

SCALE-SPACE BUDGET EQUATIONS FOR INHOMOGENEOUS (QUASI-)PERIODIC TURBULENT FLOW

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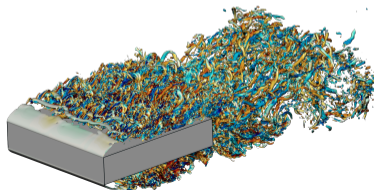
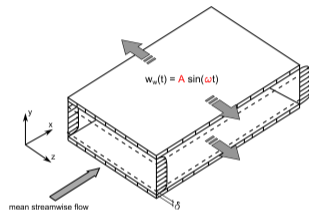
EFMC14 - 14th European Fluid Mechanics Conference

Dipartimento di Scienze e Tecnologie Aerospaziali, Politecnico di Milano

AIM OF THE WORK

We are interested in studying **turbulent flows** with **periodic features**:

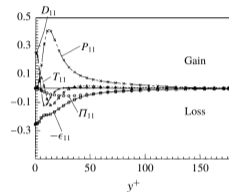
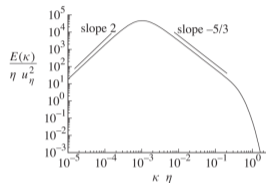
- flows subjected to an external periodic forcing
- flows which exhibit quasi-periodic features



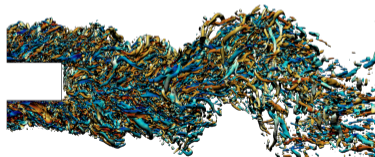
FEATURES OF THE NEW TOOL

Development of a new statistical tool to study:

- the **compound** space of scales and positions

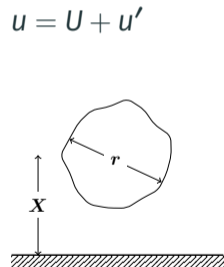
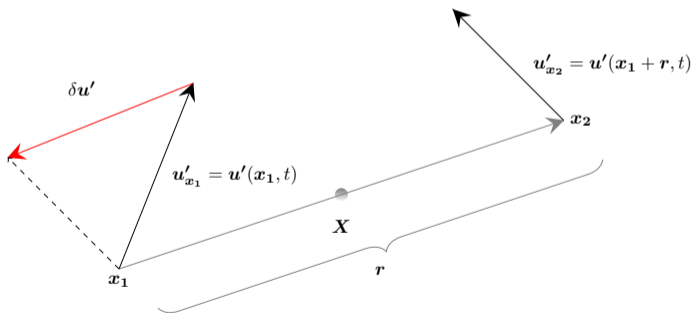


- the **interaction** between the periodic and stochastic flow fields



SECOND-ORDER STRUCTURE FUNCTION

$$\langle \delta u'_i \delta u'_j \rangle = \left\langle \left[u'_i \left(\mathbf{X} + \frac{\mathbf{r}}{2}, t \right) - u'_i \left(\mathbf{X} - \frac{\mathbf{r}}{2}, t \right) \right] \left[u'_j \left(\mathbf{X} + \frac{\mathbf{r}}{2}, t \right) - u'_j \left(\mathbf{X} - \frac{\mathbf{r}}{2}, t \right) \right] \right\rangle = \begin{bmatrix} \langle \delta u' \delta u' \rangle & \langle \delta u' \delta v' \rangle & \langle \delta u' \delta w' \rangle \\ \langle \delta u' \delta v' \rangle & \langle \delta v' \delta v' \rangle & \langle \delta v' \delta w' \rangle \\ \langle \delta u' \delta w' \rangle & \langle \delta v' \delta w' \rangle & \langle \delta w' \delta w' \rangle \end{bmatrix}$$



Turbulent stresses at the physical position \mathbf{X} and up to the scale r .

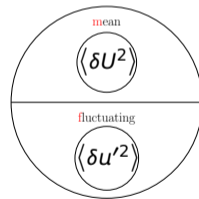
GKE: GENERALISED KOLMOGOROV EQUATION

(Kolmogorov, JFM 1941; Hill, JFM 2001)

$$\langle \delta u_i'^2 \rangle = \text{tr} \begin{bmatrix} \langle \delta u' \delta u' \rangle & \langle \delta u' \delta v' \rangle & \langle \delta u' \delta w' \rangle \\ \langle \delta u' \delta v' \rangle & \langle \delta v' \delta v' \rangle & \langle \delta v' \delta w' \rangle \\ \langle \delta u' \delta w' \rangle & \langle \delta v' \delta w' \rangle & \langle \delta w' \delta w' \rangle \end{bmatrix}$$

$$\text{1eq: } \frac{\partial \Phi_k^f}{\partial r_k} + \frac{\partial \Psi_k^f}{\partial \chi_k} = P^f + D^f \quad k = 1, 2, 3$$

- Budget equation for the **trace** of $\langle \delta u_i' \delta u_i' \rangle$: the scale energy $\langle \delta u'^2 \rangle$.
- Production, transport and dissipation of the kinetic energy in the compound space of scales and positions.



$$u = U + u'$$

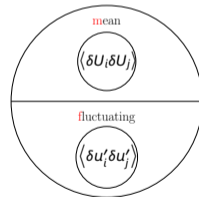
AGKE: ANISOTROPIC GENERALISED KOLMOGOROV EQUATIONS

(Gatti et al., JFM 2020)

$$\langle \delta u'_i \delta u'_j \rangle = \begin{bmatrix} \langle \delta u' \delta u' \rangle & \langle \delta u' \delta v' \rangle & \langle \delta u' \delta w' \rangle \\ \langle \delta u' \delta v' \rangle & \langle \delta v' \delta v' \rangle & \langle \delta v' \delta w' \rangle \\ \langle \delta u' \delta w' \rangle & \langle \delta v' \delta w' \rangle & \langle \delta w' \delta w' \rangle \end{bmatrix}$$

$$6 \text{ eqs: } \frac{\partial \Phi_{ij,k}^f}{\partial r_k} + \frac{\partial \Psi_{ij,k}^f}{\partial X_k} = P_{ij}^f + \Pi_{ij}^f + D_{ij}^f \quad k = 1, 2, 3$$

- Budget equations for **each component** of $\langle \delta u'_i \delta u'_j \rangle$.
- Production, transport, **redistribution** and dissipation of the turbulent stresses in the compound space of scales and positions.

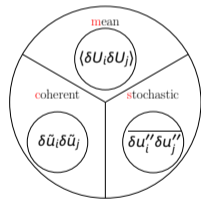


$$u = U + u'$$

ϕ AGKE: PHASE-AWARE ANISOTROPIC GENERALISED KOLMOGOROV EQUATIONS

$$6 \cdot n_{ph} \text{ eqs: } \frac{2\pi}{T} \frac{\partial \delta \tilde{u}_i \delta \tilde{u}_j}{\partial \phi} + \frac{\partial \phi_{k,ij}^c}{\partial r_k} + \frac{\partial \psi_{k,ij}^c}{\partial X_k} = p_{ij}^{mc} - p_{ij}^{cs} + \pi_{ij}^c + d_{ij}^c + \zeta_{ij}^c$$

$$6 \cdot n_{ph} \text{ eqs: } \frac{2\pi}{T} \frac{\partial \overline{\delta u_i'' \delta u_j''}}{\partial \phi} + \frac{\partial \phi_{k,ij}^s}{\partial r_k} + \frac{\partial \psi_{k,ij}^s}{\partial X_k} = p_{ij}^{ms} + p_{ij}^{cs} + \pi_{ij}^s + d_{ij}^s$$

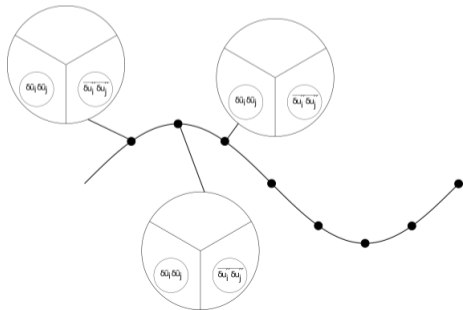


- Budget equations for each component of $\overline{\delta \tilde{u}_i \delta \tilde{u}_j}$, $\overline{\delta u_i'' \delta u_j''}$.
- Production, transport, redistribution, dissipation and **inter-phase interaction** of the turbulent stresses in the compound space of scales and positions.

$$u = U + \underbrace{\tilde{u} + u''}_{u'}$$

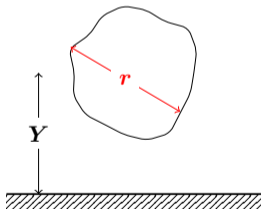
ϕ AGKE: EVOLUTION ACROSS PHASES

$$\frac{2\pi}{T} \frac{\partial \delta \tilde{u}_i \delta \tilde{u}_j}{\partial \phi} + \frac{\partial \phi_{k,ij}^c}{\partial r_k} + \frac{\partial \psi_{k,ij}^c}{\partial X_k} = p_{ij}^{mc} - p_{ij}^{cs} + \pi_{ij}^c + d_{ij}^c + \zeta_{ij}^c$$
$$\frac{2\pi}{T} \frac{\partial \delta u_i'' \delta u_j''}{\partial \phi} + \frac{\partial \phi_{k,ij}^s}{\partial r_k} + \frac{\partial \psi_{k,ij}^s}{\partial X_k} = p_{ij}^{ms} + p_{ij}^{cs} + \pi_{ij}^s + d_{ij}^s$$



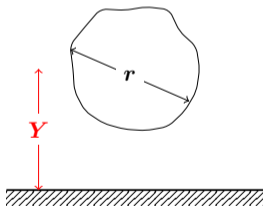
ϕ AGKE: TRANSPORT AMONG SCALES

$$\frac{2\pi}{T} \frac{\partial \delta \tilde{u}_i \delta \tilde{u}_j}{\partial \phi} + \frac{\partial \phi_{k,ij}^c}{\partial r_k} + \frac{\partial \psi_{k,ij}^c}{\partial X_k} = p_{ij}^{mc} - p_{ij}^{cs} + \pi_{ij}^c + d_{ij}^c + \zeta_{ij}^c$$
$$\frac{2\pi}{T} \frac{\partial \delta u_i'' \delta u_j''}{\partial \phi} + \frac{\partial \phi_{k,ij}^s}{\partial r_k} + \frac{\partial \psi_{k,ij}^s}{\partial X_k} = p_{ij}^{ms} + p_{ij}^{cs} + \pi_{ij}^s + d_{ij}^s$$



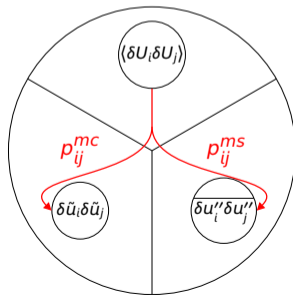
ϕ AGKE: TRANSPORT IN THE PHYSICAL SPACE

$$\frac{2\pi}{T} \frac{\partial \delta \tilde{u}_i \delta \tilde{u}_j}{\partial \phi} + \frac{\partial \phi_{k,ij}^c}{\partial r_k} + \frac{\partial \psi_{k,ij}^c}{\partial X_k} = p_{ij}^{mc} - p_{ij}^{cs} + \pi_{ij}^c + d_{ij}^c + \zeta_{ij}^c$$
$$\frac{2\pi}{T} \frac{\partial \delta u_i'' \delta u_j''}{\partial \phi} + \frac{\partial \phi_{k,ij}^s}{\partial r_k} + \frac{\partial \psi_{k,ij}^s}{\partial X_k} = p_{ij}^{ms} + p_{ij}^{cs} + \pi_{ij}^s + d_{ij}^s$$



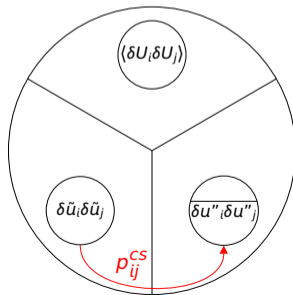
ϕ AGKE: MEAN PRODUCTION

$$\frac{2\pi}{T} \frac{\partial \delta \tilde{u}_i \delta \tilde{u}_j}{\partial \varphi} + \frac{\partial \phi_{k,ij}^c}{\partial r_k} + \frac{\partial \psi_{k,ij}^c}{\partial X_k} = p_{ij}^{mc} - p_{ij}^{cs} + \pi_{ij}^c + d_{ij}^c + \zeta_{ij}^c$$
$$\frac{2\pi}{T} \frac{\partial \overline{\delta u_i'' \delta u_j''}}{\partial \varphi} + \frac{\partial \phi_{k,ij}^s}{\partial r_k} + \frac{\partial \psi_{k,ij}^s}{\partial X_k} = p_{ij}^{ms} + p_{ij}^{cs} + \pi_{ij}^s + d_{ij}^s$$



ϕ AGKE: COHERENT-STOCHASTIC PRODUCTION

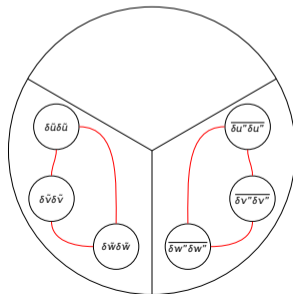
$$\frac{2\pi}{T} \frac{\partial \delta \tilde{u}_i \delta \tilde{u}_j}{\partial \phi} + \frac{\partial \phi_{k,ij}^c}{\partial r_k} + \frac{\partial \psi_{k,ij}^c}{\partial X_k} = p_{ij}^{mc} - p_{ij}^{cs} + \pi_{ij}^c + d_{ij}^c + \zeta_{ij}^c$$
$$\frac{2\pi}{T} \frac{\partial \overline{\delta u''_i \delta u''_j}}{\partial \phi} + \frac{\partial \phi_{k,ij}^s}{\partial r_k} + \frac{\partial \psi_{k,ij}^s}{\partial X_k} = p_{ij}^{ms} + p_{ij}^{cs} + \pi_{ij}^s + d_{ij}^s$$



φAGKE: PRESSURE STRAIN

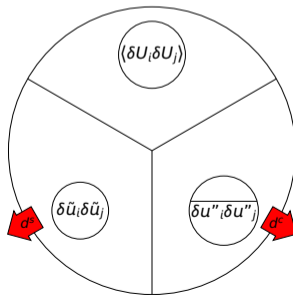
$$\frac{2\pi}{T} \frac{\partial \delta \tilde{u}_i \delta \tilde{u}_j}{\partial \varphi} + \frac{\partial \phi_{k,ij}^c}{\partial r_k} + \frac{\partial \psi_{k,ij}^c}{\partial X_k} = p_{ij}^{mc} - p_{ij}^{cs} + \pi_{ij}^c + d_{ij}^c + \zeta_{ij}^c$$

$$\frac{2\pi}{T} \frac{\partial \delta u_i'' \delta u_j''}{\partial \varphi} + \frac{\partial \phi_{k,ij}^s}{\partial r_k} + \frac{\partial \psi_{k,ij}^s}{\partial X_k} = p_{ij}^{ms} + p_{ij}^{cs} + \pi_{ij}^s + d_{ij}^s$$



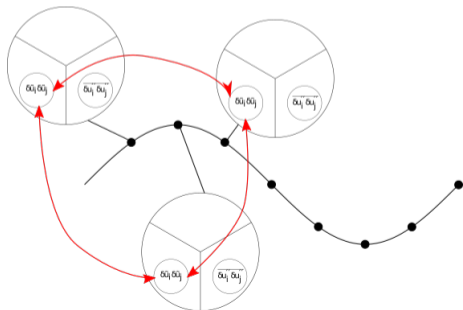
$$\frac{2\pi}{T} \frac{\partial \delta \tilde{u}_i \delta \tilde{u}_j}{\partial \varphi} + \frac{\partial \phi_{k,ij}^c}{\partial r_k} + \frac{\partial \psi_{k,ij}^c}{\partial X_k} = p_{ij}^{mc} - p_{ij}^{cs} + \pi_{ij}^c + d_{ij}^c + \zeta_{ij}^c$$

$$\frac{2\pi}{T} \frac{\partial \delta u_i'' \delta u_j''}{\partial \varphi} + \frac{\partial \phi_{k,ij}^s}{\partial r_k} + \frac{\partial \psi_{k,ij}^s}{\partial X_k} = p_{ij}^{ms} + p_{ij}^{cs} + \pi_{ij}^s + d_{ij}^s$$



ϕ AGKE: INTERACTION ACROSS PHASES

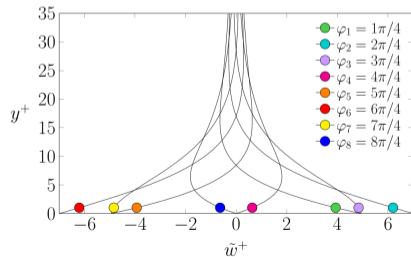
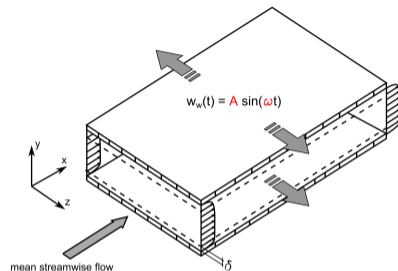
$$\frac{2\pi}{T} \frac{\partial \delta \tilde{u}_i \delta \tilde{u}_j}{\partial \phi} + \frac{\partial \phi_{k,ij}^c}{\partial r_k} + \frac{\partial \psi_{k,ij}^c}{\partial X_k} = p_{ij}^{mc} - p_{ij}^{cs} + \pi_{ij}^c + d_{ij}^c + \zeta_{ij}^c$$
$$\frac{2\pi}{T} \frac{\partial \delta u_i'' \delta u_j''}{\partial \phi} + \frac{\partial \phi_{k,ij}^s}{\partial r_k} + \frac{\partial \psi_{k,ij}^s}{\partial X_k} = p_{ij}^{ms} + p_{ij}^{cs} + \pi_{ij}^s + d_{ij}^s$$



APPLICATION: THE SPANWISE WALL OSCILLATING CHANNEL FLOW

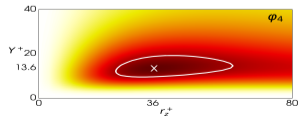
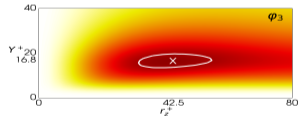
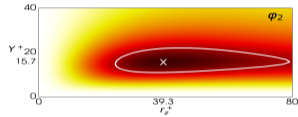
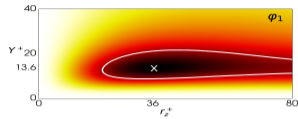
$$w_w(t) = A \sin\left(\frac{2\pi}{T}t\right)$$

- external periodic forcing
- independent variables: $(r_x, r_y, r_z, Y, \varphi)$
- $\mathbf{U}(y) = (U(y), 0, 0)$
- $\tilde{v}(y, t) = 0$

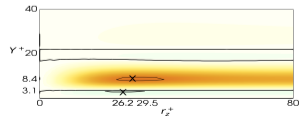
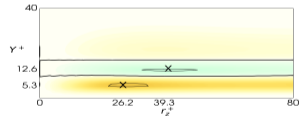
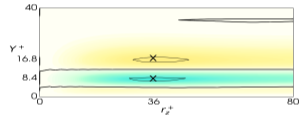
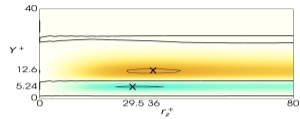


RESULTS: THE PRODUCTION TERMS

$$P_{11}^{ms} = -2\overline{\delta u'' \delta v''} \left(\frac{dU}{dy} \right)$$



$$P_{33}^{cs} = -2\overline{\delta v'' \delta w''} \left(\frac{\partial \bar{w}}{\partial y} \right)$$



We derived **phase-aware φ AGKE** exploiting a **triple decomposition** to study periodic and **quasi-periodic** turbulent flows.

Compared to the classical AGKE, φ AGKE add new features:

- the **scale-space** energy exchange among **mean, coherent, and stochastic** fields
- the mutual **interaction** of the coherent motions at **different phases**
- **no average over phases**