Effect of control discretization on streamwise traveling waves of spanwise wall velocity

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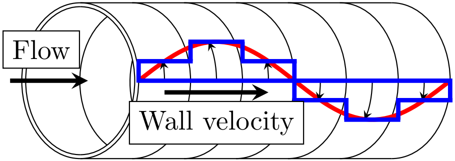
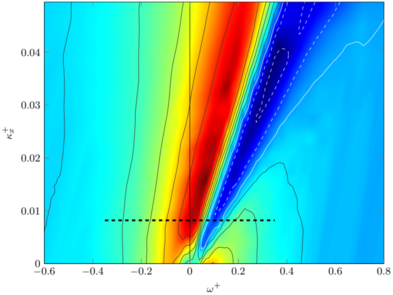


Background



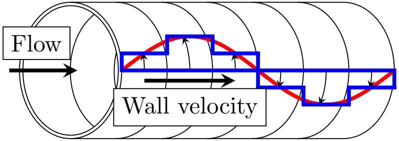
Few laboratory implementations of StTW:

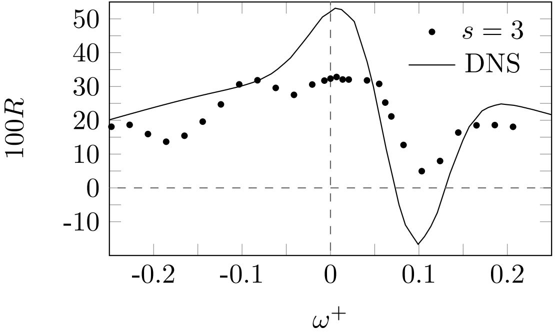
- Tensioned membrane skin (Bird et al, FTAC 2018);
- Dielectric Barrier Discharge plasma actuators (Benard et al, 2021 55^{th} 3AF ICAA);
- Moving slabs (Auteri et al, PoF 2010, Marusic et al, Nat. Comm. 2021).

Aim of the work

Experiment of Auteri et al, PoF 2010:

- Pipe flow ($Re_b = 4900$, $Re_\tau = 175$);
- Sinusoidal wave discretized with s rotating slabs, Discrete Traveling Wave (DTW);



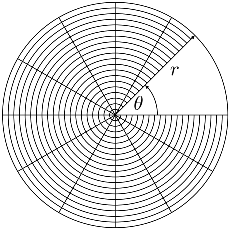


- $\mathcal{R}$ wiggles for $s = 3$;
- Maximum $\mathcal{R}$ for DNS higher than $s = 3$.

Simulations

Results of DNS (channel+StTW) and experiment (pipe+DTW) are different $\Rightarrow$ DNS for a pipe with DTW.

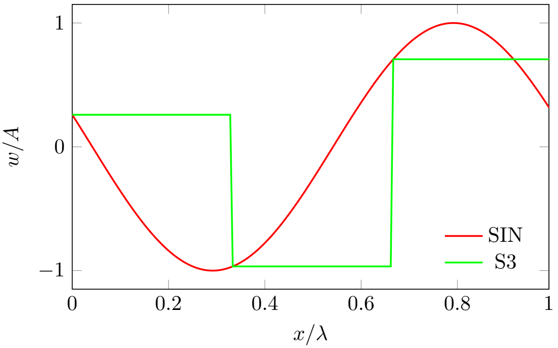
- Primitive variables in cylindrical coordinates;
- Spectral discretization in θ and x, compact FD in y;
- Implicit-explicit temporal discretization (CN for viscous term, RK for convective term).



Constant θ discretization $\Rightarrow$ center of the pipe over resolved.
Solution: (radially) varying azimuthal modes.

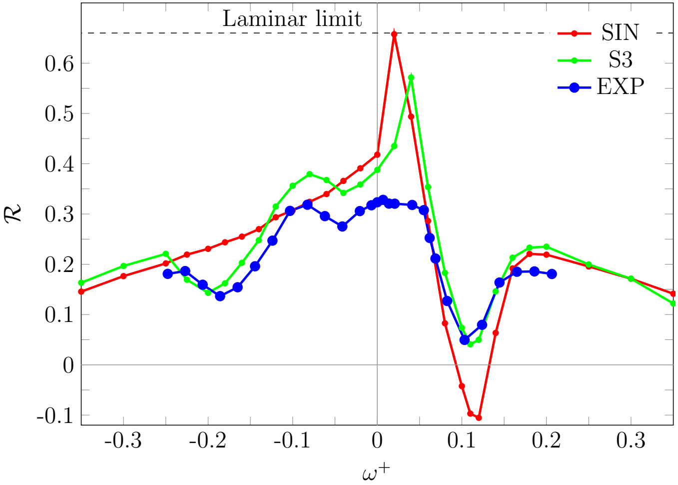
Different controls:

- StTW: $w(x,t) = A\sin(\omega t - k_x x)$ (SIN),
- DTW: with $s = 3$ (S3).



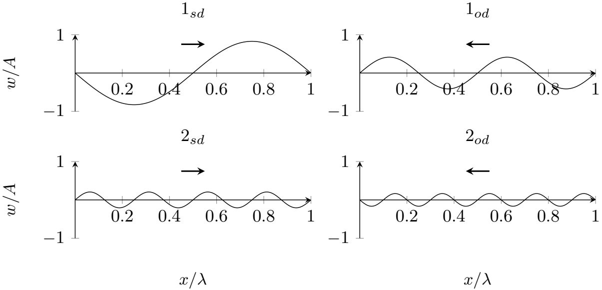
Drag reduction

- $k_x^+ = 0.0082$
- $-0.35 \leq \omega^+ \leq 0.35$



Fourier series expansion of DTW

$$w(x,t) = \frac{3\sqrt{3}}{2\pi} A \Big[\overbrace{\underbrace{\sin(\omega t - \kappa_x x)}_{1_{sd}} + \underbrace{\frac{1}{2}\sin(\omega t + 2\kappa_x x)}_{1_{od}}}^{1} \overbrace{\underbrace{- \frac{1}{4}\sin(\omega t - 4\kappa_x x)}_{2_{sd}} \underbrace{- \frac{1}{5}\sin(\omega t + 5\kappa_x x)}_{2_{od}}}^{2} \Big] \ldots$$



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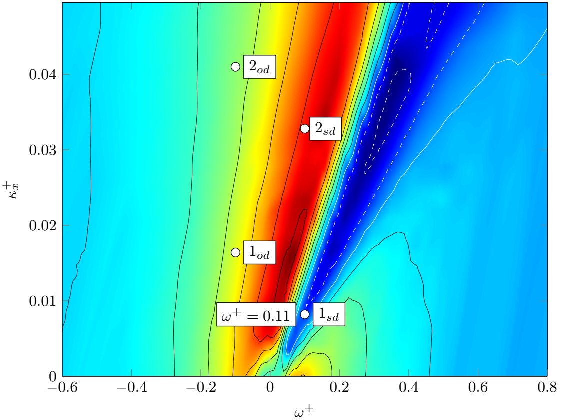

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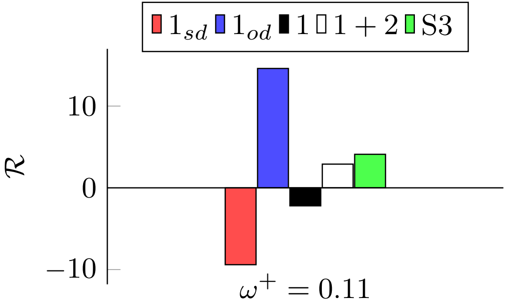
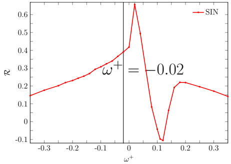
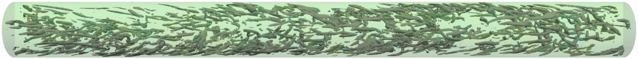


Localized turbulence

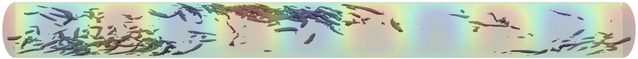
- For high DR ($\omega^+ = -0.02$) $\Rightarrow$ Localized turbulence;
- Vortices highlighted with $\lambda_2^+ = -0.022$.
- Could explain high DR peak.



Ref



High DR: $\omega^+ = -0.02$



Conclusions

- DNS to replicate (and expand) the experiment by Auteri et al, PoF 2010;
- Differences between DTW and SIN, confirmed by numerical data;
- Discretization $\Rightarrow$ Wiggles of $\mathcal{R}$;
- Localized turbulence $\Rightarrow$ High $\mathcal{R}$ peak for simulations.

The discretization of the control affects the results in terms of $\mathcal{R}$ and $\mathcal{S}$. It must be accounted when experiments are performed.

Thank you for your attention!

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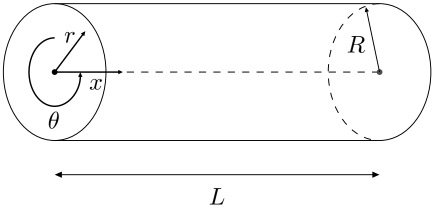
Computational details

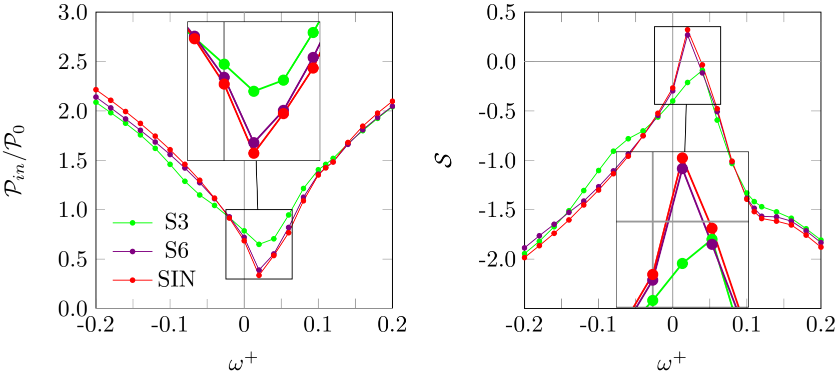


- $Re_b = 4900$
- $L = 22R$
- $N_x \times N_{\theta,max} \times N_y = 384 \times 192 \times 100$

Power budget

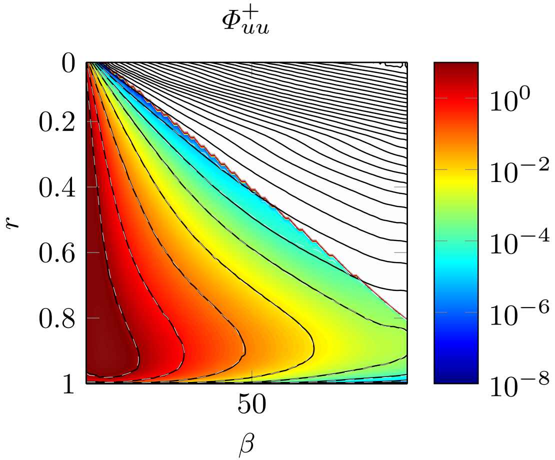
$$\mathcal{P}_{in} = \frac{1}{2\pi RLT} \int_0^T \int_0^{2\pi} \int_0^L \mu w \left(\partial w / \partial r - w/R \right) \Big|_{r=R} R \, dx \, d\theta \, dt.$$

$$\mathcal{S} = \mathcal{R} - \frac{\mathcal{P}_{in}}{\mathcal{P}_0}$$



Numerical method: variable modes

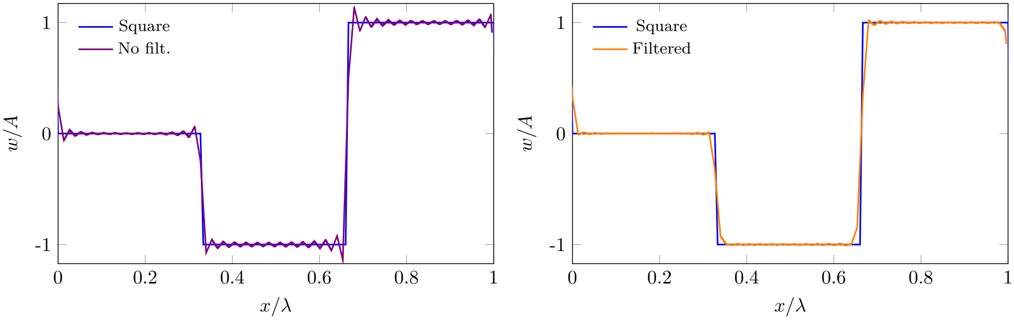
Constant azimuthal discretization ⇒ center of the pipe over resolved.
Solution: (radially) varying azimuthal modes.



Numerical method: Gibbs phenomenon

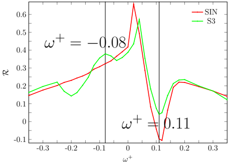
Discontinuous jump & Fourier transforms ⇒ Gibbs phenomenon.
Solution: filtering of the control wave.
Gaussian filter:

$$G(x) = \left(\frac{6}{\pi\Delta^2}\right)^{1/2} \exp\left(-\frac{6x^2}{\Delta^2}\right)$$

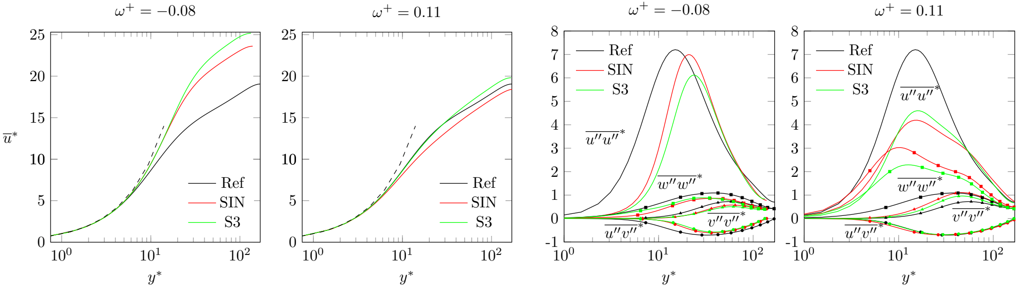


Statistics

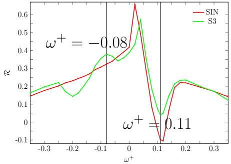
- Two frequencies, $\omega^+ = -0.08$ (DR) and $\omega^+ = 0.11$ (DI for SIN, DR for S3) for Ref, SIN and S3.
- Quantities are scaled using the actual Re_τ



$\overline{u}^*$ and variances:

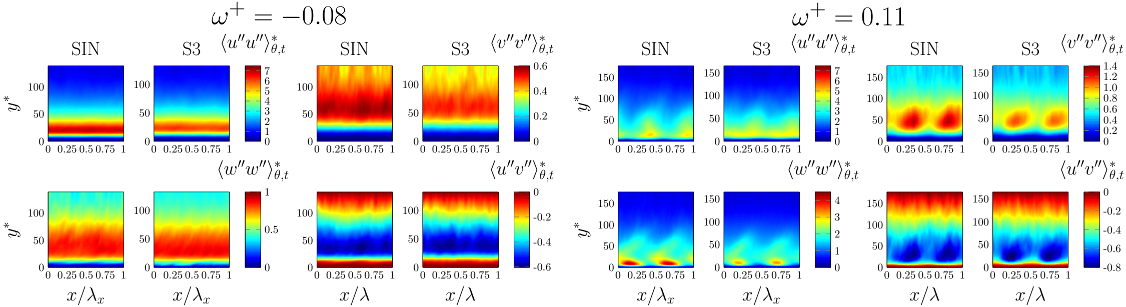


Statistics



- Two frequencies, $\omega^+ = -0.08$ and $\omega^+ = 0.11$ for SIN and S3.
- Quantities are scaled using the actual Re_τ

Average over time and azimuthal direction (θ):



Fourier series expansion of DTW

For S3:

$$W(x,t;3)=\frac{3\sqrt{3}}{2\pi}A\Big[\sin(\omega t-\kappa_x x)+\frac{1}{2}\sin(\omega t+2\kappa_x x)-\frac{1}{4}\sin(\omega t-4\kappa_x x)-\frac{1}{5}\sin(\omega t+5\kappa_x x)\Big],$$

for S6

$$W(x,t;6)=\frac{3}{\pi}A\Big[\sin(\omega t-\kappa_x x)+\frac{1}{5}\sin(\omega t+5\kappa_x x)-\frac{1}{7}\sin(\omega t-7\kappa_x x)-\frac{1}{11}\sin(\omega t+11\kappa_x x)\Big].$$

Case	S3	m0f	m0	m0+m1	S6	m0f	m0	m0+m1
$\omega^+=0.11$	4.1%	−9.4%	−2.2%	2.9%	−6.0%	−9.4%	−8.1%	−6.6%
$\omega^+=-0.08$	38.1%	30.5%	36.4%	37.5%	31.1%	31.2%	31.2%	31.8%
$\omega^+=-0.2$	14.3%	20.3%	7.5%	11.5%	25.4%	22.1%	25.3%	26.3%

Fourier series expansion of DTW: Power budget

Case	SIN	S3	m0f	m0	m0+m1	S6	m0f	m0	m0+m1
$\omega^+ = 0.11$	1.42	1.46	0.97	1.29	1.38	1.42	1.29	1.37	1.42
$\omega^+ = -0.08$	1.42	1.28	1.00	1.12	1.21	1.42	1.33	1.36	1.41
$\omega^+ = -0.2$	2.21	2.08	1.51	1.86	1.99	2.14	2.01	2.07	2.13