Karlsruhe Institute of Technology

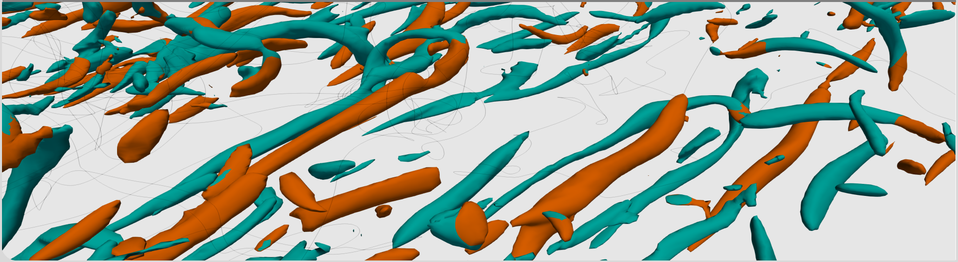
POLITECNICO MILANO 1863

UNIVERSITÀ POLITECNICA DELLE MARCHE

Production, transport and dissipation of turbulent stresses across scales and space

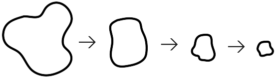
Davide Gatti, A. Chiarini, A. Cimarelli, M. Quadrio | September 6, 2018

INTERNATIONAL TURBULENCE INITIATIVE , 5 – 7 SEPTEMBER 2018, BERTINORO, ITALY

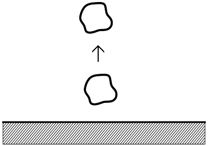


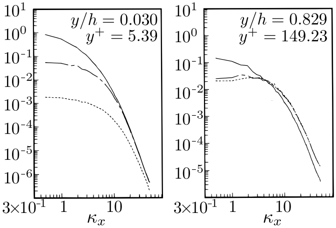
(Two) Classic approaches to turbulence

Space of scales

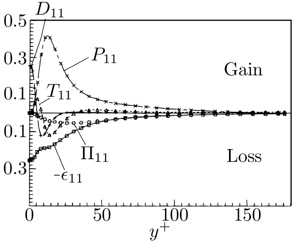


Physical space





Kim *et al.* JFM 1987



Mansour *et al.* JFM 1988

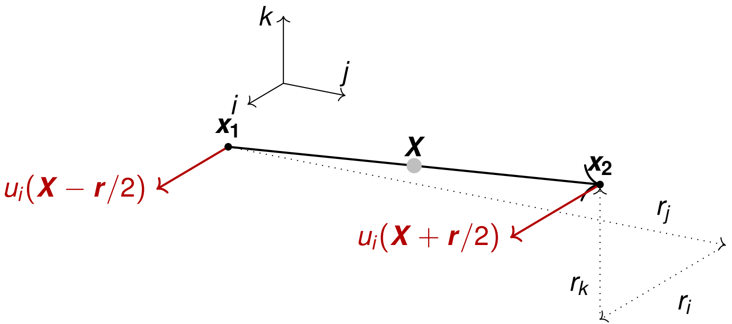
Generalized Kolmogorov Equation (GKE)

Exact budget equation for $\langle \delta u_i \delta u_i \rangle = \langle \delta u \delta u \rangle + \langle \delta v \delta v \rangle + \langle \delta w \delta w \rangle$

Generalized Kolmogorov Equation (GKE)

Exact budget equation for $\langle \delta u_i \delta u_i \rangle = \langle \delta u \delta u \rangle + \langle \delta v \delta v \rangle + \langle \delta w \delta w \rangle$

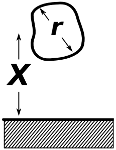
$$\delta u_i = \left(u_i \left(\boldsymbol{X} + \boldsymbol{r}/2, t \right) - u_i \left(\boldsymbol{X} - \boldsymbol{r}/2, t \right) \right)$$



dependent on:
$$\begin{cases} \boldsymbol{X} = (\boldsymbol{x_1} + \boldsymbol{x_2})/2 \\ \boldsymbol{r} = \boldsymbol{x_2} - \boldsymbol{x_1} \end{cases}$$

Generalized Kolmogorov Equation (GKE)

Amount of turbulent energy
at location $\boldsymbol{X}$ and scale (up to) $\boldsymbol{r}$
Davidson *et al.* JFM 2006



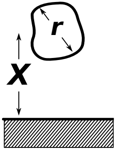
- $\langle \delta u_i \delta u_i \rangle(\boldsymbol{X}, \boldsymbol{r})$

Generalized Kolmogorov Equation (GKE)

- $\langle \delta u_i \delta u_i \rangle(\boldsymbol{X}, \boldsymbol{r})$

Amount of turbulent energy
at location $\boldsymbol{X}$ and scale (up to) $\boldsymbol{r}$

Davidson *et al.* JFM 2006



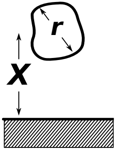
- GKE

Production, dissipation and transport
of turbulent **energy**
in both the
Space of scales & Physical space

Generalized Kolmogorov Equation (GKE)

- $\langle \delta u_i \delta u_i \rangle (\boldsymbol{X}, \boldsymbol{r})$

Amount of turbulent energy
at location ***X*** and scale (up to) ***r***
Davidson *et al.* JFM 2006



- GKE

Production, dissipation and transport
of turbulent **energy**
in both the
Space of scales & Physical space

$$\langle \delta u_i \delta u_i \rangle = \langle \delta u \delta u \rangle + \langle \delta v \delta v \rangle + \langle \delta w \delta w \rangle \rightarrow$$ **...anisotropy?**

GKE: budget for $\langle \delta u_i \delta u_i \rangle$

$$\boxed{\frac{\partial \phi_k}{\partial r_k} + \frac{\partial \psi_k}{\partial X_k} = \xi}$$

scale flux $\phi_k = \underbrace{\delta U_k \langle \delta u_i \delta u_i \rangle}_{\text{mean transport}} + \underbrace{\langle \delta u_k \delta u_i \delta u_i \rangle}_{\text{turbulent transport}} \underbrace{-2\nu \frac{\partial}{\partial r_k} \langle \delta u_i \delta u_i \rangle}_{\text{viscous diffusion}}$

space flux $\psi_k = \underbrace{\langle {u_k}^* \delta u_i \delta u_i \rangle}_{\text{turbulent transport}} + \underbrace{\frac{1}{\rho} \langle \delta p \delta u_i \rangle}_{\text{pressure transport}} \underbrace{- \frac{\nu}{2} \frac{\partial}{\partial X_k} \langle \delta u_i \delta u_i \rangle}_{\text{viscous diffusion}}$

source $\xi = \underbrace{-2 \langle {u_k}^* \delta u_i \rangle \delta \left(\frac{\partial U_i}{\partial x_k} \right)}_{\text{production}} \underbrace{-2 \langle \delta u_k \delta u_i \rangle \left(\frac{\partial U_i}{\partial x_k} \right)^*}_{\text{production}} \underbrace{-4 {\epsilon_{ii}}^*}_{\text{dissipation}}$

Anisotropic GKEs (AGKEs): budget for $\langle \delta u_i \delta u_j \rangle$

$$\boxed{\frac{\partial \phi_{k,ij}}{\partial r_k} + \frac{\partial \psi_{k,ij}}{\partial X_k} = \xi_{ij}}$$

scale flux $\quad \phi_{k,ij} = \underbrace{\delta U_k \langle \delta u_i \delta u_j \rangle}_{\text{mean transport}} + \underbrace{\langle \delta u_k \delta u_i \delta u_j \rangle}_{\text{turbulent transport}} \underbrace{-2\nu \frac{\partial}{\partial r_k} \langle \delta u_i \delta u_j \rangle}_{\text{viscous diffusion}}$

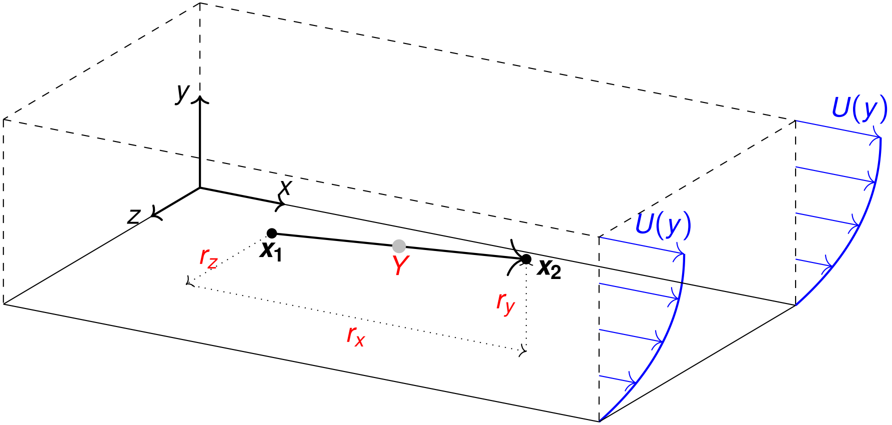
space flux $\quad \psi_{k,ij} = \underbrace{\langle u_k{}^* \delta u_i \delta u_j \rangle}_{\text{turbulent transport}} + \underbrace{\frac{1}{\rho}\langle \delta p \delta u_i \rangle \delta_{kj} + \frac{1}{\rho}\langle \delta p \delta u_j \rangle \delta_{ki}}_{\text{pressure transport}} \underbrace{-\frac{\nu}{2}\frac{\partial}{\partial X_k}\langle \delta u_i \delta u_j \rangle}_{\text{viscous diffusion}}$

source $\quad \xi_{ij} = \underbrace{-\langle u_k{}^* \delta u_j \rangle \delta\left(\frac{\partial U_i}{\partial x_k}\right) - \langle u_k{}^* \delta u_i \rangle \delta\left(\frac{\partial U_j}{\partial x_k}\right)}_{\text{production}} +$

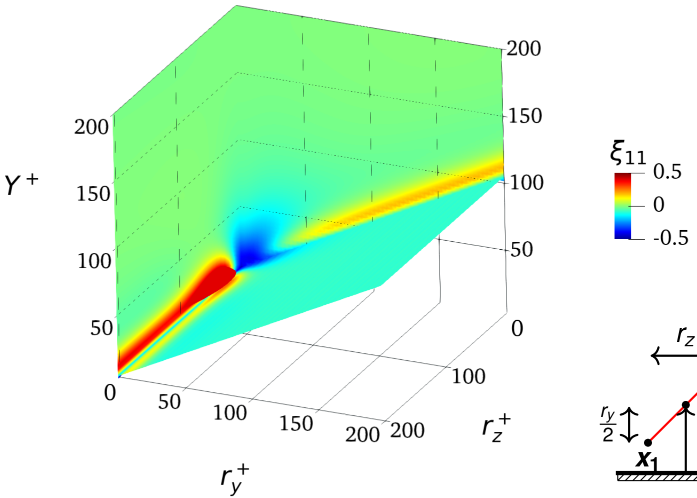
$$\underbrace{-\langle \delta u_k \delta u_j \rangle \left(\frac{\partial U_i}{\partial x_k}\right)^* - \langle \delta u_k \delta u_i \rangle \left(\frac{\partial U_j}{\partial x_k}\right)^*}_{\text{production}} \underbrace{+\frac{1}{\rho}\left\langle \delta p \frac{\partial \delta u_i}{\partial X_j}\right\rangle + \frac{1}{\rho}\left\langle \delta p \frac{\partial \delta u_j}{\partial X_i}\right\rangle}_{\text{pressure strain}} \underbrace{-4\epsilon_{ij}{}^*}_{\text{dissipation}}$$

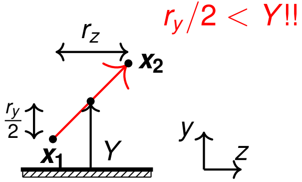
AGKEs for indefinite plane channels

$$\langle \delta u_i \delta u_j \rangle(\boldsymbol{X}, \boldsymbol{r}) \rightarrow \langle \delta u_i \delta u_j \rangle(Y, r_x, r_y, r_z)$$

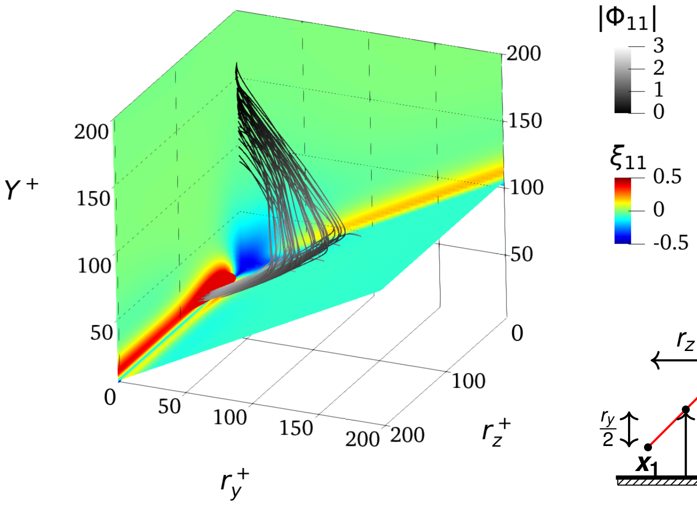


Turbulent channel ($Re_\tau = 200$): $\langle \delta u \delta u \rangle$ in $r_x = 0$ space

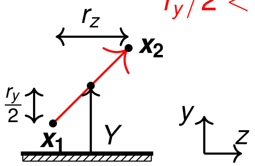




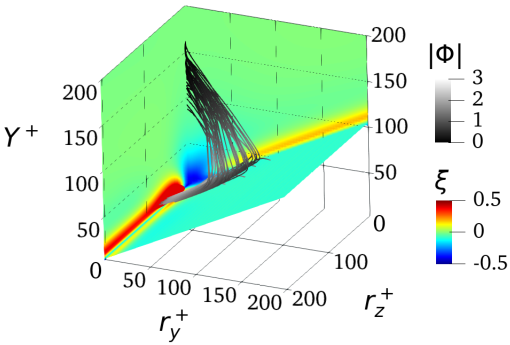
Turbulent channel ($Re_\tau = 200$): $\langle \delta u \delta u \rangle$ in $r_x = 0$ space



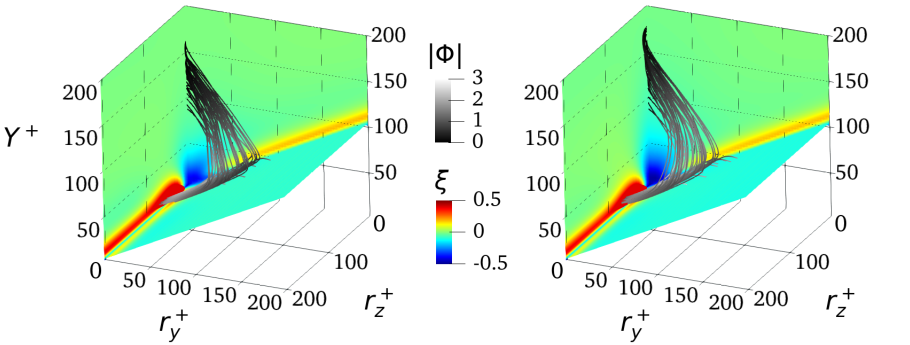
$r_y/2 < Y$!!



$\langle \delta u \delta u \rangle$ **versus** $\langle \delta u_i \delta u_i \rangle$

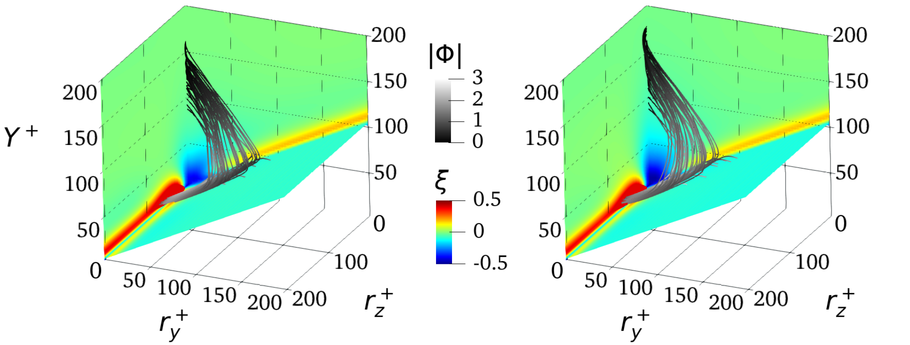


$\langle \delta u \delta u \rangle$ **versus** $\langle \delta u_i \delta u_i \rangle$



$$\langle \delta u \delta u \rangle \approx \langle \delta u_i \delta u_i \rangle$$

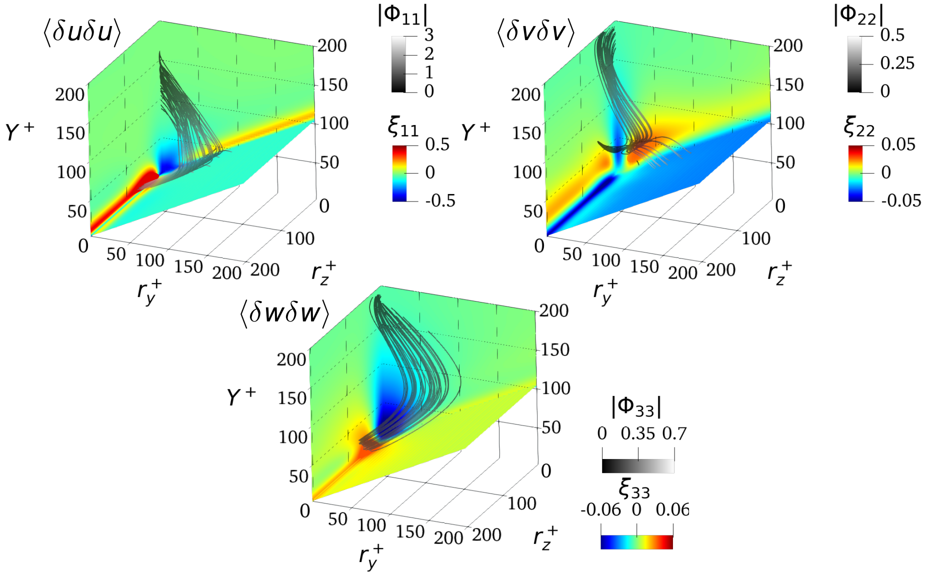
$\langle \delta u \delta u \rangle$ **versus** $\langle \delta u_i \delta u_i \rangle$



$$\langle \delta u \delta u \rangle \approx \langle \delta u_i \delta u_i \rangle$$

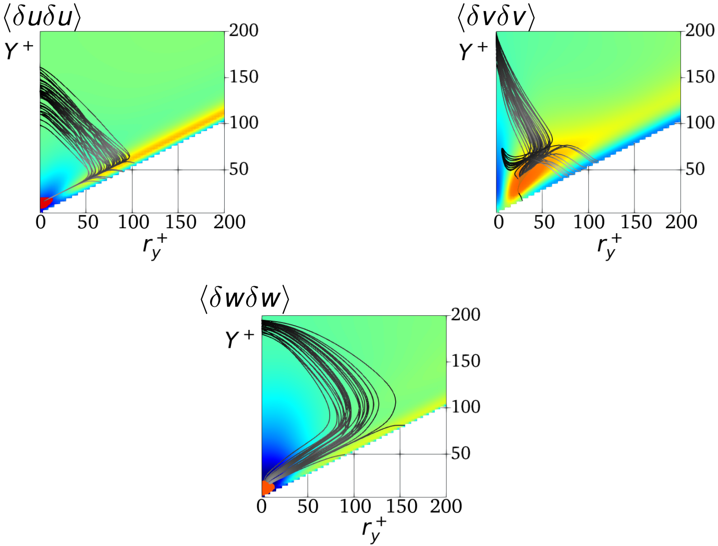
what does AGKE add to GKE?

Anisotropy

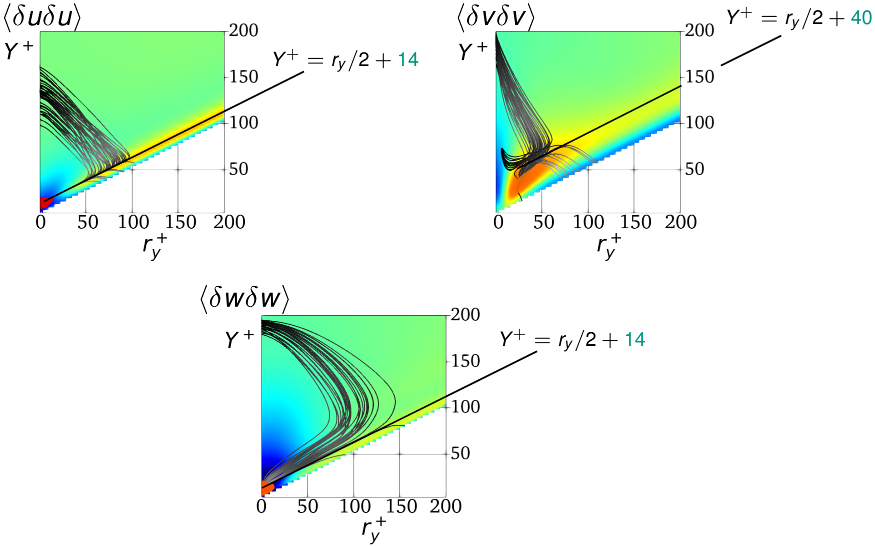


⟨δuδu⟩
|Φ11|
3
2
1
0
ξ11
0.5
0
-0.5
Y+
r_y+
r_z+
⟨δvδv⟩
|Φ22|
0.5
0.25
0
ξ22
0.05
0
-0.05
⟨δwδw⟩
|Φ33|
0 0.35 0.7
ξ33
-0.06 0 0.06

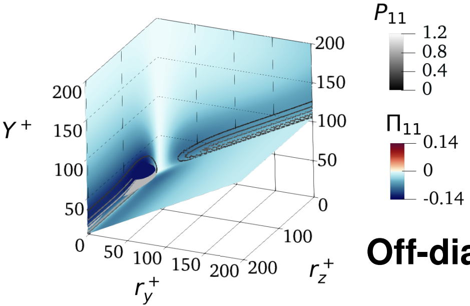
Anisotropy


$\langle \delta u \delta u \rangle$
Y^+
200
150
100
50
0
50
100
150
200
r_y^+
$\langle \delta v \delta v \rangle$
Y^+
200
150
100
50
0
50
100
150
200
r_y^+
$\langle \delta w \delta w \rangle$
Y^+
200
150
100
50
0
50
100
150
200
r_y^+

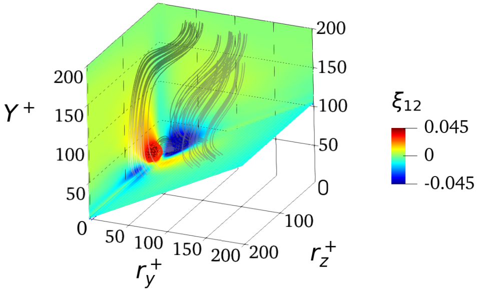
Anisotropy of attached scales

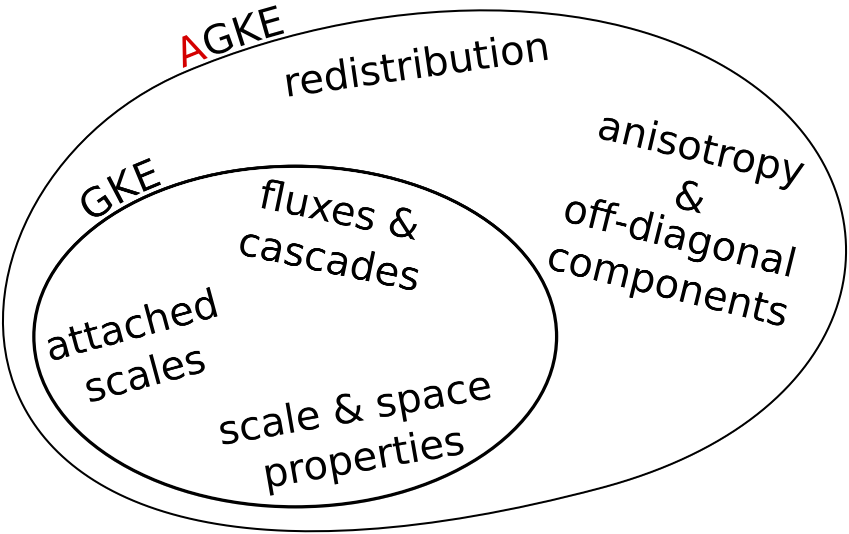


Redistribution: pressure strain



Off-diagonal component $\langle \delta u \delta v \rangle$

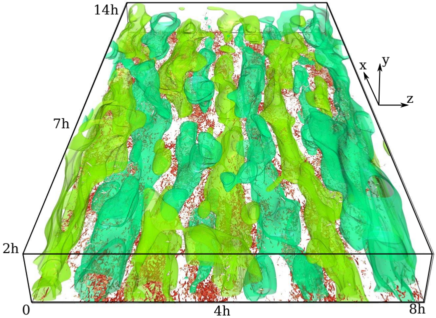



AGKE
redistribution
anisotropy
&
off-diagonal
components
GKE
fluxes &
cascades
attached
scales
scale & space
properties

How will I use the AGKE?

Role and occurrence of large scales

- Very Large Scale Motions at high *Re*

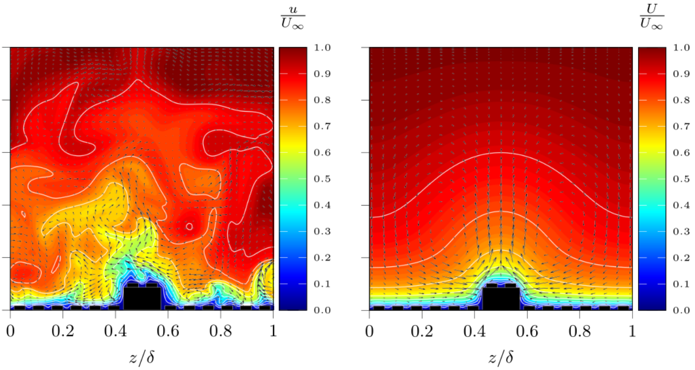


Gatti *et al.* FTaC 2018

How will I use the AGKE?

Role and occurrence of large scales

- Very Large Scale Motions at high *Re*
- Secondary Motions of Prandtl second kind



Stroh *et al.* JFM submitted

THANKS

for your kind attention!

for questions and suggestions:

davide.gatti@kit.edu
maurizio.quadrio@polimi.it
alessandro.chiarini@polimi.it
andrea.cimarelli@unimore.it

Conclusion

We presented the AGKE: exact budget equations for $\langle \delta u_i \delta u_j \rangle$

- In addition to GKE:
  - anisotropy
  - off-diagonal components
  - redistribution
- In addition to spectral Reynolds stress budgets:
  - no need for homogeneity
  - allows scales in inhomogeneous directions
  - possible “fluxes” interpretation
- probably interesting for your research too!

Conclusion

We presented the AGKE: exact budget equations for $\langle \delta u_i \delta u_j \rangle$

- In addition to GKE:
  - anisotropy
  - off-diagonal components
  - redistribution
- In addition to spectral Reynolds stress budgets:
  - no need for homogeneity
  - allows scales in inhomogeneous directions
  - possible “fluxes” interpretation
- probably interesting for your research too!

Conclusion

We presented the AGKE: exact budget equations for $\langle \delta u_i \delta u_j \rangle$

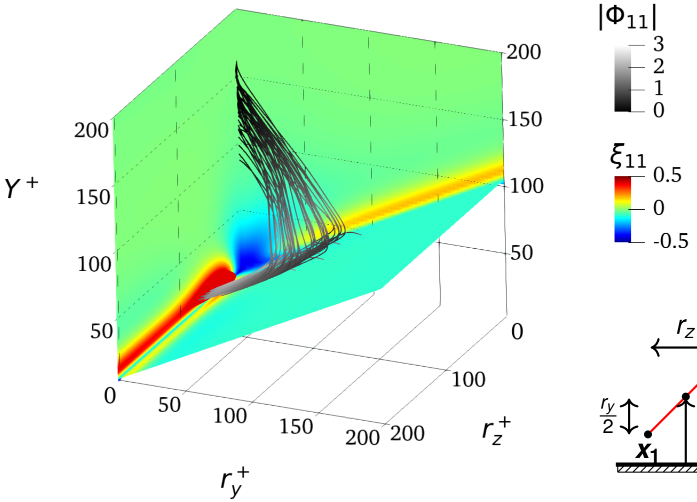
- In addition to GKE:
  - anisotropy
  - off-diagonal components
  - redistribution
- In addition to spectral Reynolds stress budgets:
  - no need for homogeneity
  - allows scales in inhomogeneous directions
  - possible “fluxes” interpretation
- probably interesting for your research too!

Conclusion

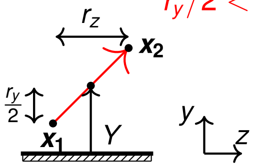
We presented the AGKE: exact budget equations for $\langle \delta u_i \delta u_j \rangle$

- In addition to GKE:
  - anisotropy
  - off-diagonal components
  - redistribution
- In addition to spectral Reynolds stress budgets:
  - no need for homogeneity
  - allows scales in inhomogeneous directions
  - possible “fluxes” interpretation
- probably interesting for your research too!

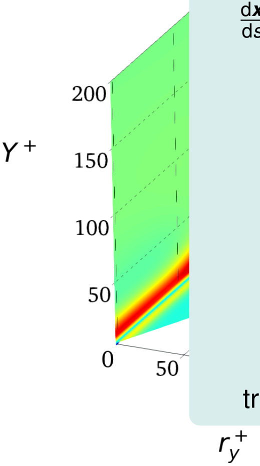
Fluxes, field lines



$r_y/2 < Y!!$

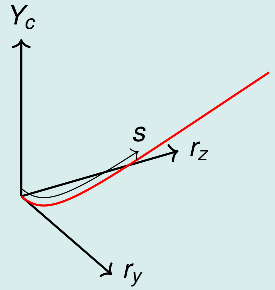


Fluxes, field lines

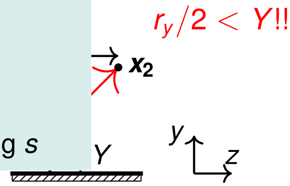


Field lines

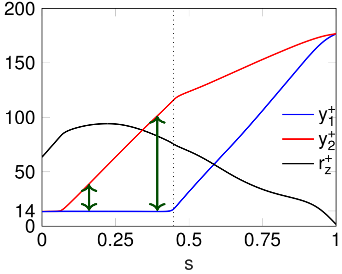
$$\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}s} = \phi_{i,j} \qquad \text{where } \boldsymbol{x} = (Y_c, r_y, r_z)$$

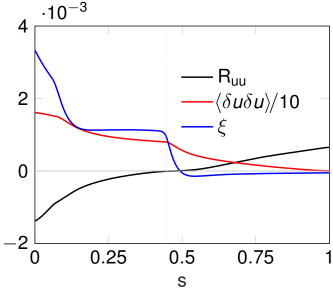


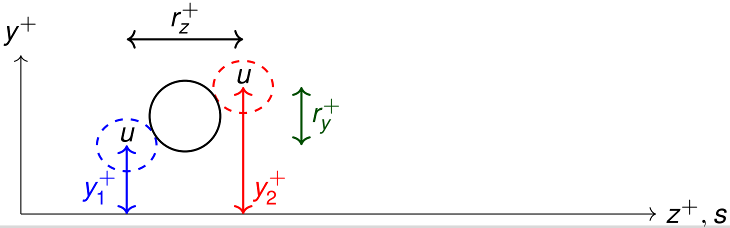
track AGKE terms and R_{ij} along s



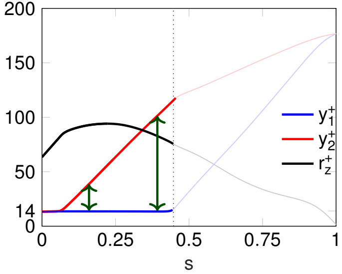
Fluxes, field lines: attached & detached scales

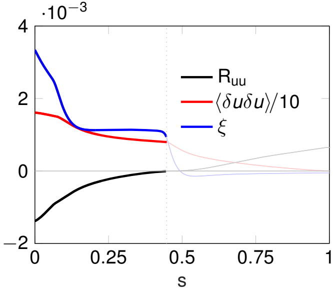


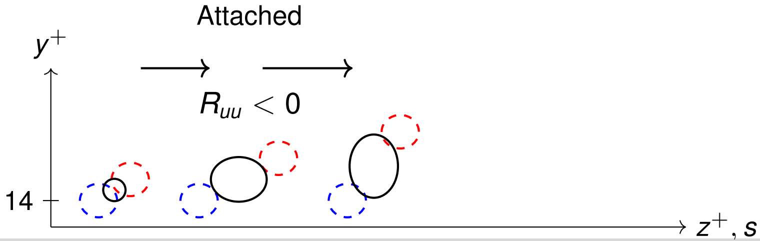




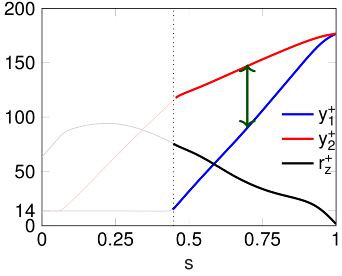
Fluxes, field lines: attached & detached scales

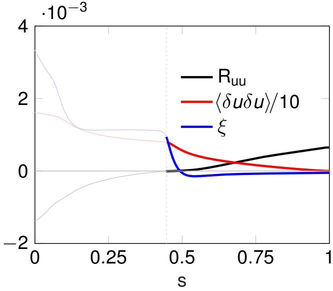


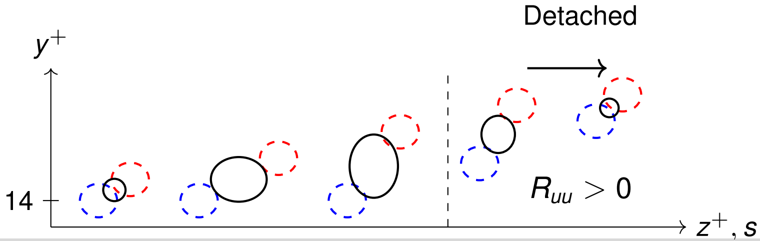




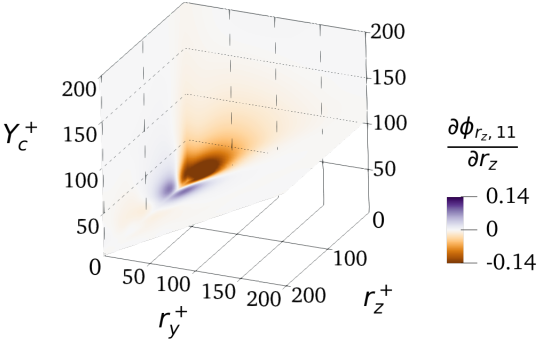
Fluxes, field lines: attached & detached scales







Fluxes, divergence: donor & receiver scales



contribution of various physical processes to $\langle \delta u_i \delta u_j \rangle$
(e.g. nonlinear turbulent transport)

$\langle \delta u_i \delta u_j \rangle$: relationship with correlation $R_i j$

$$\langle \delta u_i \delta u_j \rangle (Y_c, r_x, r_y, r_z) = \underbrace{\langle u_i u_j \rangle|_{y_1} + \langle u_i u_j \rangle|_{y_2}}_{sum\ of\ variances} \underbrace{-2 R_{u_i|_{y_1} u_j|_{y_2}}(r_x, r_z)}_{Cross-correlation}$$

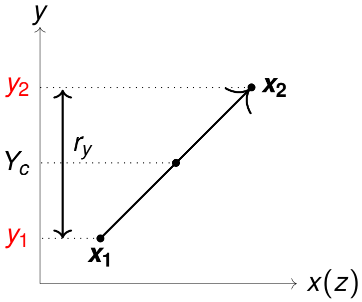
$\langle \delta u_i \delta u_j \rangle$: relationship with correlation $R_i j$

$$\langle \delta u_i \delta u_j \rangle(Y_c, r_x, r_y, r_z) = \underbrace{\langle u_i u_j \rangle|_{y_1} + \langle u_i u_j \rangle|_{y_2}}_{sum\ of\ variances} - \underbrace{2R_{u_i|_{y_1} u_j|_{y_2}}(r_x, r_z)}_{Cross-correlation}$$

$$y_1 = Y_c - r_y/2$$

$$y_2 = Y_c + r_y/2$$

$$(Y_c, r_x, r_y, r_z) \leftrightarrow (y_1, y_2, r_x, r_z)$$



$\langle \delta u_i \delta u_j \rangle$: relationship with correlation R_ij

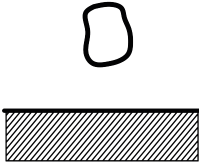
$$\langle \delta u_i \delta u_j \rangle (Y_c, r_x, r_y, r_z) = \underbrace{\langle u_i u_j \rangle|_{y_1} + \langle u_i u_j \rangle|_{y_2}}_{sum\ of\ variances} - \underbrace{2 R_{u_i|_{y_1} u_j|_{y_2}} (r_x, r_z)}_{Cross-correlation}$$

$$\langle \delta u_i \delta u_j \rangle (Y_c, r_x, r_y, r_z) \neq \langle u_i u_j \rangle|_{y_1} + \langle u_i u_j \rangle|_{y_2}$$

$$\downarrow$$

$$R_{u_i|_{y_1} u_j|_{y_2}} (r_x, r_z) \neq 0$$

Coherent structures!

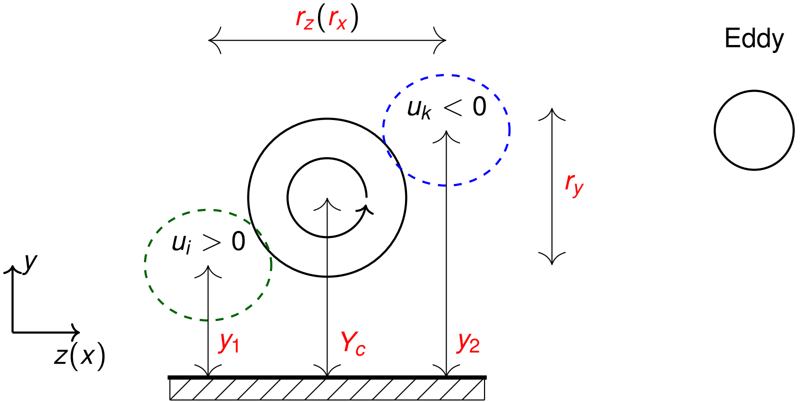


$\langle \delta u_i \delta u_j \rangle$: relationship with correlation R_ij

$$\langle \delta u \delta u \rangle (Y_c, 0, r_y, r_z) > \underbrace{\langle uu \rangle|_{y_1} + \langle uu \rangle|_{y_2}}_{sum\ of\ variances} \underbrace{-2R_{u|_{y_1} u|_{y_2}}(0, r_z)}_{Cross-correlation}$$

$\langle \delta u_i \delta u_j \rangle$: relationship with correlation $R_i j$

$$\langle \delta u \delta u \rangle (Y_c, 0, r_y, r_z) = \underbrace{\langle uu \rangle|_{y_1} + \langle uu \rangle|_{y_2}}_{sum\ of\ variances} - \underbrace{2R_{u|_{y_1} u|_{y_2}}(0, r_z)}_{Cross-correlation} < 0$$



$\langle \delta u_i \delta u_j \rangle$: relationship with correlation R_ij

$$\langle \delta u_i \delta u_j \rangle (Y_c, r_x, r_y, r_z) \leftrightarrow \underbrace{\langle u_i u_j \rangle|_{y_1} + \langle u_i u_j \rangle|_{y_2}}_{\text{sum of variances}} \underbrace{-2 R_{u_i|_{y_1} u_j|_{y_2}}(r_x, r_z)}_{\text{Cross-correlation}}$$