

MEAN IMPULSE RESPONSE IN A TURBULENT CHANNEL FLOW

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INTRODUCTION

The linear response of a turbulent channel flow to an impulsive body force is measured through direct numerical simulations. This mathematical object can be used for many different purposes, such as the design or optimization of flow control techniques acting through body forces like plasma actuators [1, 2], the understanding of how perturbations propagate in turbulent flows [3], or the assessment of turbulence models for the closure of the Reynolds-averaged Navier–Stokes equation [4].

Focusing on flow control applications, the impulse response is an effective tool to perform a sort of sensitivity analysis, and determine in a statistical sense *where* and *in which direction* a localized volume force exerts its maximal influence in a turbulent flow. Moreover the time evolution of the response can also describe in detail *where* and *at which delay* the effects of the volume force are bound to affect the flow.

In this work we provide for the first time a detailed and complete spatio-temporal description of the response of both laminar and turbulent flows to an impulsive (in space and time) volume force. Particular attention is given to the wall-normal distance at which the body force is applied, which is an important parameter in flow control applications based on plasma actuators.

RESPONSE MEASUREMENT

Even though the Navier–Stokes equations are non-linear, a linear response of a flow to external perturbations can be defined, provided the perturbation has vanishing amplitude. Under linearity hypothesis, the response of a flow is described through an unique mathematical object which relates the whole velocity field to the input forcing. The response is a function of the streamwise, spanwise and temporal separation, as well as of the wall-normal distance from the position y_f where forcing is applied in the form of a Dirac delta function. The response has nine independent components \mathcal{H}_{ij} resulting from the influence analysis of every forcing component f_j on the velocity component u_i .

We measure the response via Direct Numerical Simulation (DNS). The measurement is particularly difficult in turbulence; first of all, it becomes meaningful only in a statistical sense; moreover, since the turbulent fluctuations act as noise and overwhelm the small (to enforce linearity) external forcing, measuring the true mean response requires unpractically long averaging times.

Taking advantages of the pseudo-spectral formulation of

the DNS solver, the impulse response \mathcal{H}_{ij} can be considered as follows:

$$u_i(\alpha, y, \beta, t; y_f) = \int \mathcal{H}_{ij}(\alpha, y - y_f, \beta, t - \tau) f_j(\alpha, \beta, t) d\tau$$

where α and β are the wavenumbers in the homogeneous directions, t is time, y is the wall-normal coordinate and y_f is the wall-normal location where the forcing is applied.

In the present work, the measurement is carried out by extending the approach introduced by Luchini *et al.* [5] for the response to wall-based perturbations. A zero-mean white-noise volume forcing is used to probe the turbulent flow, and the impulse response function is obtained by accumulating the space-time correlation between the white forcing and the whole velocity field. In this way the permitted forcing amplitude is larger, and the averaging time needed for the deterministic part of the response to emerge becomes manageable. We note that the impulsive body force is introduced as a Dirac delta function in the wall-normal direction, so that the influence of wall-normal forcing distance y_f for each point of the discretization is studied independently.

The DNS solver is that described by Luchini and Quadrio in [6], and two main simulation campaigns in the geometry of the plane channel flow have been carried out, one for the laminar case with prescribed Poiseuille profile at the Reynolds value $Re_P = 2000$ and one for the turbulent case at $Re_\tau = 150$. Length and width of the computational domain are respectively $4\pi h$ and $2\pi h$, where h is half the channel width. The spatial resolution (before de-aliasing) is $64 \times 100 \times 32$ for the laminar case and $192 \times 150 \times 96$ for the turbulent one. The impulse response is observed on a subset of wavenumbers in the homogeneous directions. This helps, jointly with the implementation of a non uniform time sampling of the input-output correlation, reducing the storage space requirements, which in the end are about 300 GB for each one of the two cases.

The averaging time required to reach convergence of the mean input-output correlation is estimated to be 10^3 viscous time units for the laminar case while, for the turbulent case, more than 10^4 time units are required to extract the mean deterministic response from the turbulent noise.

VALIDATION

The linear response of the laminar channel flow has been previously analytically computed within a linearized approach by Jovanović and Bamieh in [3] for the Reynolds value of $Re_P = 2000$. Their approach is somewhat less informative,

as details regarding wall-normal distances and time are not available and a single forcing position y_f cannot be isolated. However, for validation purposes it is possible to integrate our measurements and compare with their analytical result.

As an example, figure 1 shows the H_2 norm for the response component \mathcal{H}_{12} presented in [3] and the one computed with the approach employed in the present work, after proper integration in y and t . The two approaches show a good quantitative agreement, with the small irregularities in the lower figure simply due to the short averaging time and the interpolation of the discrete data onto a Cartesian grid for plotting.

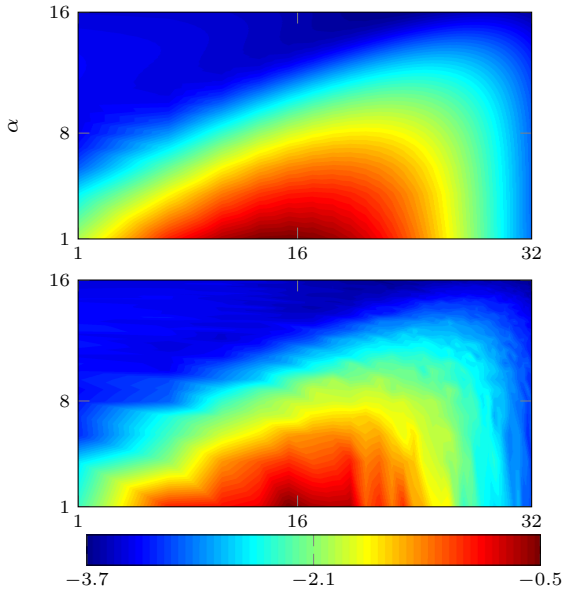


Figure 1: Comparison of the H_2 norm of \mathcal{H}_{12} for the laminar simulation at $Re_p = 2000$. Top: analytical solution after [3]; bottom: present results, after proper integration in wall-normal direction and time.

RESULTS

The impulse response in the channel flow presents a rich phenomenology that will be discussed at the meeting. Thanks to the innovative measurement technique used in this work, the analysis can be performed both in the space domain and in the wavenumber space. Both approaches provide valuable information about which forcing component has the largest impact on which velocity component.

For example, figure 2 shows how \mathcal{H}_{ij} for two of the nine components changes with y_f . In particular, it can be observed that for some components such as \mathcal{H}_{11} the body forcing has an influence which remains almost constant with respect to y_f . On the contrary, other components such as \mathcal{H}_{22} show an increasing effect of the body forcing with increasing values of y_f . Such analysis could provide useful insight in the design of plasma actuators for flow control applications. The study of the maximal values of the impulse response with respect to the time can give valuable information about decay time of the volume perturbation and also about non linear phenomena like transient growth, as found also in [3].

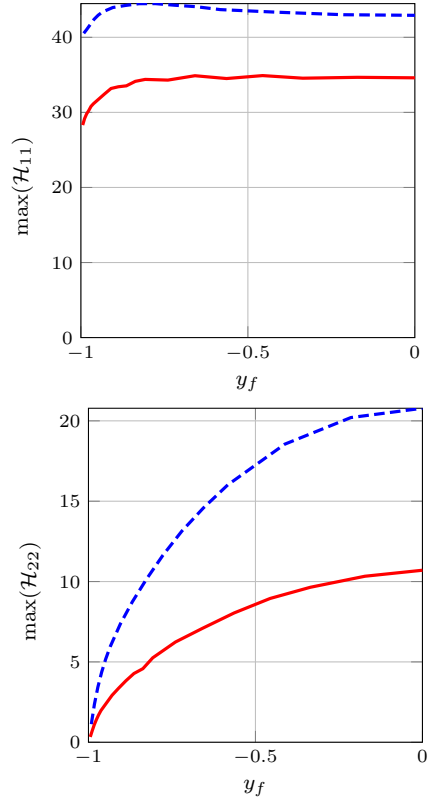


Figure 2: Maximum of the impulse response \mathcal{H}_{ij} in the physical space with respect to different forcing distances y_f . (---) laminar, (—) turbulent. Top: \mathcal{H}_{11} , bottom: \mathcal{H}_{22}

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