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TOWARDS RELIABLE AND COST-EFFECTIVE DNS OF TURBULENT FLOW OVER RIBLETS: ANALYTICAL CORRECTION OF CORNER SINGULARITY

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BACKGROUND

Flow control for skin friction drag reduction aims at mitigating the negative effects of turbulence near the wall to cut energy consumption and boost cost-effectiveness and environmental efficiency. Drag reduction strategies can be classified into two categories, passive and active. Among the latter, riblets are the most promising and considered for application on commercial airplanes. Riblets are small two-dimensional surface protrusions aligned with the direction of the flow, that produce an anisotropic roughness: their particular shape blocks the near-wall spanwise flow more effectively than the streamwise one.

Studies by Luchini et al. [4] clarified their drag reducing mechanism, which produces a vertical shift of the velocity profile in the turbulent region of the boundary layer once plotted in the law-of-the-wall form. It was demonstrated that, when the spanwise period of the riblets is small (i.e. in the so-called viscous regime), the drag reduction (DR) rate depends on the protrusion height only. The protrusion height is the difference (Δh) between the longitudinal ($h_{\parallel}$) and the transverse ($h_{\perp}$) protrusion heights, which represent the virtual origin of the longitudinal and transverse velocity profile, each measurable from an arbitrary origin (e.g. the riblet tip). When Δh is positive, the spanwise flow induced by the overlying turbulent streamwise vortices is hampered more than the longitudinal one, thus reducing turbulent activity and eventually drag.

Riblets have been extensively studied over the last 50 years, particularly by experimental works. A few geometries have been tested and V-grooves in which riblets have sharp triangular ridges have been found particularly effective, guaranteeing a maximum DR of $7-8\%$. More difficult is the corresponding numerical verification of the riblets performance, which requires an high-fidelity numerical approach such as Direct Numerical Simulation (DNS).

The geometric singularity at the tip that characterizes riblets for any cross-sectional shape is directly responsible for its drag reduction capabilities, but at the same time makes numerical simulations extremely challenging. In this contribution we will show how a brute-force approach where a large number of grid points in the spanwise direction are placed near the tip increases the computational cost at an extreme level, while still providing unacceptable performance. Hence, we introduce an analytical correction of the instantaneous solution near the corner, that combines solution accuracy and computational efficiency.

METHODS

The local loss of accuracy of the numerical solution near the geometric singularity made by the riblets tip makes the measure of the friction drag reduction over surfaces with riblets a major challenge. Since properly resolving the tip by locally refining the grid is computationally prohibitive, we exploit the numerical method proposed by Luchini [2] for the analytical correction of the corner singularity. The method hinges upon the knowledge of the local behaviour of the solution close to the singularity, which is analytically determined and compensated for. Near the tip the velocity gradients become infinitely large, viscous effects are dominant over non-linear effects, and the local solution can be obtained from the analytically determined Stokes flow in polar coordinates (see figure 1). Let us

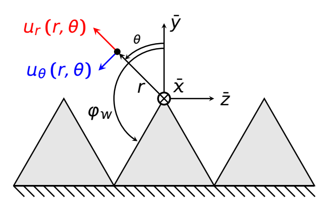

Figure 1: Local reference frame for a straight V-shape riblet.

consider two-dimensional riblets first. Actually the solution is provided by two distinct problems. Parallel to the riblets edge, the pressure is not affected and solving a Laplace problem provides the solution of the streamwise component of the velocity:

$$u = r^{\pi/(2\varphi_w)} \cos\left(\pi/(2\varphi_w)\theta\right).$$

In the plane of the cross-section of the riblets, a two-dimensional Stokes problem provides the local solution for the wall-normal and spanwise components of the velocity, and a

Poisson problem provides the pressure:

$$u_r\left(r,\theta\right) = -r^{\gamma}\left[A\left(\gamma+1\right)\sin\left(\left(\gamma+1\right)\theta\right) + B\left(\gamma-1\right)\sin\left(\left(\gamma-1\right)\theta\right)\right]$$
$$u_\theta(r,\theta) = -(\gamma+1)r^{\gamma}\left[A\cos((\gamma+1)\theta) + B\cos((\gamma-1)\theta)\right]$$
$$\frac{1}{\nu}p\left(r,\theta\right) = -4\gamma B r^{\gamma-1}\sin\left(\left(\gamma-1\right)\theta\right),$$

where γ is the first mode of the spectrum of exact solutions and the constants A and B are provided enforcing the boundary conditions. The corner correction is enforced thanks to a tight integration into the DNS solver, which is based on an implicit immersed-boundary method. The correction is present close to the tip, and vanishes far from it. The DNS code was introduced by Luchini [3], and solves the incompressible Navier–Stokes equations in primitive variables. An implicit immersed-boundary method, implemented on staggered Cartesian grid, continuous with respect to boundary crossing and numerically stable at all distances from the boundary, describes the geometry of the non-planar wall.

We carry out a DNS at $Re_\tau = 200$ with a Constant Pressure Gradient strategy of a fully developed turbulent half-channel flow with the wall covered by 2D riblets with a triangular V-shaped cross-section with height to spacing ratio $h/s = \sqrt{3}/2$. The riblets spacing in viscous units is $s^+ = 16$, i.e. about the optimal value for reducing skin friction drag. The computational domain is $(L_x, L_y, L_z) = (7.5, 2.08, 1)$ with a computational grid with a number of points of $(N_x, N_y, N_z) = (240, 416, 186)$ in the streamwise, spanwise and wall-normal direction. Periodic boundary conditions are enforced in both the streamwise and spanwise directions, whereas an anti-symmetric boundary condition for the wall-normal velocity component is imposed at the centerline; no slip and no penetration boundary conditions are enforced at the wall.

RESULTS

Figure 2 shows the significant enhancement of the reliability of the DNS results (in terms of value of the protrusion heights) when the corner correction is applied. The protrusion heights computed with/without the corner correction are compared to the analytical solution [4] for a laminar case. For the highest resolution with 16 grid points per riblets in spanwise direction, the error in computing of Δh drops from 32.5 % to 1.7 % when the correction is applied. This error can be thought of as a relative error in computing drag reduction. The skin-friction

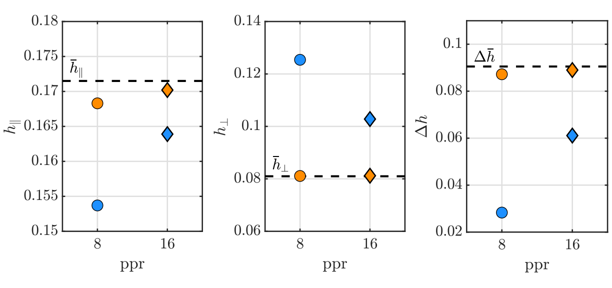

Figure 2: Protrusion heights of triangular riblets computed with (orange) and without (blue) corner correction. Dashed lines are the analytical values taken from [4]. The horizontal axis represents the number of grid points per single riblets (ppr) in the spanwise direction.

drag of the channel covered by riblets is compared to the one of a reference smooth channel at the same Reynolds number to compute the drag reduction performance of riblets. The computed reduction of drag with the finer computational grid changes from 0.1 % without corner correction to 4.8 % with corner correction, which resembles the experimental results of Bechert et al.[1] who found a reduction of drag of 5 % for the same geometry.

After assessing the validity of the corner correction for straight riblets, its extension to three-dimensional riblets is considered. If the sharp edge of the riblets is not straight the decoupling of the local solution in the Laplace and 2D Stokes problems fails in a global reference frame. Decoupling is still obtained in a reference frame aligned with the local cross-section, but the components of the velocity vector become intermixed, and an implicit discretization becomes impossible. Even an explicit treatment would be difficult if the mesh is staggered. Thus, in this preliminary attempt we neglect the mixed terms in the correction, limiting it to the diagonal components of the correction matrix, under the assumption that the local misalignment of three-dimensional riblets is small.

At the Meeting, we will present preliminary results obtained with our DNS code for the case of sinusoidal riblets, a particular class of 3D configurations where the riblet tip varies its spanwise position along the streamwise direction following a sinusoidal law, and the spanwise spacing between the riblets remains constant (see figure 3). A few attempts have been made in the past to characterize this geometry, with scattered results. We consider sinusoidal riblets with a wavelength $\lambda_x^+ = 1500$ and a maximum angle between the streamwise direction and the riblets crest $\beta_{max} = 2°$. Sinusoidal riblets have been found to outperform straight riblets, bumping up DR from 4.8% to 6.4%. Although the result is still preliminary, and the full corner correction needs to be implemented, given our methodological approach, we are inclined to believe that such geometry can in fact perform better than straight riblets, albeit moderately so as long as the cross-sectional shape of the riblets remains unchanged along the streamwise direction.

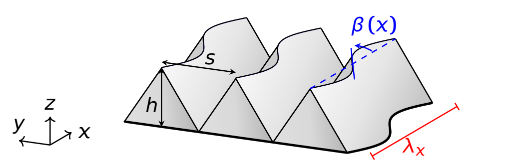

Figure 3: Sinusoidal riblets with geometric characteristics highlighted: h is height, s the spacing, $\beta(x)$ is the angle between the crests and the streamwise direction and λ_x is the wavelength. Straight riblets have $\beta(x) = 0$.

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