Impulse Response in Turbulent Channel Flow

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Motivation

Impulse Response Features

It describes the Input-Output relationship of a dynamic system.
- Perturbation propagation
- Flow control application (plasma actuators)
- Insights for development and testing of turbulent models

Background

M. Jovanović and B. Bamieh, *Componentwise energy amplification in channel flows* - J. Fluid Mech., 2004

Impulse response for linearized laminar channel flow

Goal

Extend Jovanović’s work and provide the impulse response in the turbulent case

1) S. Russo, P. Luchini - J. Fluid Mech., 2016
Description

Impulse response to volume force $H_{ij}(k_x, y, k_z, \omega)$

- linearized laminar base flow
- results averaged in the wall-normal direction
- forcing uniformly applied among the channel height

Current work

Impulse response to volume force $H_{ij}(x, y, z, t; y_f)$

- turbulent base flow (DNS)
- physical space and time evolution
- influence of the wall-normal distance of the forcing $y_f$
Plasma Actuators

- Embedded electrode
- Dielectric
- Plasma induced flow
- DBD configuration

Introduction
Measurement technique
Validation
Impulse Response
Conclusion

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Impulse Response

1D definition:

\[ u(t) = \int H(t - t') f(t) dt' \]

Impulse response \( H \) (fluid dynamics)

Relationship between the body forcing input \( f(x, t) \) and the velocity output \( u(x, t) \):

\[ u_i(x, t) = \int H_{ij}(x - x', t - t') f_j(x', t) dx' dt' \]
Impulse Response Measurement

Three possible techniques:

**Impulse Response**
- ✔ easy implementation
- ✗ linear response $\Rightarrow$ small perturbation $\Rightarrow$ small S/N ratio

**Frequency Response**
- ✔ distributed force
- ✗ only one space-time frequency at once

**Input-Output correlation**
- ✔ tested for the wall blowing/suction input
- ✔ more homogeneous force distribution, all time-space frequency at once

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2) P. Luchini, M. Quadrio, S. Zuccher - Phys. Fluids, 2006
Impulse Response Measurement

Input-Output correlation\(^1\) $\mathcal{R}_{\text{in},\text{out}}$

\[
\mathcal{R}_{\text{in, in}} \xrightarrow{\mathcal{H}} \mathcal{R}_{\text{in, out}}
\]

\[
\mathcal{R}_{\text{in, out}}(t) = \int \mathcal{H}(t - \tau) \mathcal{R}_{\text{in, in}}(\tau) d\tau
\]

White noise input:

\[
\mathcal{R}_{\text{in, in}}(\tau) = \delta(\tau)
\]

\[
\Rightarrow \mathcal{R}_{\text{in, out}}(\tau) = \mathcal{H}(\tau)
\]

1) P. Luchini, M. Quadrio, S. Zuccher - Phys. Fluids, 2006
3D Mean Impulse Response

DNS of Turbulent channel flow at $Re = 150$

Volume force applied at a certain wall normal distance $y_f$

$$f_j(\alpha, y, \beta, t) = \epsilon f_j(\alpha, \beta, t)\delta(y - y_f)$$

Measurement formulation

$$\mathcal{H}_{ij}(\alpha, y, \beta, T; y_f) = \frac{\langle u_i(\alpha, y, \beta, t)f_j^*(\alpha, \beta, t - T)\rangle}{\epsilon^2}$$

- 4+1 variables describe the impulse response
- $\mathcal{H}_{ij}$ is a 3x3 tensor
- phase-locked averaged (mean) impulse response
Results from Jovanović’s work

$H_2$ norm: ensemble average energy density

$$\forall y_f \quad \| \mathcal{H} \|_2^2 \equiv \int_0^H \int_0^\infty \| \mathcal{H}(\alpha, y, \beta, t) \|_{HS}^2 dt dy$$

uniform forcing across the height

M. Jovanovic, B. Bamieh - J. Fluid Mech., 2004
Response component $\mathcal{H}_{ux}$ with laminar flow at $Re_P = 2000$.

Channel resolution:

$L_x = 4\pi H, \ L_z = 2\pi H, \ 128x100x128$

Response resolution:

$64x100x64$

with 100 time step from $\tilde{t} = 0$ to $\tilde{t} = 100$
Validation

Linearity test

\[ f_j(\alpha, y, \beta, t) = \epsilon f_j(\alpha, \beta, t) \delta(y - y_f) \]

Forcing distance: \( y_f = 0.1H \)
$H_2$ Norm, Laminar case

\begin{align*}
\mathcal{H}_{ux} & \quad \mathcal{H}_{uy} & \quad \mathcal{H}_{uz} \\
\alpha & \quad \beta & \\
1 & 1 & 1 \\
16 & 32 & 32
\end{align*}

\begin{align*}
\mathcal{H}_{vx} & \quad \mathcal{H}_{vy} & \quad \mathcal{H}_{vz} \\
\alpha & \quad \beta & \\
1 & 1 & 1 \\
16 & 32 & 32
\end{align*}

\begin{align*}
\mathcal{H}_{wx} & \quad \mathcal{H}_{wy} & \quad \mathcal{H}_{wz} \\
\alpha & \quad \beta & \\
1 & 1 & 1 \\
16 & 32 & 32
\end{align*}

$Re_p = 2000$

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Influence of the forcing distance $y_f$

Response component $\mathcal{H}_{uz}$

$y_f = 0.01h$

$y_f = 0.1h$

$y_f = 0.2h$

$y_f = 0.3h$

$y_f = 0.4h$

$y_f = 0.5h$

$Re_P = 2000$

$\alpha$

$\beta$

$-12.7$ $-8.6$ $-4.4$
$H_2$ Norm, Turbulent case

$\mathcal{H}_{ux}$  

$\mathcal{H}_{uy}$  

$\mathcal{H}_{uz}$  

$\mathcal{H}_{vx}$  

$\mathcal{H}_{vy}$  

$\mathcal{H}_{vz}$  

$\mathcal{H}_{wx}$  

$\mathcal{H}_{wy}$  

$\mathcal{H}_{wz}$  

Re$_\tau$ = 150

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Maxima of $\mathcal{H}_{ij}$ vs. forcing distance $y_f$

(- - -) Laminar, (---) Turbulent
Conclusion

- Successful validation of new response measurement technique.
- First turbulent characterization almost done (just averaging).
- Analysis of the $\|\mathcal{H}\|_2$ show that $\mathcal{H}_{uy}$ and $\mathcal{H}_{uz}$ are the most influent components.
- Influence of the forcing wall-normal distance.

Outlook

- Further averaging turbulent simulations.
- Response measurements at higher Reynolds numbers.
Thank you for your attention!

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