

TOWARDS RELIABLE AND COST-EFFECTIVE DNS OVER RIBBLETS

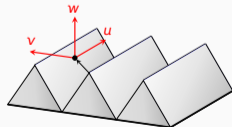
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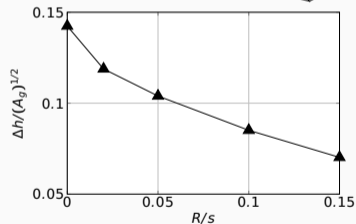
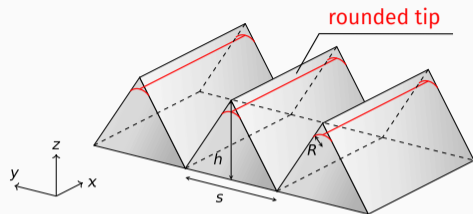
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DR PERFORMANCE: IMPORTANCE OF THE SHARP CORNER

The drag reduction performance of the riblets depends on the **sharpness of their tip**.

Consequences for DNS:
An extremely fine grid is required near the tip.



Adapted from Garcia-Mayoral & Jimenez, Phil. Trans. R. Soc. A (2011)

ANALYTICAL CORRECTION OF THE CORNER SINGULARITY

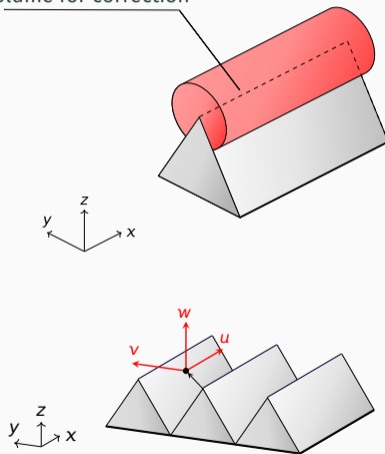
Luchini, "Higher-order difference approximations of the Navier-Stokes equations", J. Comput. Phys. (1991)

Stokes Problem:

$$\cancel{\frac{\partial \mathbf{u}}{\partial t}} + (\mathbf{u} \cdot \nabla) \mathbf{u} - \frac{1}{\rho} \nabla P = \nu \nabla^2 \mathbf{u}$$
$$\nabla \cdot \mathbf{u} = 0$$

↓
Analytical solution

volume for correction



Two uncoupled problems

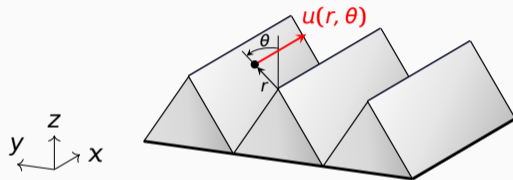
ANALYTICAL CORRECTION OF THE CORNER SINGULARITY

- || to the edge

$$0 = \nu \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$$



1D Laplace problem



$u(r, \theta)$

ANALYTICAL CORRECTION OF THE CORNER SINGULARITY

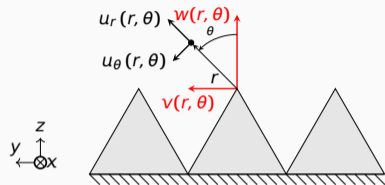
- \perp to the edge

$$\frac{\partial P}{\partial y} = \nu \left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right)$$

$$\frac{\partial P}{\partial z} = \nu \left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right)$$



2D **Stokes** problem



$$v(r, \theta) = u_r \sin(\theta) + u_\theta \cos(\theta)$$

$$w(r, \theta) = u_r \cos(\theta) - u_\theta \sin(\theta)$$

$$p(r, \theta)$$

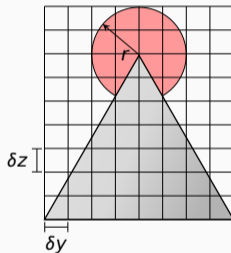
ANALYTICAL SOLUTION: PROCEDURE

- Switch from cartesian to polar coordinates
- Assume variables separation
- Impose the boundary conditions
- Choose in the spectrum of exact solutions the dominant one, uniquely identified by the requirement that it must reduce with continuity to a linear velocity when the surface is flat

ANALYTICAL SOLUTION: IMPLEMENTATION INTO A DNS SOLVER

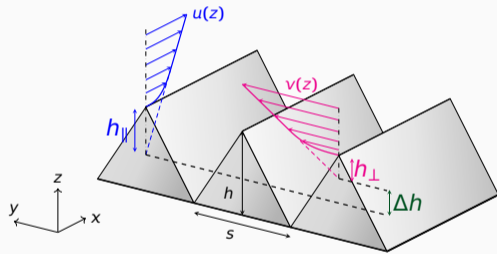
Integration of the analytical corner correction with a IBM solver (Luchini, Eur. J. Mech. B Fluids (2016))

- correction imposed to $\nu \nabla^2 \mathbf{u}$ and ∇P
- correction imposed implicitly
- $r = 2\delta y$



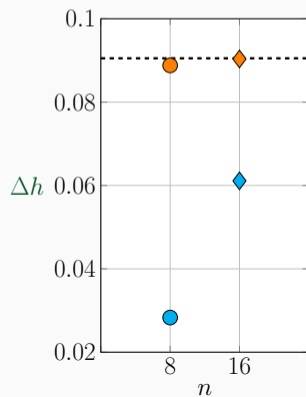
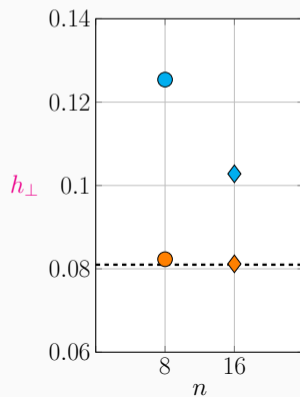
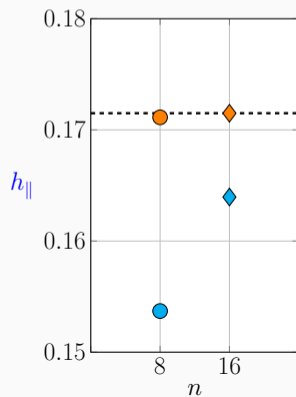
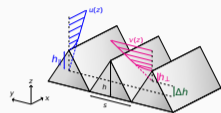
VALIDATION: PROTRUSION HEIGHTS

- For laminar flows, the protrusion height $\Delta h = h_{\parallel} - h_{\perp}$ can be computed exactly
- For turbulence flows, drag reduction performance is proportional to Δh



VALIDATION: PROTRUSION HEIGHTS

Protrusion heights **without** and **with** corner correction with 8 (●) and 16 (◆) points per riblet (n):

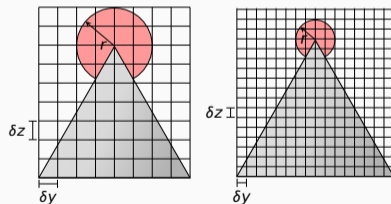


SIMULATION PARAMETERS

We performed two sets of DNS of a turbulent half channel flow with the wall covered by riblets at CPG by an IBM code written in CPL language.

Re_τ	L_x^+	L_y^+	δx^+	n	δy^+	$\delta z_{max}/\delta z_{min}$	h/s	s^+	r
200	1500	416	6.3	16 (8)	1 (2)	1.3	$\sqrt{3}/2$	16	2

	n_x	n_y	n_z
$n = 8$	240	208	94
$n = 16$	240	416	186



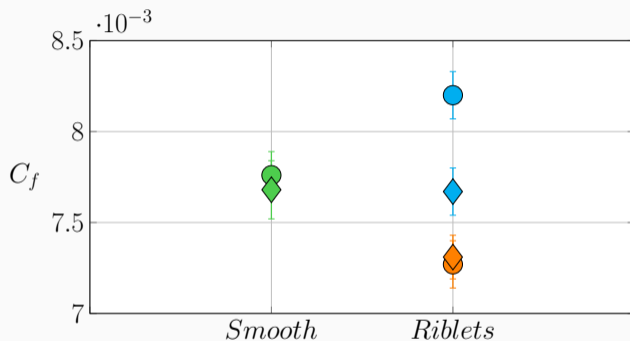
Experimental result: $DR = 5\%$ (Bechert et al., J. Fluid Mech. (1997))

TURBULENT RESULTS: DRAG REDUCTION PERFORMANCES

Friction coefficient for the cases

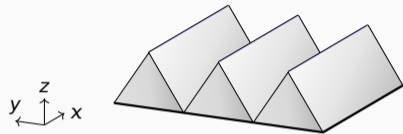
- smooth
- with riblets
 - without corner correction
 - with corner correction

with 8 (●) and 16 (◆) points per riblet (n).

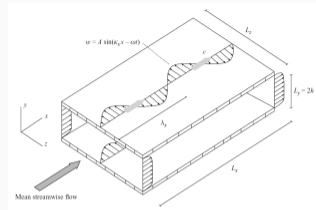


DNS with analytical correction: $DR = 4.8\%$

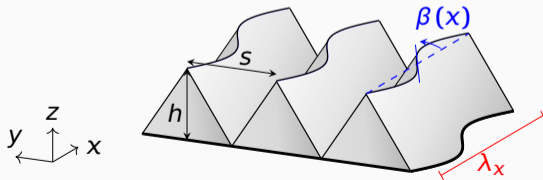
PRELIMINARY EXTENSION TO 3D SINUSOIDAL RIBLETS



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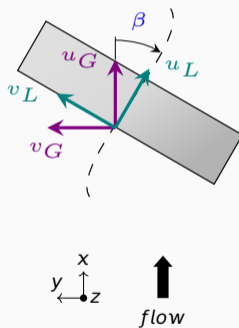


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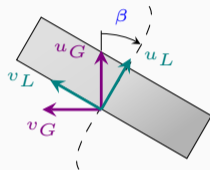


3D RIBLETS: ISSUES

- **Global** reference frame: decoupling into 1D Laplace and 2D Stokes problems fails
- **Local** reference frame: decoupling is possible, but velocity components are intermixed
 - discretization becomes explicit
 - discretization becomes challenging due to staggered grid



3D RIBLETS: PROVISIONAL SOLUTION



Assumption: local misalignment of the riblets section is small

$$(\beta(x))_{max} = 2^\circ, \lambda_x^+ = 1500)$$

$$\begin{Bmatrix} u_G \\ v_G \end{Bmatrix} = \begin{bmatrix} f(\beta, c_{lap}, c_{st}) & \cancel{g(\beta, c_{tap}, c_{st})} \\ \cancel{p(\beta, c_{tap}, c_{st})} & q(\beta, c_{lap}, c_{st}) \end{bmatrix} \begin{Bmatrix} u_L \\ v_L \end{Bmatrix}$$

Solution: limitation to the diagonal components of the correction matrix

3D RIBLETS: PRELIMINARY RESULTS

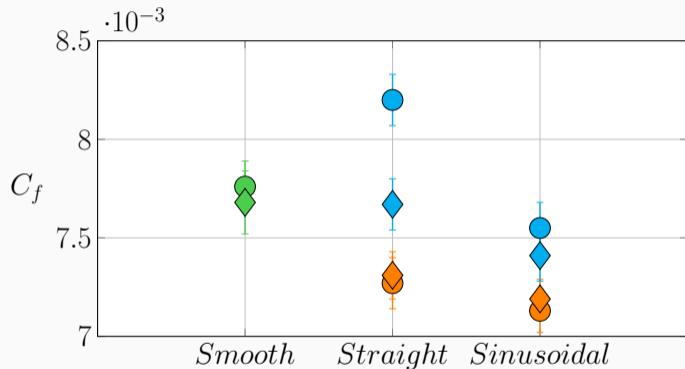
Friction coefficient for
the cases

- smooth
- with riblets
 - without corner correction
 - with corner correction

with

- 8 (●)
- 16 (◆)

points per riblet



An analytical correction for the corner singularity was applied to the turbulent flow over riblets

- **reliable**: increased accuracy in computing Δh
- **effective**: much fewer points per riblets are needed for a given accuracy

obtaining

- **DR of 4.8%** for the reference configuration
- **DR of +30%** for sinusoidal riblets compared to the reference configuration.

ANALYTICAL CORNER CORRECTION: STOKES PROBLEM WITH STREAMFUNCTION-VORTICITY FORMULATION

$$\begin{cases} \nabla \cdot \mathbf{u} = 0 \\ \nabla^2 \mathbf{u} - \nu^{-1} \nabla p = 0 \end{cases} \implies \begin{cases} \nabla^2 \psi = \omega \\ \nabla^2 \omega = 0. \end{cases}$$

The steady $\psi - \omega$ Stokes system in polar coordinates is

$$\begin{aligned} \frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r} \frac{\partial \psi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \psi}{\partial \theta^2} &= \omega \\ \frac{\partial^2 \omega}{\partial r^2} + \frac{1}{r} \frac{\partial \omega}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \omega}{\partial \theta^2} &= 0. \end{aligned}$$

ANALYTICAL CORNER CORRECTION: POLAR COORDINATES

By imposing a variable separation for $\psi(r, \theta) = P(r)F(\theta)$ and $\omega(r, \theta) = R(r)G(\theta)$, calling $\chi = G''/G$ and $k = -\sqrt{\chi} < 0$:

$$\begin{aligned} r^2 R'' + rR' - \chi R &= 0 \\ G'' + \chi G &= 0 \end{aligned} \quad \Rightarrow \quad R = ar^{-\sqrt{\chi}} + br^{\sqrt{\chi}} = ar^k$$

since $r \ll 1$, we obtain:

$$\omega(r, \theta) = r^k [C_1 \cos(k\theta) + C_2 \sin(k\theta)].$$

$$\psi(r, \theta) = r^{k+2} [D_1 \cos((k+2)\theta) + D_2 \sin((k+2)\theta) + D_3 \cos(k\theta) + D_4 \sin(k\theta)].$$

ANALYTICAL CORNER CORRECTION: BOUNDARY CONDITIONS

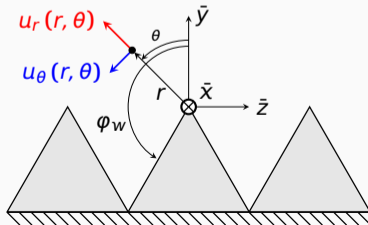
The coefficients D_j are given after the following boundary and symmetry conditions are provided:

$$u_r(r, \pm\phi_w) = 0 \quad \text{no penetration}$$

$$u_\theta(r, \pm\phi_w) = 0 \quad \text{no-slip}$$

$$u_r(r, \theta) = -u_r(r, -\theta) \quad u_r \text{ odd in } \theta$$

$$u_\theta(r, \theta) = u_\theta(r, -\theta) \quad u_\theta \text{ even in } \theta.$$



ANALYTICAL CORNER CORRECTION: BOUNDARY CONDITIONS

The symmetry conditions lead to $D_2 = D_4 = 0$, and the definition of the stream-function gives u_r and u_θ depending on $\gamma = k + 1$ as

$$u_r(r, \theta) = \frac{1}{r} \frac{\partial \psi}{\partial \theta} = -r^\gamma [D_1 (\gamma + 1) \sin((\gamma + 1)\theta) + D_3 (\gamma - 1) \sin((\gamma - 1)\theta)]$$
$$u_\theta(r, \theta) = -\frac{\partial \psi}{\partial r} = -(\gamma + 1)r^\gamma [D_1 \cos((\gamma + 1)\theta) + D_3 \cos((\gamma - 1)\theta)].$$

The boundary conditions are used to find the ratio between the coefficients D_3 and D_1 , that is

$$\frac{D_3}{D_1} = \frac{\cos((\gamma + 1)\varphi_w)}{\cos((\gamma - 1)\varphi_w)}. \quad (1)$$

We set $D_1 = 1$.

ANALYTICAL CORNER CORRECTION: BOUNDARY CONDITIONS

The last constant to find is γ , whose value is given solving numerically $\det(\mathbf{Q}(\gamma)) = 0$.

$$\underbrace{\begin{bmatrix} (\gamma + 1) \sin((\gamma + 1) \varphi_w) & (\gamma - 1) \sin((\gamma - 1) \varphi_w) \\ \cos((\gamma + 1) \varphi_w) & \cos((\gamma - 1) \varphi_w) \end{bmatrix}}_{\mathbf{Q}(\gamma)} \begin{bmatrix} D_1 \\ D_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

The solution depends on the geometry considered: for the problem at hand, with $\varphi_w = \pi/6$, the result is $\gamma \approx 0.51222$.

The last unknown for the Stokes problem is the pressure:

$$\begin{aligned} \nu \left[\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r} \frac{\partial u_r}{\partial r} + \frac{1}{r^2} \left(\frac{\partial^2 u_r}{\partial \theta^2} - 2 \frac{\partial u_\theta}{\partial \theta} - u_r \right) \right] - \frac{1}{\rho} \frac{\partial p}{\partial r} &= 0 \\ \nu \left[\frac{\partial^2 u_\theta}{\partial r^2} + \frac{1}{r} \frac{\partial u_\theta}{\partial r} + \frac{1}{r^2} \left(\frac{\partial^2 u_\theta}{\partial \theta^2} + 2 \frac{\partial u_r}{\partial \theta} - u_\theta \right) \right] - \frac{1}{\rho} \frac{1}{r} \frac{\partial p}{\partial \theta} &= 0 \\ \frac{1}{\nu} p(r, \theta) &= -4\gamma D_3 r^{\gamma-1} \sin((\gamma-1)\theta). \end{aligned}$$

ANALYTICAL CORNER CORRECTION: PRESSURE

The expression for p can not be used itself, because it is not guaranteed that p is symmetric and continuous inside the body. A correction can be implemented to choose a continuous branch for the solution, considering $\tilde{\theta} = \theta f(\theta)$ where $f(\theta) \neq 1$ only if $|\theta| > \varphi_w$, so that p is given by

$$\frac{1}{\nu} p(r, \theta) = -4\gamma D_3 r^{\gamma-1} \sin((\gamma-1)\tilde{\theta})$$
$$f(\theta) = \begin{cases} 1 + \frac{|\theta| - \pi}{\pi - \varphi_w} \left(\frac{1}{\gamma-1} - 1 \right) & \text{if } |\theta| > \varphi_w \\ 1 & \text{otherwise.} \end{cases}$$

ANALITICAL CORRECTION: LAPLACE PROBLEM

The Laplace problem reads:

$$\nabla^2 u = 0 \implies \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} = 0,$$

and a variable separation leads to the general solution

$$u(r, \theta) = r^m [C \cos(m\theta) + D \sin(m\theta)].$$

No-slip boundary conditions, namely $u(r, \pm\phi_w) = 0$, lead to $\cos(m\phi_w) = 0$ and so $m\phi_w = \pi/2$. The symmetry condition, $u(r, \theta) = u(r, -\theta)$, gives $D = 0$ and the final expression for u , namely

$$u = Cr^m \cos(m\theta).$$

C here is a free constant that can be set to 1 to have a unique solution.

ANALYTICAL CORNER CORRECTION: IMPLEMENTATION

$$u^{(t+\Delta t)} = u^{(t)} + (lapl + NL + \nabla p) \Delta t - u^{(t+\Delta t)} imbc \Delta t \implies u^{(t+\Delta t)} = \frac{u^{(t)} + RHS \Delta t}{1 + imbc \Delta t}$$

Being u_{loc} and p_{loc} the analytical solutions for the velocity and the pressure respectively, considering the problem for the x-direction one gets

$$d_u = \underbrace{\left(\frac{lapl(u_{loc}(x, \cdot))}{Re} - \frac{p_{loc}(x + \Delta x, \cdot) - p_{loc}(x, \cdot)}{\Delta x} \right)}_{corr_{stokes}} \frac{1}{u_{loc}(x, \cdot)} u(x, \cdot),$$

where $lapl()$ is the laplacian corrected with the true distance from the body. The Navier-Stokes problem here is not so different: the terms to add inside $imbc$ are a contribution from the Laplace problem in u , $corr_{lapl}$, and from the Stokes problem in v and w , $corr_{stokes}$.

ANALYTICAL CORNER CORRECTION: ROTATION

Considering (u', v') in the local reference frame and (u, v) in the global one, the following additional rotation should be performed:

$$u' = \cos(\beta) u + \sin(\beta) v, \quad v' = \cos(\beta) v - \sin(\beta) u.$$

The *imbc* coefficients in the local reference frame were already found for the straight riblets as

$$d_{u'} = \text{corr}_{lapl} u', \quad d_{v'} = \text{corr}_{stokes} v',$$

but to define the corrections in the cartesian global reference frame the two components get mixed into the 2×2 non-diagonal system.

ANALYTICAL CORNER CORRECTION: ROTATION

$$\begin{bmatrix} d_u \\ d_v \end{bmatrix} = \begin{bmatrix} \cos^2(\beta) \text{corr}_{lapl} + \sin^2(\beta) \text{corr}_{stokes} & (\text{corr}_{lapl} - \text{corr}_{stokes}) \sin(2\beta)/2 \\ (\text{corr}_{stokes} - \text{corr}_{lapl}) \sin(2\beta)/2 & \cos^2(\beta) \text{corr}_{stokes} + \sin^2(\beta) \text{corr}_{lapl} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}.$$

$$\begin{cases} d_u = (\cos^2(\beta) \text{corr}_{lapl} + \sin^2(\beta) \text{corr}_{stokes}) u \\ d_v = (\cos^2(\beta) \text{corr}_{stokes} + \sin^2(\beta) \text{corr}_{lapl}) v. \end{cases}$$

PROTRUSION HEIGHTS

	nppr	h_{\parallel}	(err%)	h_{\perp}	(err%)	Δh	(err%)
Standard	8	0.1537	(-10.4)	0.1254	(+54.8)	0.02831	(-68.7)
+ Correction	8	0.1683	(-1.9)	0.0811	(+0.2)	0.0872	(-3.7)
Standard	16	0.1639	(-4.4)	0.1028	(+26.9)	0.06111	(-32.5)
+ Correction	16	0.1702	(-0.7)	0.0812	(+0.3)	0.0890	(-1.7)

Table 1: Results of the validation for straight riblets with the immersed boundary correction only (Standard) and with the addition of the corner correction (+ Correction). Errors are estimated as $(h - \bar{h})/\bar{h}$.

\bar{h}_{\parallel}	\bar{h}_{\perp}	$\Delta\bar{h}$
0.17150	0.08099	0.09051

Table 2: Protrusion heights reference values for $h/s = \sqrt{3}/2$.

SKIN FRICTION COEFFICIENT AND U_b - STRAIGHT

	n	U_b	$(\Delta U_b^+ \%)$	$C_f \times 10^3$	$(\Delta C_f / C_{f,0} \%)$
Standard	8	15.62	(-2.7)	8.20	(+5.7)
+ Correction	8	16.58	(+3.3)	7.27	(-6.3)
Standard	16	16.14	(+0.1)	7.67	(-0.1)
+ Correction	16	16.54	(+2.6)	7.31	(-4.8)

Table 3: U_b^+ and C_f for the straight case. ΔU_b^+ and ΔC_f are evaluated considering the smooth channel simulation with the same δy^+ of the case considered.

SKIN FRICTION COEFFICIENT AND U_b - SINUSOIDAL

	n	U_b	$(\Delta U_b^+ \%)$	$C_f \times 10^3$	$(\Delta C_f / C_{f,0} \%)$
L Standard	8	16.28	(+1.4)	7.55	(-2.7)
L + Correction	8	16.75	(+4.4)	7.13	(-8.1)
L Standard	16	16.43	(+1.9)	7.41	(-3.5)
L + Correction	16	16.67	(+3.4)	7.19	(-6.4)

Table 4: U_b^+ and C_f for the sinusoidal cases. ΔU_b^+ and ΔC_f are evaluated considering the smooth channel simulation with the same δy^+ of the case considered.