Skin-friction drag reduction described via the Anisotropic Generalised Kolmogorov Equations

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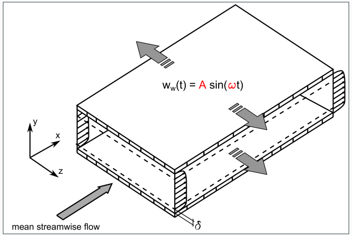
How do DR techniques affect

the production, transport and dissipation

of turbulent stresses

among scales and in space?

Turbulent channel flow forced via spanwise oscillating walls

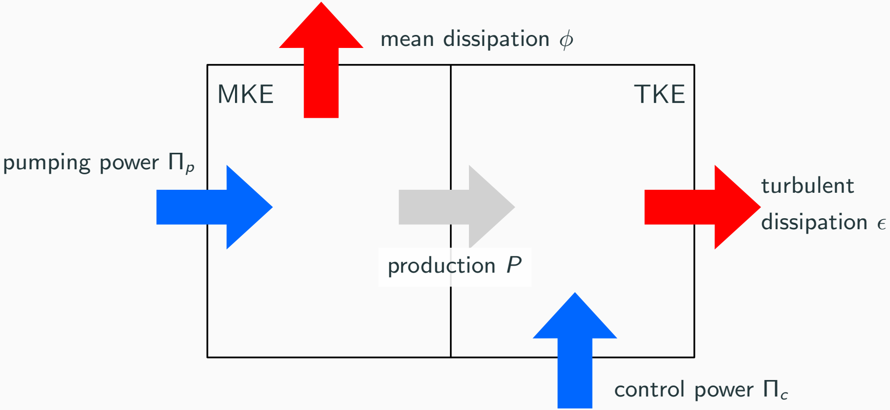


Controlled channel vs Reference channel

At Constant Power Input

Constant Power Input: an alternative to CFR and CPG

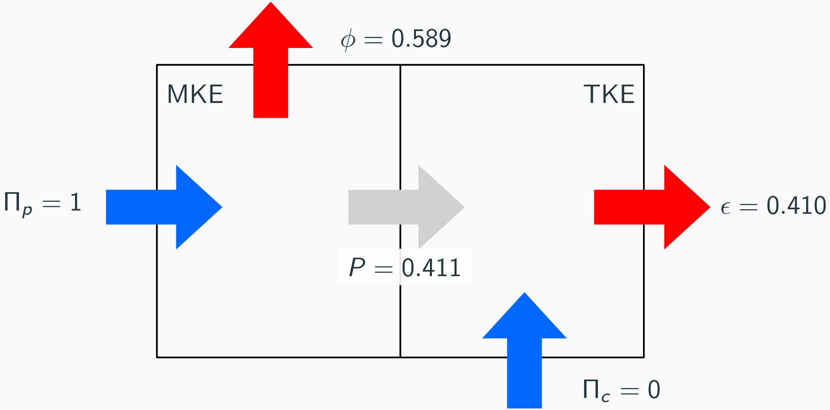
The input power is kept constant



Gatti *et al.* JFM 2018

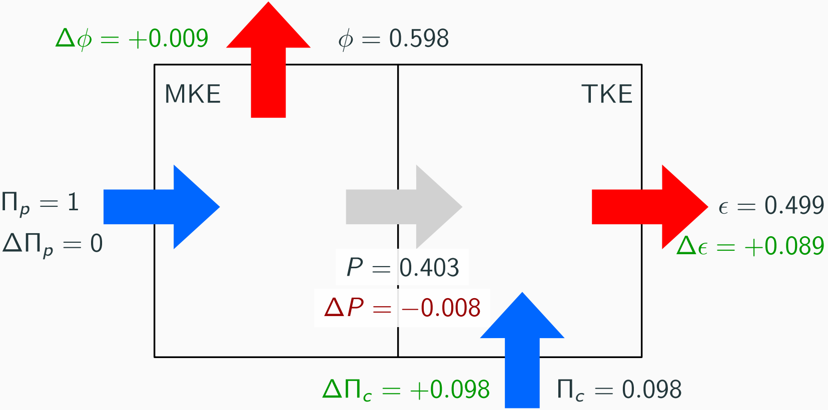
Starting point: OW effect on the global energy fluxes

Reference channel at $Re_\tau = 200$



Starting point: OW effect on the global energy fluxes

Controlled channel via OW with $A^+ = 4.5$ and $T^+ = 125$



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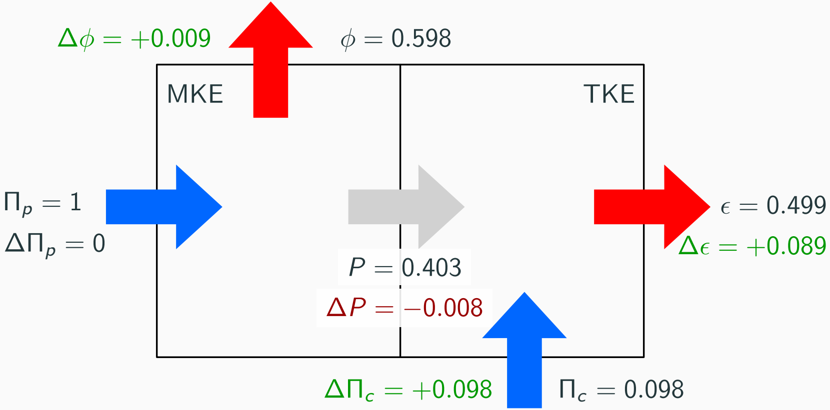
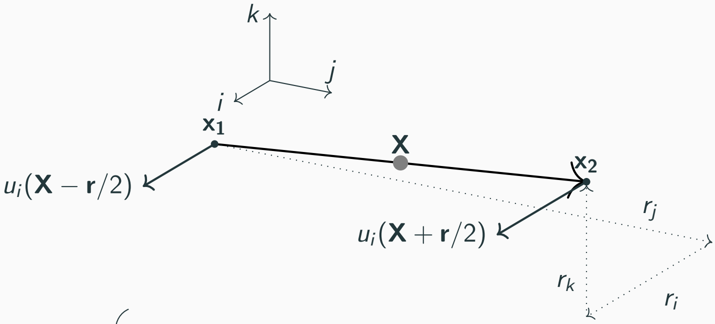


Global variations $\rightarrow$ detailed changes

Anisotropic Generalised Kolmogorov Equations

AGKE: Exact budget equation for $\langle \delta u_i \delta u_j \rangle$

$$\delta u_i = \left(u_i \left(\mathbf{X} + \mathbf{r}/2, t \right) - u_i \left(\mathbf{X} - \mathbf{r}/2, t \right) \right)$$



Dependent on:

$$\begin{cases} \mathbf{X} = (\mathbf{x_1} + \mathbf{x_2})/2 \\ \mathbf{r} = \mathbf{x_2} - \mathbf{x_1} \end{cases}$$

AGKE: extension of the Generalised Kolmogorov Equation to anisotropy

GKE: Exact budget equation for the scale energy

$$\langle \delta u_i \delta u_i \rangle = tr \begin{bmatrix} \langle \delta u \delta u \rangle & \langle \delta u \delta v \rangle & \langle \delta u \delta w \rangle \\ & \langle \delta v \delta v \rangle & \langle \delta v \delta w \rangle \\ sym & & \langle \delta w \delta w \rangle \end{bmatrix} = \langle \delta u \delta u \rangle + \langle \delta v \delta v \rangle + \langle \delta w \delta w \rangle$$

AGKE: extension of the Generalised Kolmogorov Equation to anisotropy

GKE: Exact budget equation for the scale energy

$$\langle \delta u_i \delta u_i \rangle = tr \begin{bmatrix} \langle \delta u \delta u \rangle & \langle \delta u \delta v \rangle & \langle \delta u \delta w \rangle \\ & \langle \delta v \delta v \rangle & \langle \delta v \delta w \rangle \\ sym & & \langle \delta w \delta w \rangle \end{bmatrix} = \langle \delta u \delta u \rangle + \langle \delta v \delta v \rangle + \langle \delta w \delta w \rangle$$

What if $\langle \delta u \delta u \rangle \gg \langle \delta v \delta v \rangle, \langle \delta w \delta w \rangle$?

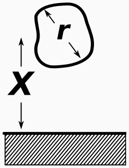
The GKE does not account for anisotropy..

..but the AGKE do!

AGKE: interpretation

- $\langle \delta u_i \delta u_j \rangle(\mathbf{X}, \mathbf{r})$

Amount of turbulent stresses
at location **X** and scale (up to) **r**



JFM, in preparation

- AGKE

Production, transport and dissipation
of turbulent stresses
in both the
Space of scales & Physical space

AGKE

$$\boxed{\frac{\partial \phi_{k,ij}}{\partial r_k} + \frac{\partial \psi_{k,ij}}{\partial X_k} = \xi_{ij}}$$

AGKE

$$\boxed{\frac{\partial \phi_{k,ij}}{\partial r_k} + \frac{\partial \psi_{k,ij}}{\partial X_k} = \xi_{ij}}$$

$$\phi_{k,ij} = \underbrace{\langle \delta U_k \delta u_i \delta u_j \rangle}_{\text{mean transport}} + \underbrace{\langle \delta u_k \delta u_i \delta u_j \rangle}_{\text{turbulent transport}} \underbrace{- 2\nu \frac{\partial}{\partial r_k} \langle \delta u_i \delta u_j \rangle}_{\text{viscous diffusion}}$$

flux of $\langle \delta u_i \delta u_j \rangle$ throughout scales $\mathbf{r}$

AGKE

$$\boxed{\frac{\partial \phi_{k,ij}}{\partial r_k} + \frac{\partial \psi_{k,ij}}{\partial X_k} = \xi_{ij}}$$

$$\psi_{k,ij} = \underbrace{\langle U_k^* \delta u_i \delta u_j \rangle}_{\text{mean transport}} + \underbrace{\langle u_k^* \delta u_i \delta u_j \rangle}_{\text{turbulent transport}} + \underbrace{\frac{1}{\rho} \langle \delta p \delta u_i \rangle \delta_{kj} + \frac{1}{\rho} \langle \delta p \delta u_j \rangle \delta_{ki}}_{\text{pressure transport}} - \underbrace{\frac{\nu}{2} \frac{\partial}{\partial X_k} \langle \delta u_i \delta u_j \rangle}_{\text{viscous diffusion}}$$

flux of $\langle \delta u_i \delta u_j \rangle$ in space $\mathbf{X}$

AGKE

$$\boxed{\frac{\partial \phi_{k,ij}}{\partial r_k} + \frac{\partial \psi_{k,ij}}{\partial X_k} = \xi_{ij}}$$

source/sink of $\langle \delta u_i \delta u_j \rangle$ at scale $\mathbf{r}$ and location $\mathbf{X}$

$$\xi_{ij} = \underbrace{-\langle u_k^* \delta u_j \rangle \delta \left(\frac{\partial U_i}{\partial x_k} \right) - \langle u_k^* \delta u_i \rangle \delta \left(\frac{\partial U_j}{\partial x_k} \right) - \langle \delta u_k \delta u_j \rangle \left(\frac{\partial U_i}{\partial x_k} \right)^* - \langle \delta u_k \delta u_i \rangle \left(\frac{\partial U_j}{\partial x_k} \right)}_{\text{production}} +$$

$$\underbrace{+ \frac{1}{\rho} \left\langle \delta p \frac{\partial \delta u_i}{\partial X_j} \right\rangle + \frac{1}{\rho} \left\langle \delta p \frac{\partial \delta u_j}{\partial X_i} \right\rangle}_{\text{pressure strain}} \underbrace{-4\epsilon_{ij}^*}_{\text{dissipation}}$$

AGKE

$$\boxed{\frac{\partial \phi_{k,ij}}{\partial r_k} + \frac{\partial \psi_{k,ij}}{\partial X_k} = \xi_{ij}}$$

$$\phi_{k,ij} = \underbrace{\langle \delta U_k \delta u_i \delta u_j \rangle}_{\text{mean transport}} + \underbrace{\langle \delta u_k \delta u_i \delta u_j \rangle}_{\text{turbulent transport}} \underbrace{- 2\nu \frac{\partial}{\partial r_k} \langle \delta u_i \delta u_j \rangle}_{\text{viscous diffusion}}$$

$$\psi_{k,ij} = \underbrace{\langle U_k^* \delta u_i \delta u_j \rangle}_{\text{mean transport}} + \underbrace{\langle u_k^* \delta u_i \delta u_j \rangle}_{\text{turbulent transport}} \underbrace{+ \frac{1}{\rho} \langle \delta p \delta u_i \rangle \delta_{kj} + \frac{1}{\rho} \langle \delta p \delta u_j \rangle \delta_{ki}}_{\text{pressure transport}} \underbrace{- \frac{\nu}{2} \frac{\partial}{\partial X_k} \langle \delta u_i \delta u_j \rangle}_{\text{viscous diffusion}}$$

$$\xi_{ij} = \underbrace{- \langle u_k^* \delta u_j \rangle \delta \left(\frac{\partial U_i}{\partial x_k} \right) - \langle u_k^* \delta u_i \rangle \delta \left(\frac{\partial U_j}{\partial x_k} \right) - \langle \delta u_k \delta u_j \rangle \left(\frac{\partial U_i}{\partial x_k} \right)^* - \langle \delta u_k \delta u_i \rangle \left(\frac{\partial U_j}{\partial x_k} \right)}_{\text{production}} +$$

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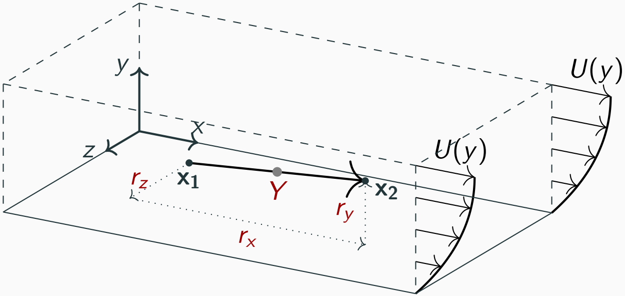
AGKE tailored to channel flow

$$\langle \delta u_i \delta u_j \rangle(\mathbf{X}, \mathbf{r}) \rightarrow \langle \delta u_i \delta u_j \rangle(Y, r_x, r_y, r_z)$$



$$\boxed{\frac{\partial \phi_{k,ij}}{\partial r_k} + \frac{\partial \psi_{ij}}{\partial Y} = \xi_{ij}}$$

How do DR techniques affect

the production, transport and dissipation

of turbulent stresses

among scales and in space?

We investigate the changes of the AGKE terms

Numerical Data

- Two Direct Numerical Simulations (with and without wall oscillation) at CPI

  $Re_\tau = 200$ for the uncontrolled case

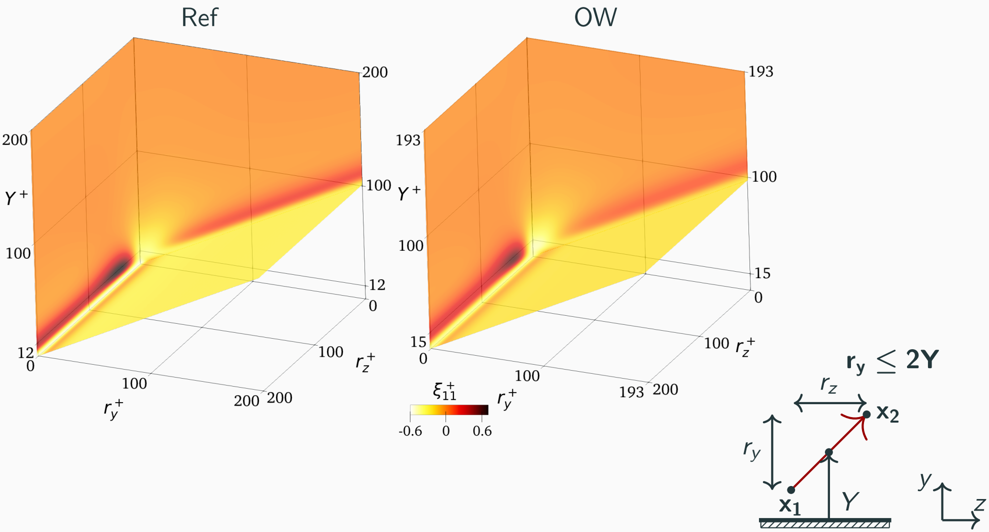
  Wall oscillation parameters: $A^+ = 4.5,\ T^+ = 125.5$

  Quadrio & Ricco JFM 2004

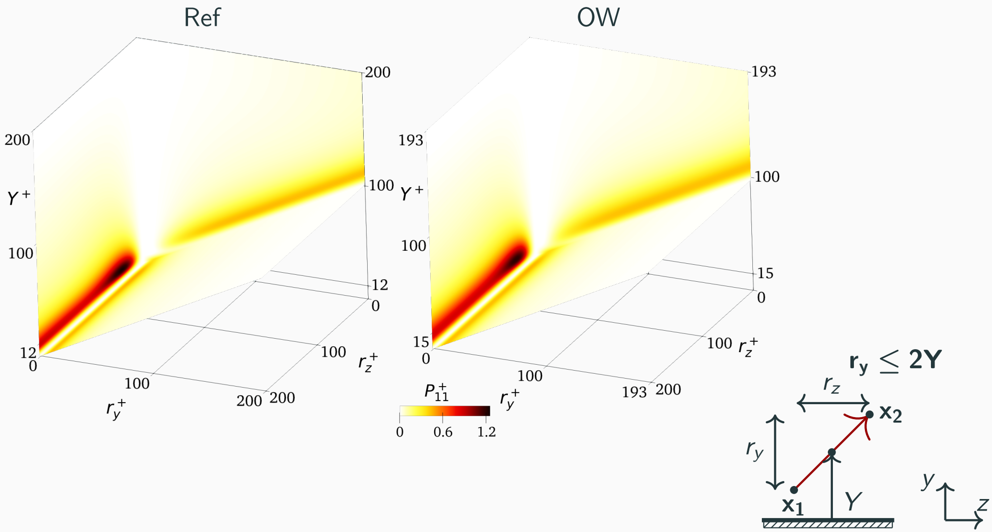
- Six smaller Direct Numerical Simulations at CPI

  $A^+ \in (0, 30),\ T^+ \in (100, 125)$

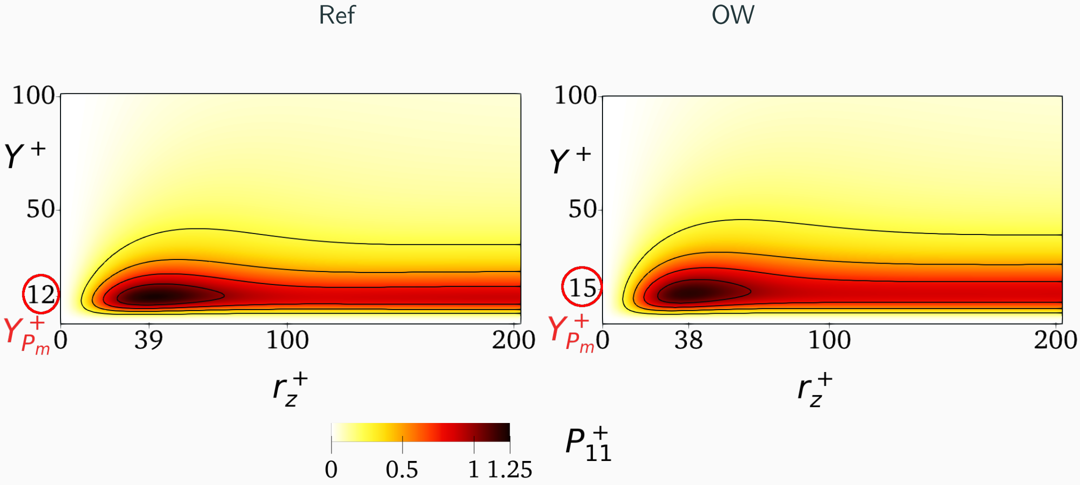
Source of $\langle \delta u \delta u \rangle$ in the $r_x = 0$ space



Production of $\langle \delta u \delta u \rangle$



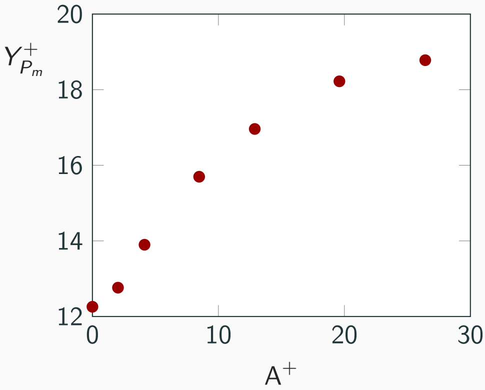
Production of $\langle \delta u \delta u \rangle$



Shift of $P_{11,m}$ ($\Delta Y^+_{P_m} \sim 3$)

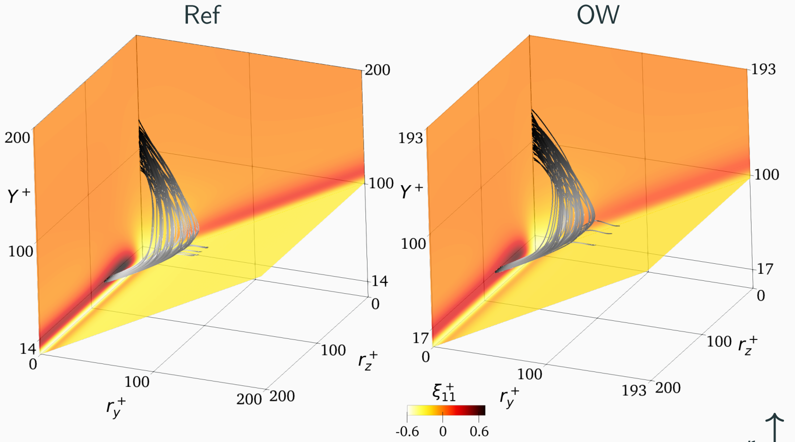
Production of $\langle \delta u \delta u \rangle$

What if A^+ is incremented?

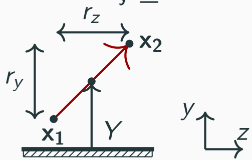


$\Delta Y^+_{P_m}$ increases with A^+ (%DR)

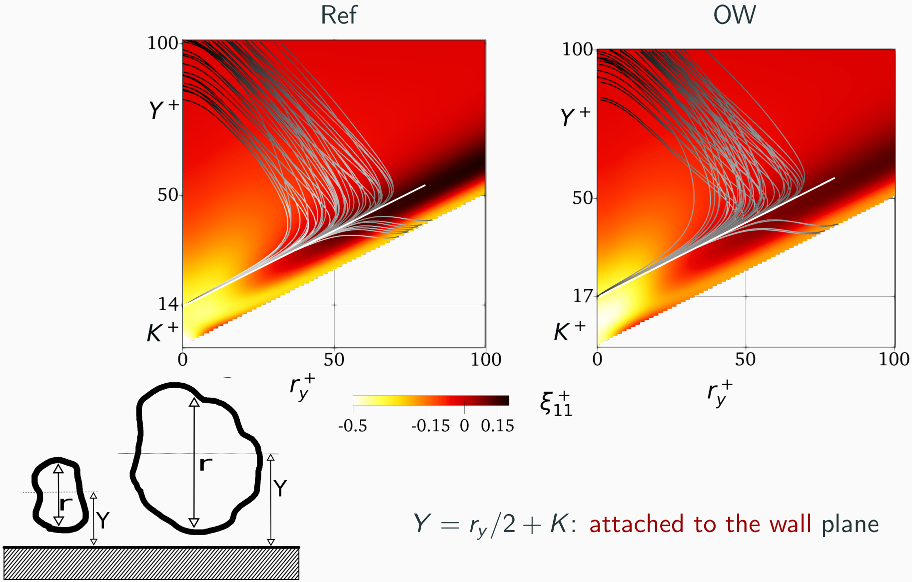
Fluxes of $\langle \delta u \delta u \rangle$: $\phi_{k,11}$ and ψ_{11}



$r_y \leq 2Y$

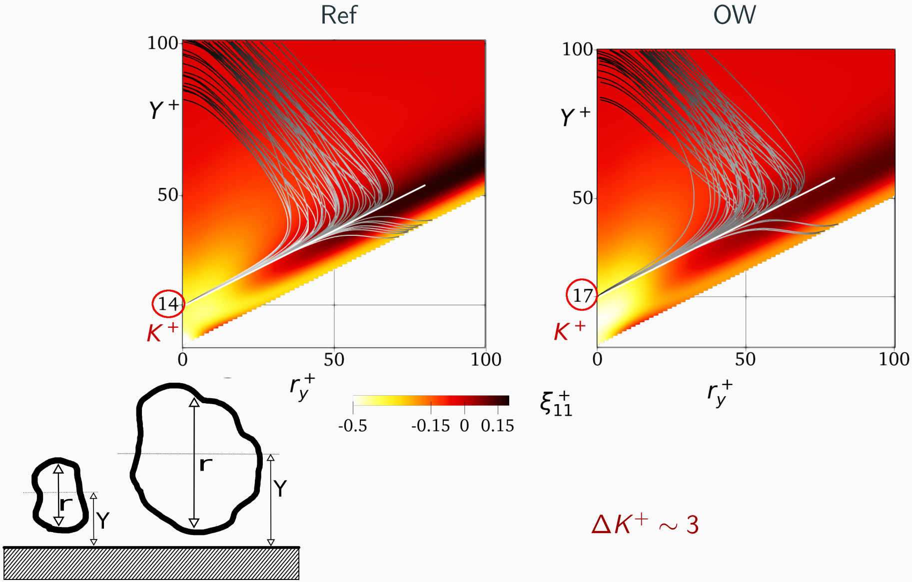


Fluxes of $\langle \delta u \delta u \rangle$: $\phi_{k,11}$ and ψ_{11}



$Y = r_y/2 + K$: attached to the wall plane

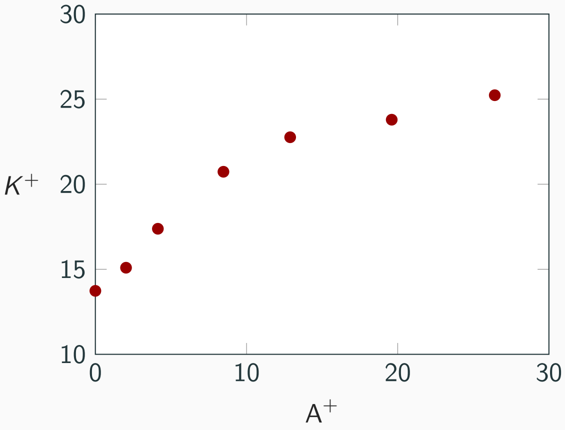
Fluxes of $\langle \delta u \delta u \rangle$: $\phi_{k,11}$ and ψ_{11}



$\Delta K^+ \sim 3$

Fluxes of $\langle \delta u \delta u \rangle$: $\phi_{k,11}$ and ψ_{11}

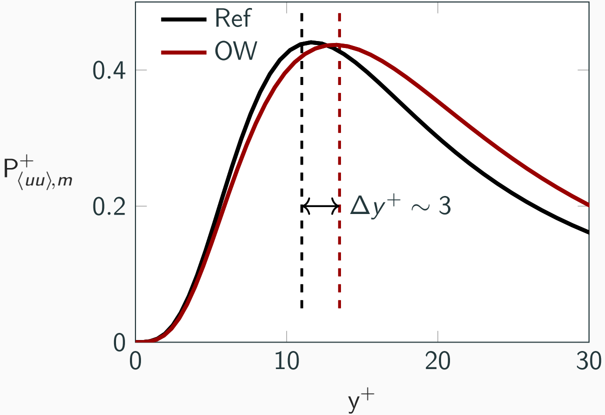
What if A^+ is incremented?



ΔK^+ increases with A^+ ($\%DR$)

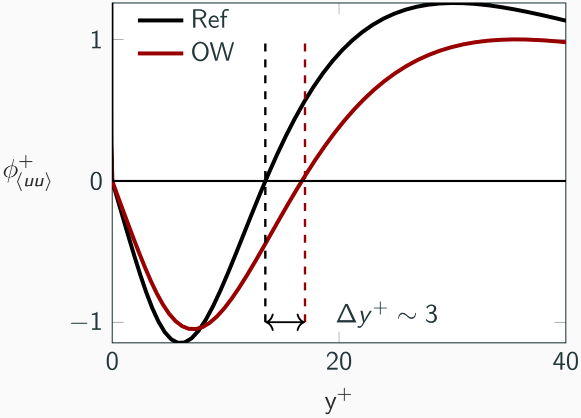
Links with well-known results?

Vertical shift Δy of the maximum of $P_{\langle uu \rangle}$



Links with well-known results?

Vertical shift Δy of $\phi_{\langle uu \rangle} = 0$



Interpretation

Single-point statistics

- Shift of $\langle uu \rangle_m$
- Shift of $P_{\langle uu \rangle, m}$
- Shift of $\phi_{\langle uu \rangle} = 0$

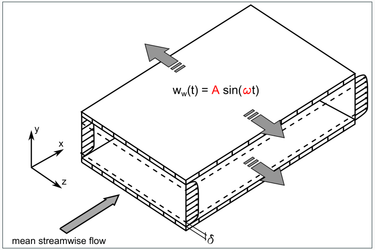
Two-points statistics

- Shift of $\langle \delta u \delta u \rangle_m$
- Shift of $P_{\langle \delta u \delta u \rangle, m}$
- Shift of the attached to the wall plane

OW $\rightarrow$ Virtual shift of the wall

Conclusion

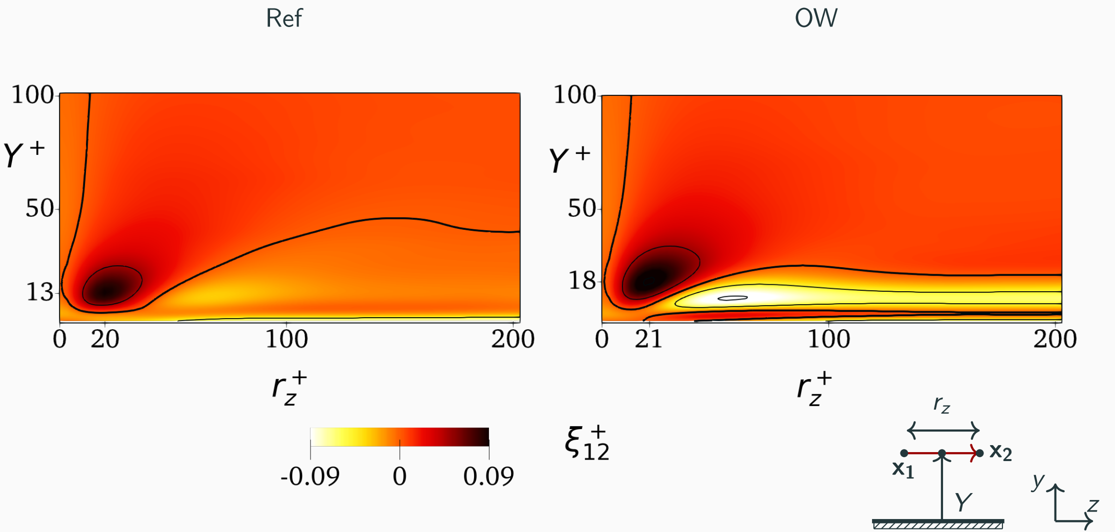
AGKE terms in turbulent channel forced via spanwise oscillating walls



For the streamwise normal stress..

- Shift of the production activity of $\langle \delta u \delta u \rangle$ towards larger wall-distances
- Shift of the main transport of $\langle \delta u \delta u \rangle$ towards larger wall-distances
- Both shifts increase with A

Outlook: source of $\langle -\delta u \delta v \rangle$



Thanks

for your kind attention!

For questions or suggestions:

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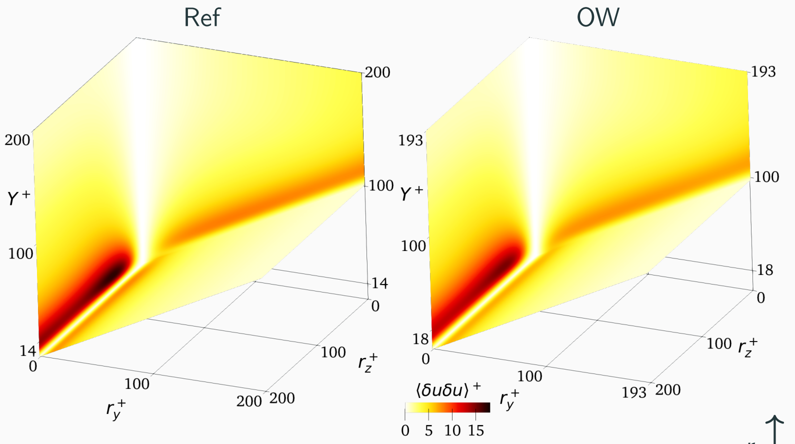
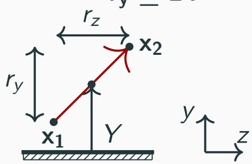
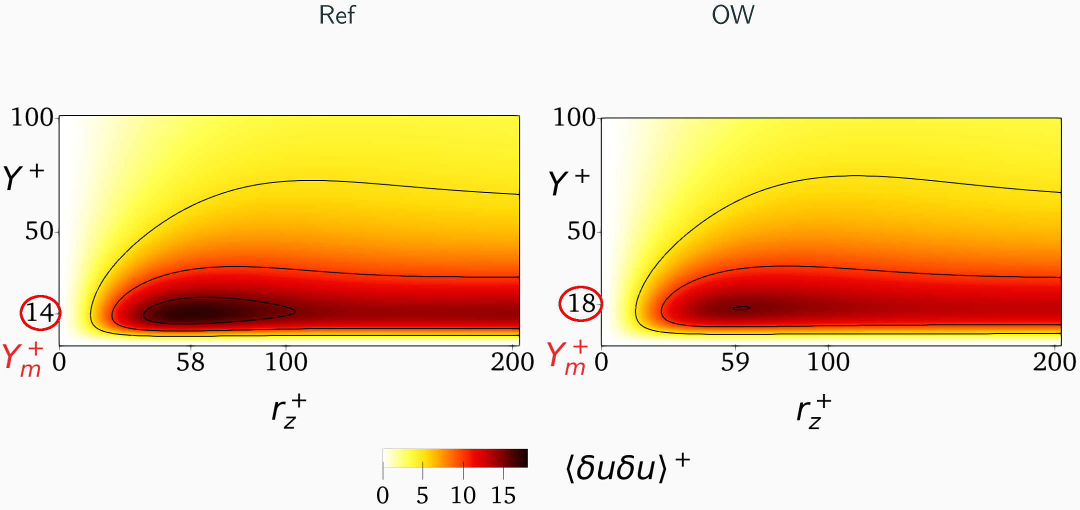
$\langle \delta u \delta u \rangle$ in the $r_x = 0$ space



$\mathbf{r_y \leq 2Y}$

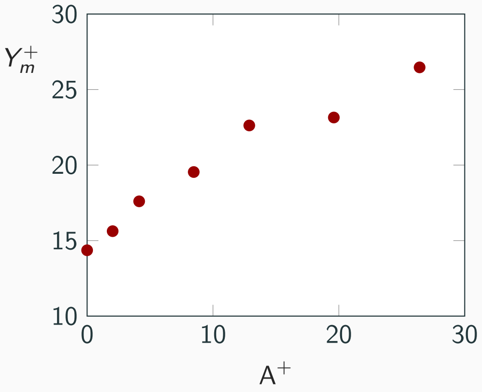


$\langle \delta u \delta u \rangle$ in the $r_x = 0$ space



$\langle \delta u \delta u \rangle$ in the $r_x = 0$ space

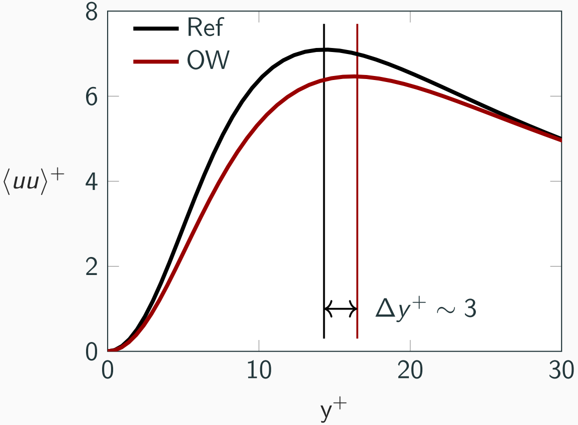
What if A^+ is incremented?



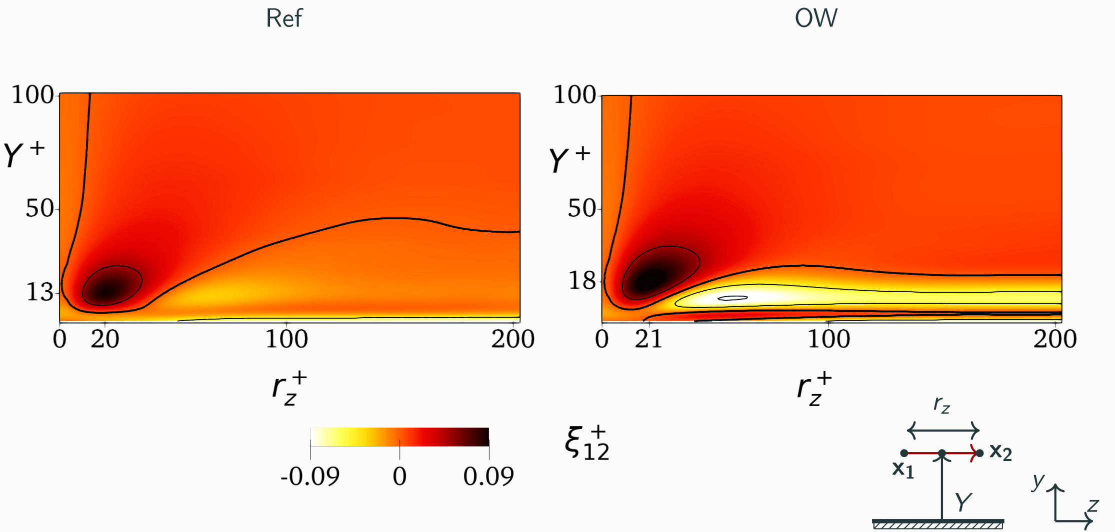
ΔY_m^+ increases with A^+ (%DR)

$\langle uu \rangle$

Vertical shift Δy of $\phi = 0$



Source of $\langle -\delta u \delta v \rangle$



Source of $\langle -\delta u \delta v \rangle$

$$\underbrace{\xi_{12}}_{\text{source}} = \underbrace{P_{12}}_{\text{production}} + \underbrace{\Pi_{12}}_{\text{pressure strain}} + \underbrace{\epsilon_{12}}_{\text{dissipation}}$$

$P_{12} > 0$ in all the domain

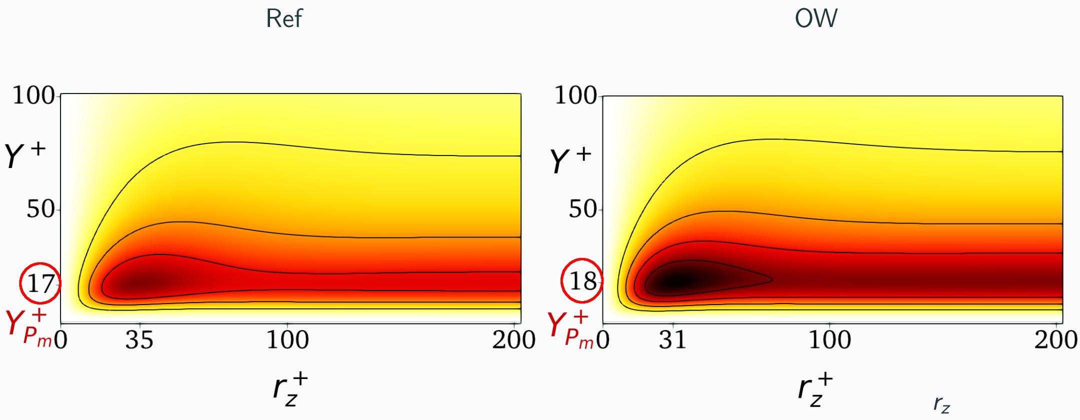
$\Pi_{12} < 0$ in (almost) all the domain

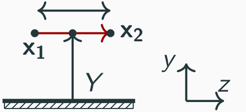
ϵ_{12} is negligible in all the domain

Source of $\langle -\delta u \delta v \rangle$

$$\underbrace{\xi_{12}}_{\text{source}} \sim \underbrace{P_{12}}_{\text{production}} + \underbrace{\Pi_{12}}_{\text{pressure strain}} + \underbrace{\epsilon_{12}}_{\text{dissipation}}$$

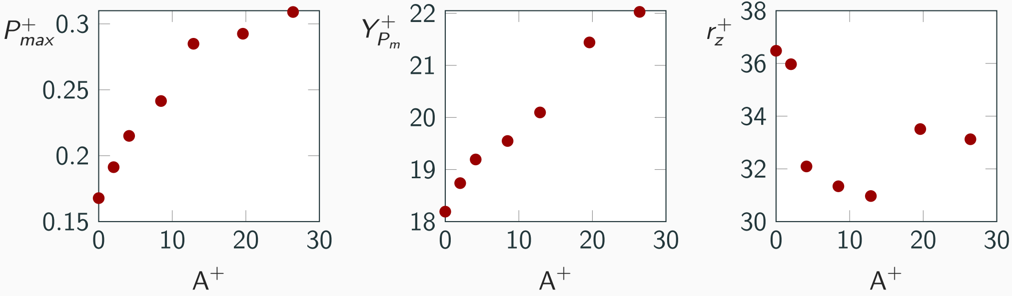
Production of $\langle -\delta u \delta v \rangle$



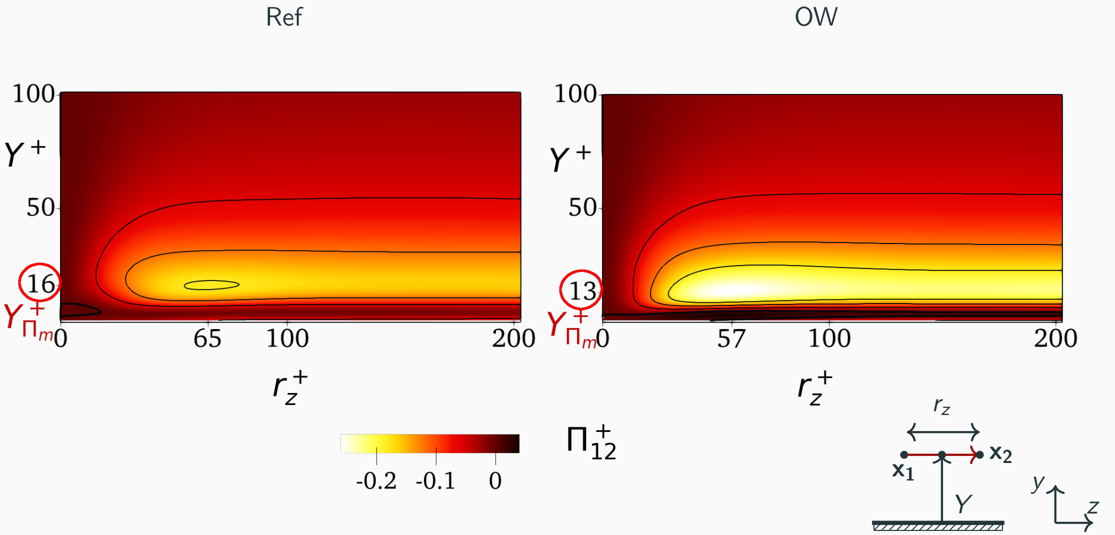


Production of $\langle -\delta u \delta v \rangle$

How does P_{max} changes with A^+?

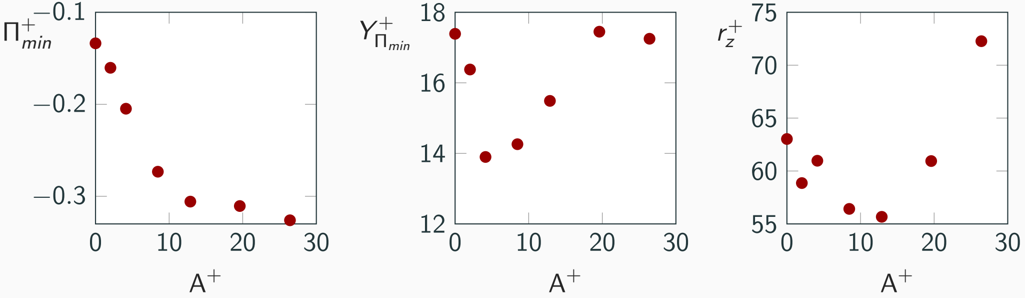


Pressure strain of $\langle -\delta u \delta v \rangle$

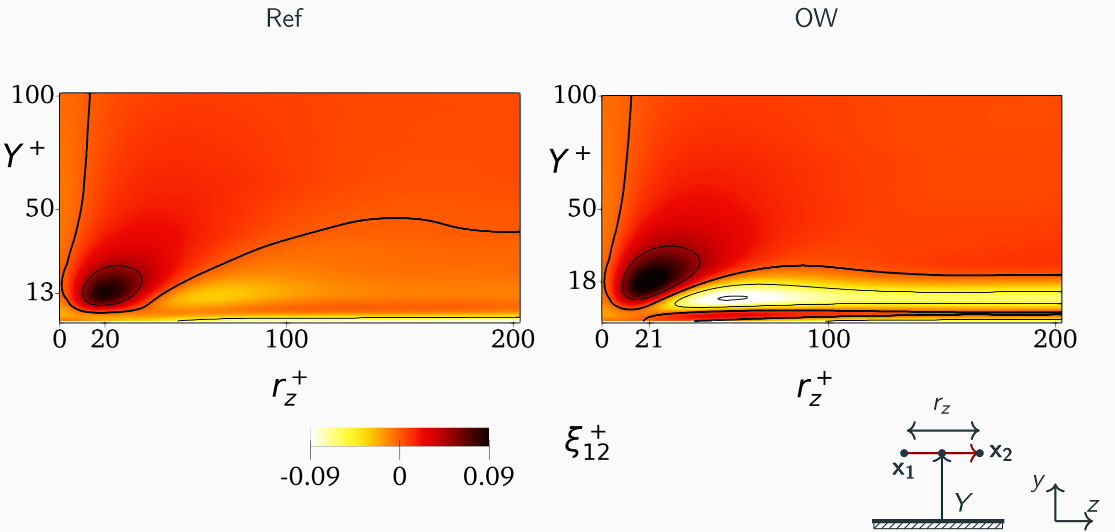


Pressure strain of $\langle -\delta u \delta v \rangle$

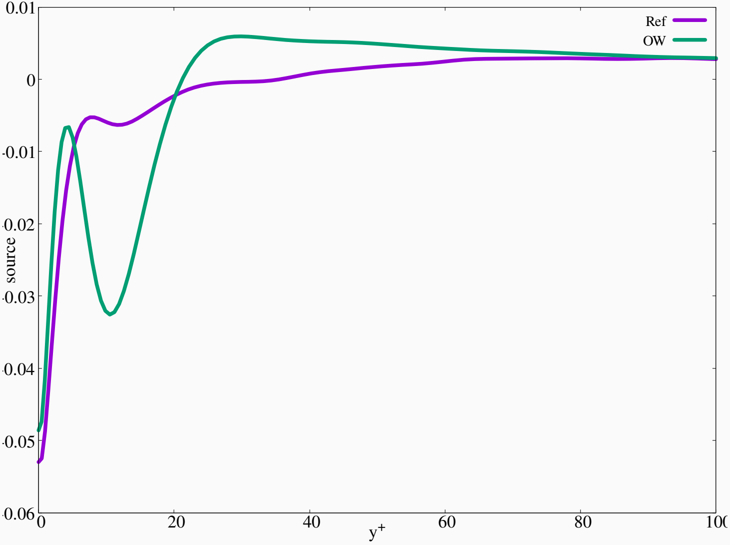
How does Π_{min} changes with A^+?



Source of $\langle -\delta u \delta v \rangle$



ξ_{-uv}



Conclusion

How do DR techniques affect

the production, transport and dissipation

of turbulent stresses

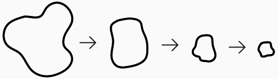
among scales and in space?

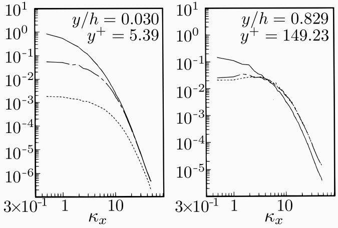
Conclusion

- $\langle \delta u \delta u \rangle$
  - The maximum production occurs at larger wall-distances
  - The largest transports are shifted towards larger wall-distances
- $\langle -\delta u \delta v \rangle$
  - The maximum of the production increases and occurs at larger wall-distances
  - The negative minimum of the pressure strain decreases

AGKE: unifies two Classic approaches to turbulence

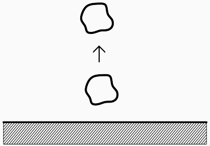
Space of scales

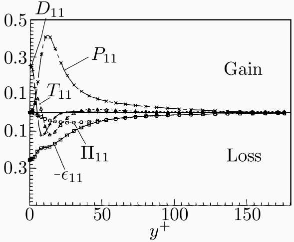




Kim *et al.* JFM 1987

Physical space

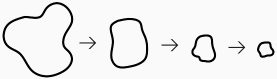




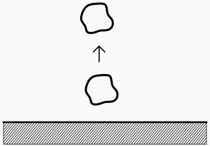
Mansour *et al.* JFM 1988

AGKE: unifies two Classic approaches to turbulence

Space of scales



Physical space

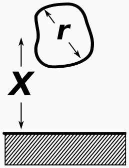


The AGKE consider together the space of scales (**r**) and the physical space (**X**)

Generalised Kolmogorov Equation

- $\langle \delta u_i \delta u_i \rangle (\mathbf{X}, \mathbf{r})$

Amount of turbulent energy
at location **X** and scale (up to) **r**



Davidson *et al.* JFM 2006

- GKE

Production, transport and dissipation
of turbulent energy
in both the
Space of scales & Physical space