end{aligned} \quad \Longrightarrow \quad \mathrm{R} = ar^{-\sqrt{\chi}} + br^{\sqrt{\chi}} = ar^k$$

since $r \ll 1$, we obtain:

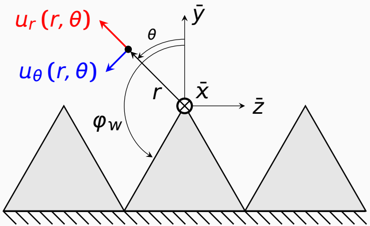
$$\omega(r,\theta) = r^k \left[C_1 \cos(k\theta) + C_2 \sin(k\theta)\right].$$

$$\psi(r,\theta) = r^{k+2}\left[D_1 \cos((k+2)\,\theta) + D_2 \sin((k+2)\,\theta) + D_3 \cos(k\theta) + D_4 \sin(k\theta)\right].$$

Analytical Corner Correction: boundary conditions

The coefficients D_i are given after the following boundary and symmetry conditions are provided:

$$
\begin{aligned}
u_r(r, \pm\varphi_w) &= 0 && \text{no penetration}\\
u_\theta(r, \pm\varphi_w) &= 0 && \text{no-slip}\\
u_r(r, \theta) &= -u_r(r, -\theta) && u_r \text{ odd in } \theta\\
u_\theta(r, \theta) &= u_\theta(r, -\theta) && u_\theta \text{ even in } \theta.
\end{aligned}
$$



The symmetry conditions lead to $D_2 = D_4 = 0$, and the definition of the stream-function gives u_r and u_θ depending on $\gamma = k + 1$ as

$$u_r(r,\theta) = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = -r^\gamma\left[D_1(\gamma+1)\sin((\gamma+1)\theta) + D_3(\gamma-1)\sin((\gamma-1)\theta)\right]$$

$$u_\theta(r,\theta) = -\frac{\partial\psi}{\partial r} = -(\gamma+1)r^\gamma\left[D_1\cos((\gamma+1)\theta) + D_3\cos((\gamma-1)\theta)\right].$$

The boundary conditions are used to find the ratio between the coefficients D_3 and D_1, that is

$$\frac{D_3}{D_1} = \frac{\cos((\gamma+1)\varphi_w)}{\cos((\gamma-1)\varphi_w)}. \tag{1}$$

We set $D_1 = 1$.

Analytical Corner Correction: boundary conditions

The last constant to find is γ, whose value is given solving numerically $det(\mathbf{Q}(\gamma)) = 0$.

$$\underbrace{\begin{bmatrix} (\gamma+1)\sin((\gamma+1)\varphi_w) & (\gamma-1)\sin((\gamma-1)\varphi_w) \\ \cos((\gamma+1)\varphi_w) & \cos((\gamma-1)\varphi_w) \end{bmatrix}}_{\mathbf{Q}(\gamma)} \begin{bmatrix} D_1 \\ D_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

The solution depends on the geometry considered: for the problem at hand, with $\varphi_w = \pi/6$, the result is $\gamma \approx 0.51222$.

The last unknown for the Stokes problem is the pressure:

$$\nu\left[\frac{\partial^2 u_r}{\partial r^2}+\frac{1}{r}\frac{\partial u_r}{\partial r}+\frac{1}{r^2}\left(\frac{\partial^2 u_r}{\partial \theta^2}-2\frac{\partial u_\theta}{\partial \theta}-u_r\right)\right]-\frac{1}{\rho}\frac{\partial p}{\partial r}=0$$
$$\nu\left[\frac{\partial^2 u_\theta}{\partial r^2}+\frac{1}{r}\frac{\partial u_\theta}{\partial r}+\frac{1}{r^2}\left(\frac{\partial^2 u_\theta}{\partial \theta^2}+2\frac{\partial u_r}{\partial \theta}-u_\theta\right)\right]-\frac{1}{\rho}\frac{1}{r}\frac{\partial p}{\partial \theta}=0$$

$$\frac{1}{\nu}p\left(r,\theta\right)=-4\gamma D_3 r^{\gamma-1}\sin\left(\left(\gamma-1\right)\theta\right).$$

The expression for p can not be used itself, because it is not guaranteed that p is symmetric and continuous inside the body. A correction can be implemented to choose a continuous branch for the solution, considering $\tilde{\theta} = \theta f(\theta)$ where $f(\theta) \neq 1$ only if $|\theta| > \varphi_w$, so that p is given by

$$\frac{1}{\nu} p\left(r, \theta\right) = -4\gamma D_3 r^{\gamma-1} \sin\left(\left(\gamma - 1\right) \theta f\left(\theta\right)\right)$$

$$f(\theta) = \begin{cases} 1 + \dfrac{|\theta| - \pi}{\pi - \varphi_w}\left(\dfrac{1}{\gamma - 1} - 1\right) & \text{if } |\theta| > \varphi_w \\ 1 & \text{otherwise.} \end{cases}$$

The Laplace problem reads:

$$\nabla^2 u = 0 \Longrightarrow \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0,$$

and a variable separation leads to the general solution

$$u(r,\theta) = r^m\big[C\cos(m\theta) + D\sin(m\theta)\big].$$

No-slip boundary conditions, namely $u(r, \pm\varphi_w) = 0$, lead to $\cos(m\varphi_w) = 0$ and so $m\varphi_w = \pi/2$. The symmetry condition, $u(r,\theta) = u(r,-\theta)$, gives $D = 0$ and the final expression for u, namely

$$u = Cr^m\cos(m\theta)\,.$$

C here is a free constant that can be set to 1 to have a unique solution.

$$u^{(t+\Delta t)} = u^{(t)} + (lapl + \mathrm{NL} + \nabla p)\,\Delta t - u^{(t+\Delta t)} imbc\Delta t \Longrightarrow u^{(t+\Delta t)} = \frac{u^{(t)} + RHS\Delta t}{1 + imbc\Delta t}$$

Being u_{loc} and p_{loc} the analytical solutions for the velocity and the pressure respectively, considering the problem for the x-direction one gets

$$d_u = \underbrace{\left(\frac{lapl\,(u_{loc}\,(x,\cdot))}{Re} - \frac{p_{loc}\,(x+\Delta x,\cdot) - p_{loc}\,(x,\cdot)}{\Delta x}\right)\frac{1}{u_{loc}\,(x,\cdot)}}_{\mathrm{corr}_{stokes}} u\,(x,\cdot)\,,$$

where $lapl()$ is the laplacian corrected with the true distance from the body. The Navier-Stokes problem here is not so different: the terms to add inside $imbc$ are a contribution from the Laplace problem in u, corr_{lapl}, and from the Stokes problem in v and w, corr_{stokes}.

Analytical Corner Correction: rotation

Considering (u', v') in the local reference frame and (u, v) in the global one, the following additional rotation should be performed:

$$u' = \cos(\beta)\, u + \sin(\beta)\, v, \qquad v' = \cos(\beta)\, v - \sin(\beta)\, u.$$

The *imbc* coefficients in the local reference frame were already found for the straight riblets as

$$d_{u'} = \mathrm{corr}_{lapl}\, u', \qquad d_{v'} = \mathrm{corr}_{stokes}\, v',$$

but to define the corrections in the cartesian global reference frame the two components get mixed into the 2×2 non-diagonal system.

Analytical Corner Correction: rotation

$$\begin{bmatrix} d_u \\ d_v \end{bmatrix} = \begin{bmatrix} \cos^2(\beta)\,\mathrm{corr}_{lapl} + \sin^2(\beta)\,\mathrm{corr}_{stokes} & \left(\mathrm{corr}_{lapl} - \mathrm{corr}_{stokes}\right)\sin(2\beta)/2 \\ \left(\mathrm{corr}_{stokes} - \mathrm{corr}_{lapl}\right)\sin(2\beta)/2 & \cos^2(\beta)\,\mathrm{corr}_{stokes} + \sin^2(\beta)\,\mathrm{corr}_{lapl} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}.$$

$$\begin{cases} d_u = \left(\cos^2(\beta)\,\mathrm{corr}_{lapl} + \sin^2(\beta)\,\mathrm{corr}_{stokes}\right) u \\ d_v = \left(\cos^2(\beta)\,\mathrm{corr}_{stokes} + \sin^2(\beta)\,\mathrm{corr}_{lapl}\right) v. \end{cases}$$

Protrusion Heights

	nppr	$h_{\parallel}$	(err%)	$h_{\perp}$	(err%)	Δh	(err%)
Standard	8	0.1537	(-10.4)	0.1254	(+54.8)	0.02831	(-68.7)
+ Correction	8	0.1683	(-1.9)	0.0811	(+0.2)	0.0872	(-3.7)
Standard	16	0.1639	(-4.4)	0.1028	(+26.9)	0.06111	(-32.5)
+ Correction	16	0.1702	(-0.7)	0.0812	(+0.3)	0.0890	(-1.7)

Table 1: Results of the validation for straight riblets with the immersed boundary correction only (Standard) and with the addition of the corner correction (+ Correction). Errors are estimated as $\left(h-\bar{h}\right)/\bar{h}$.

Protrusion Heights

$\bar{h}_{\parallel}$	$\bar{h}_{\perp}$	$\Delta\bar{h}$
0.17150	0.08099	0.09051

Table 2: Protrusion heights reference values for $h/s = \sqrt{3}/2$.

	n	U_b	$(\Delta U_b^+\%)$	$C_f \times 10^3$	$(\Delta C_f/C_{f,0}\%)$
Standard	8	15.62	(-2.7)	8.20	(+5.7)
+ Correction	8	16.58	(+3.3)	7.27	(-6.3)
Standard	16	16.14	(+0.1)	7.67	(-0.1)
+ Correction	16	16.54	(+2.6)	7.31	(-4.8)

Table 3: U_b^+ and C_f for the straight case. ΔU_b^+ and ΔC_f are evaluated considering the smooth channel simulation with the same δy^+ of the case considered.

	n	U_b	$(\Delta U_b^+\%)$	$C_f \times 10^3$	$(\Delta C_f/C_{f,0}\%)$
L Standard	8	16.28	(+1.4)	7.55	(-2.7)
L + Correction	8	16.75	(+4.4)	7.13	(-8.1)
L Standard	16	16.43	(+1.9)	7.41	(-3.5)
L + Correction	16	16.67	(+3.4)	7.19	(-6.4)

Table 4: U_b^+ and C_f for the sinusoidal cases. ΔU_b^+ and ΔC_f are evaluated considering the smooth channel simulation with the same δy^+ of the case considered.