



Impact of Drag Reduction Control on Energy Box of a Fully Developed Turbulent Channel Flow

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Background

- A Fully Developed Channel Flow
 - ✓ Essential physics of near-wall turbulence
 - ✓ Flow control (**drag reduction**, mixing enhancement, etc.)
- Global Energy Budget

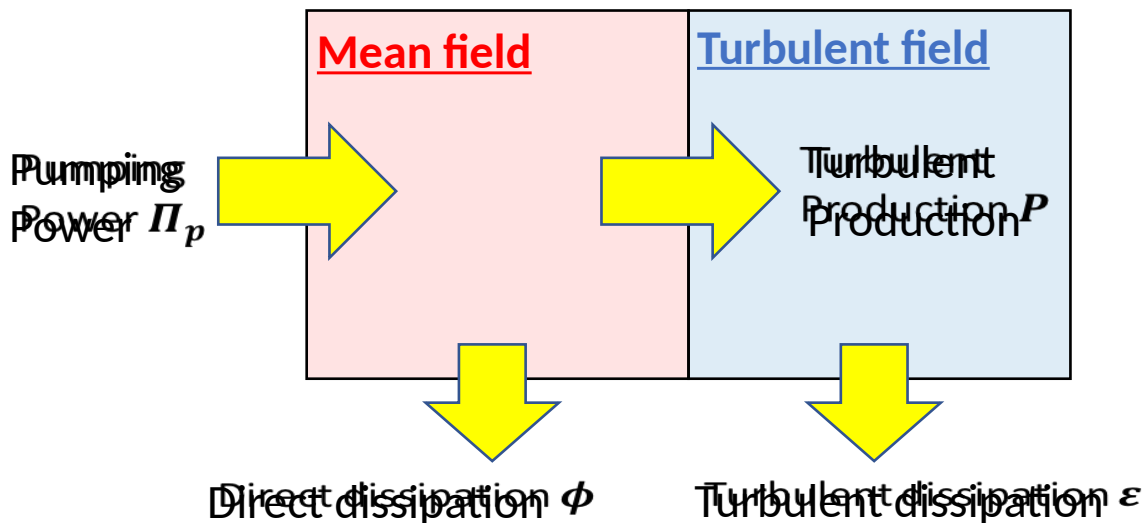
$$\text{Total kinetic energy} = \underbrace{\frac{1}{2} \bar{u}^2}_{\text{Mean}} + \underbrace{\frac{1}{2} \overline{u'_i u'_i}}_{\text{Turbulent}}$$

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Energy Box



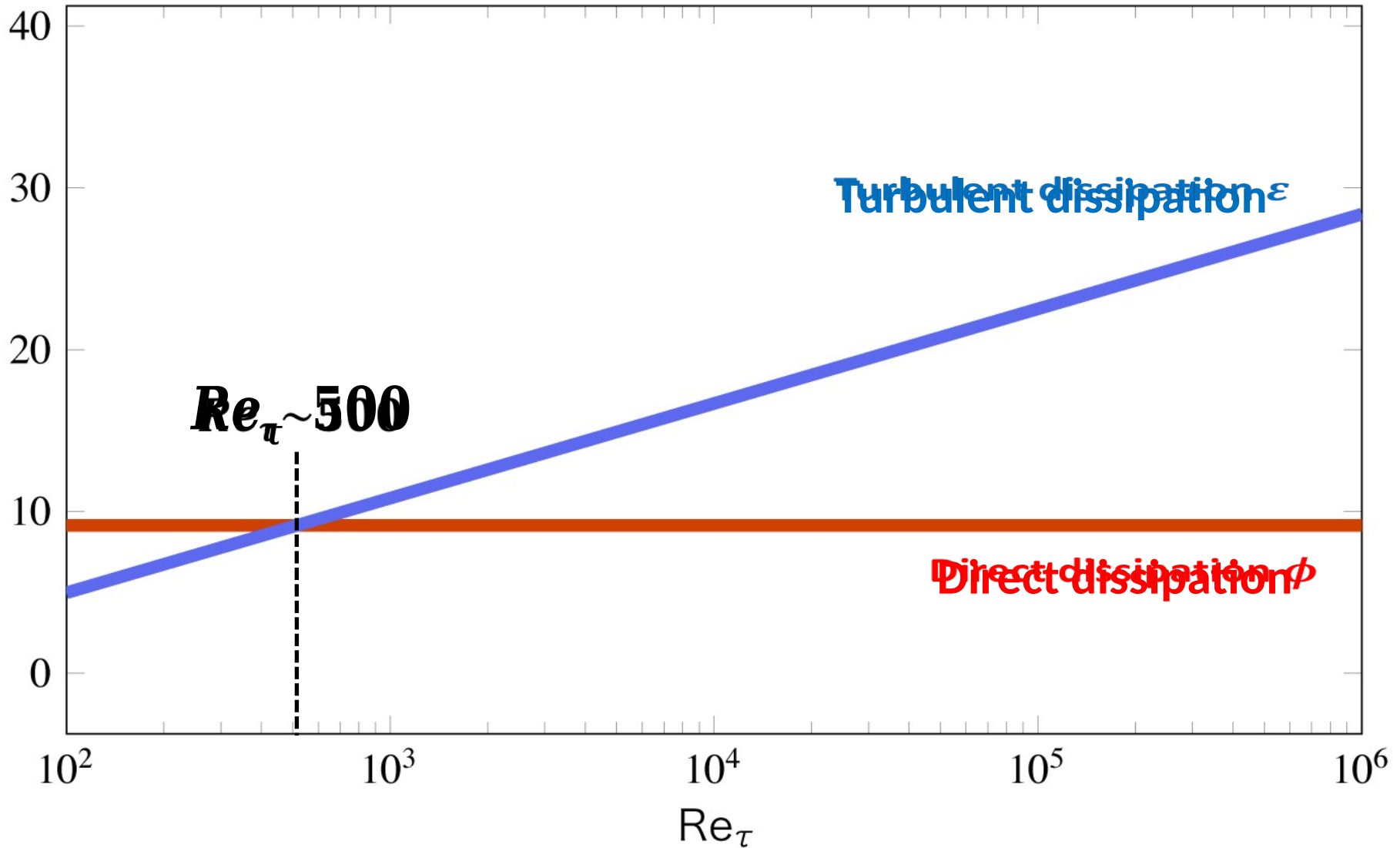
Direct dissipation (diss. of mean field)

$$\phi = \frac{1}{Re} \left(\frac{d\bar{u}}{dy} \right)^2$$

Turbulent dissipation

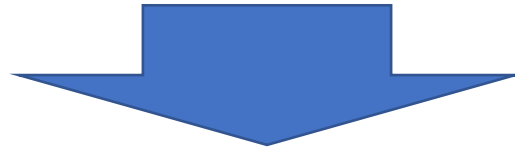
$$\varepsilon = \frac{11}{Re} \left(\frac{du''_i}{dx_{jj}} \frac{du'_i}{dx_j} \right)^2$$

Direct and Turbulent Dissipation in Uncontrolled Flow



Objectives

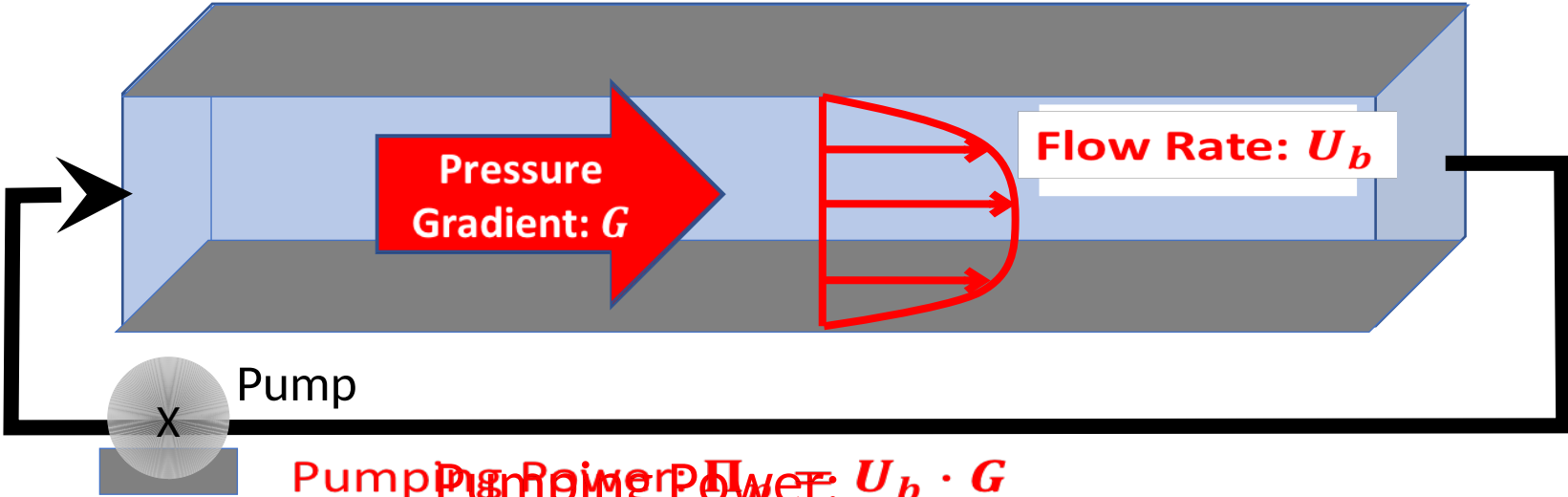
- Drag reduction control is somewhat similar to make the “effective” Re lower
 - Then, **the direct dissipation increases**, while **the turbulent dissipation decreases** ?
- Ultimate control: complete relaminarization
 - All input energy should be dissipated by **the direct dissipation** only.



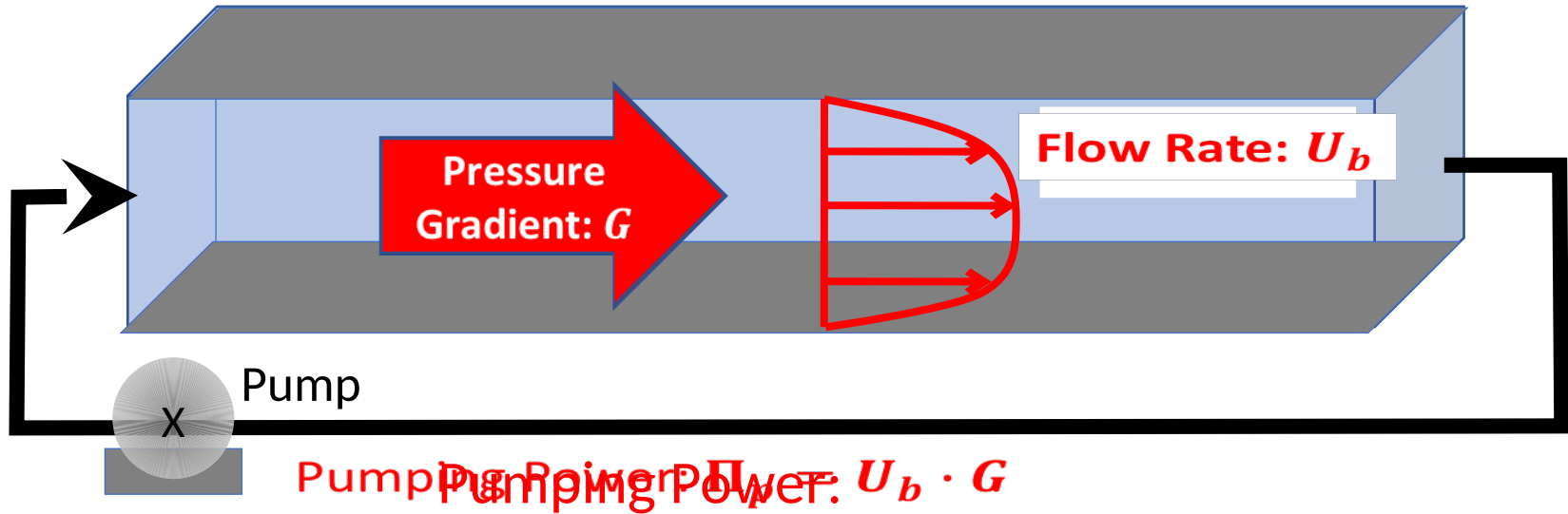
Questions

- ✓ Is there an unique relationship between the changes in direct/turbulent dissipation and a drag reduction effect ?

Flow Conditions



Flow Conditions

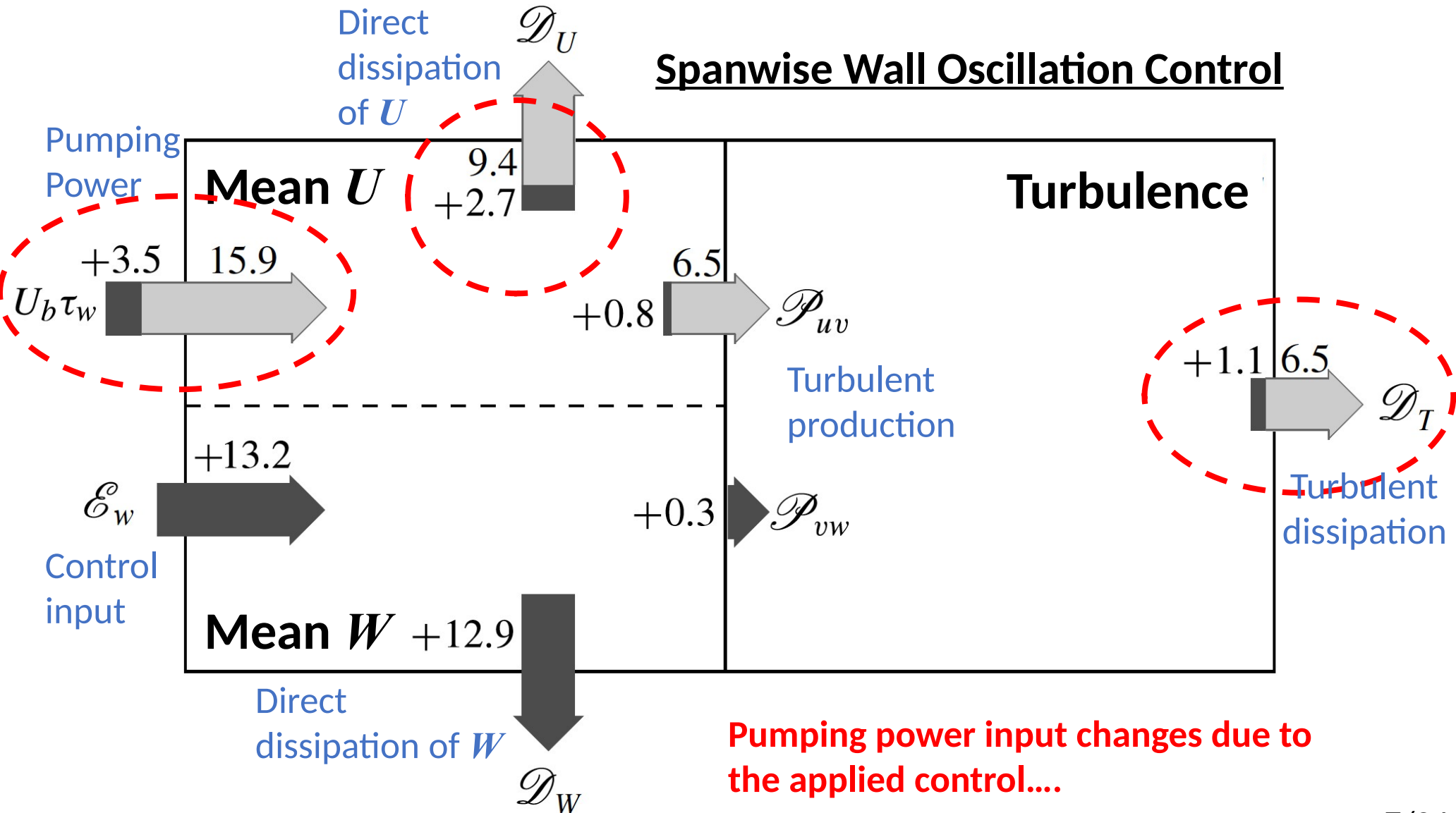


	Flow Rate: U_b	Pressure Gradient: G	Pumping Power: Π_p	Flow Rate: U_b	Pressure Gradient: G	Pumping Power: Π_p	Flow Rate: U_b	Pressure Gradient: G	Pumping Power: Π_p
Constant Flow Rate (CFR)	Constant			Constant Flow Rate (CFR)	Constant		Constant Flow Rate (CFR)	Constant	
Constant Pressure Gradient (CPG)		Constant		Constant Pressure Gradient (CPG)		Constant	Constant Pressure Gradient (CPG)		Constant
Constant Flow Rate (CFR)	Constant								
Constant Pressure Gradient (CPG)					Constant				

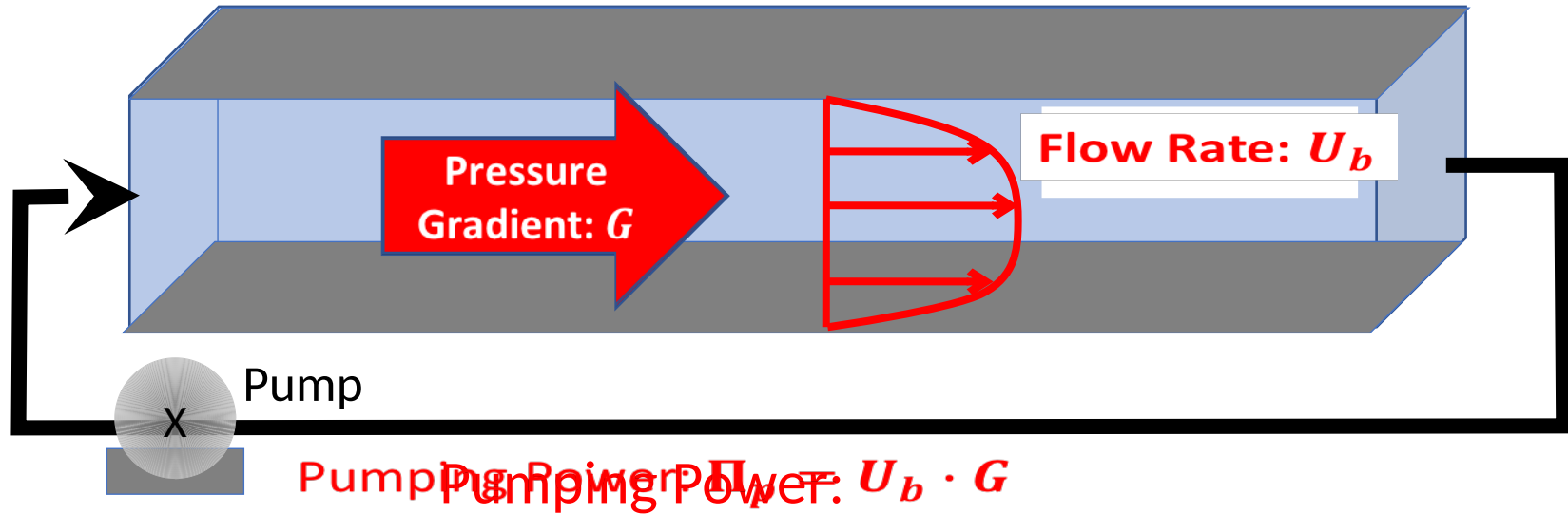
Energy Box Under CPG

Ricco et al. JFM (2011)

Spanwise Wall Oscillation Control



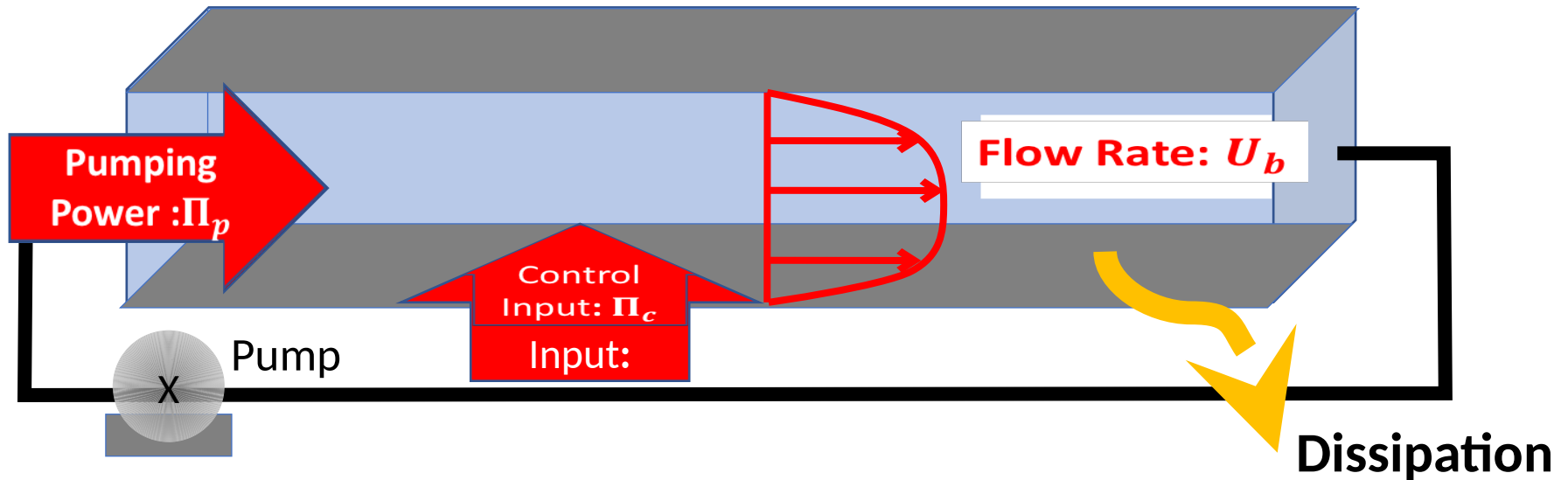
Flow Conditions



	Flow Rate: U_b	Pressure Gradient: G	Pumping Power: Π_p	Flow Rate: U_b	Pressure Gradient: G	Pumping Power: Π_p	Flow Rate: U_b	Pressure Gradient: G	Pumping Power: Π_p
Constant Flow Rate (CFR)	Constant			Constant			Constant		
Constant Pressure Gradient (CPG)		Constant			Constant			Constant	
Constant Power Input (CPI)			Constant			Constant			Constant
Constant Flow Rate (CFR)	Constant			↓			↓		
Constant Pressure Gradient (CPG)	↑			Constant			↑		
Constant Power Input (CPI)	↑			↓			Constant		

Concept of Constant Power Input (CPI)

(Frohnafel et al. JFM 2012, Hasegawa et al. JFM 2014)



- Total Power Input

$$\Pi_t^* = \Pi_p^* + \Pi_c^* = \epsilon \text{ const.}$$

- Fraction of Control Power Input

$$\gamma = \Pi_c^* / \Pi_t^*$$

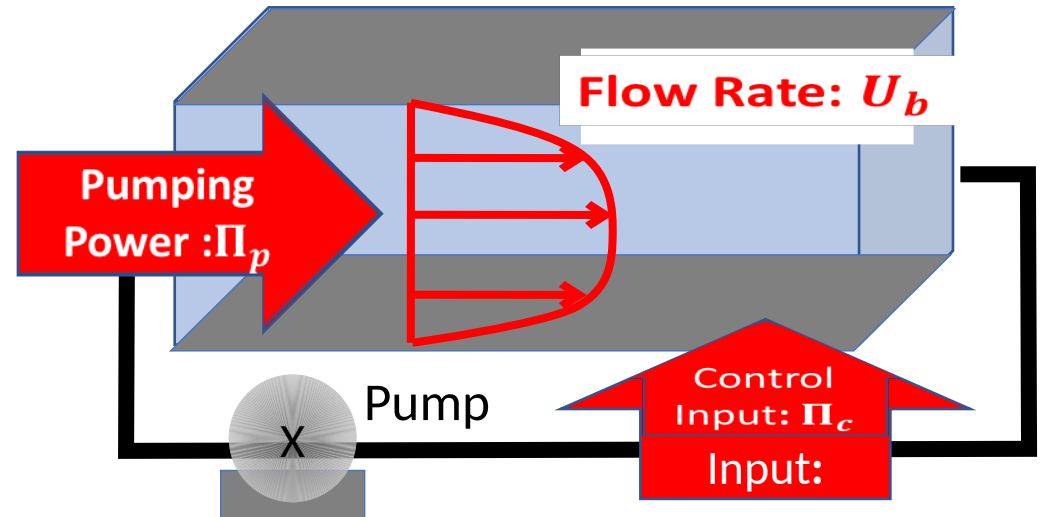
- Max. Achievable Flow Rate
- Power-based Reynolds Number

$$U_{II}^* \equiv \sqrt{\frac{\Pi_t^* \delta^*}{3\mu^*}}$$

$$Re_{II} \equiv \frac{U_{II}^* \delta^*}{\nu^*}$$

Example

- $Re_{\Pi} = 6500$
 $= Re_{CPG} \approx 200$ (CPG)
 $= Re_{CFR} \approx 3176$ (CFR)

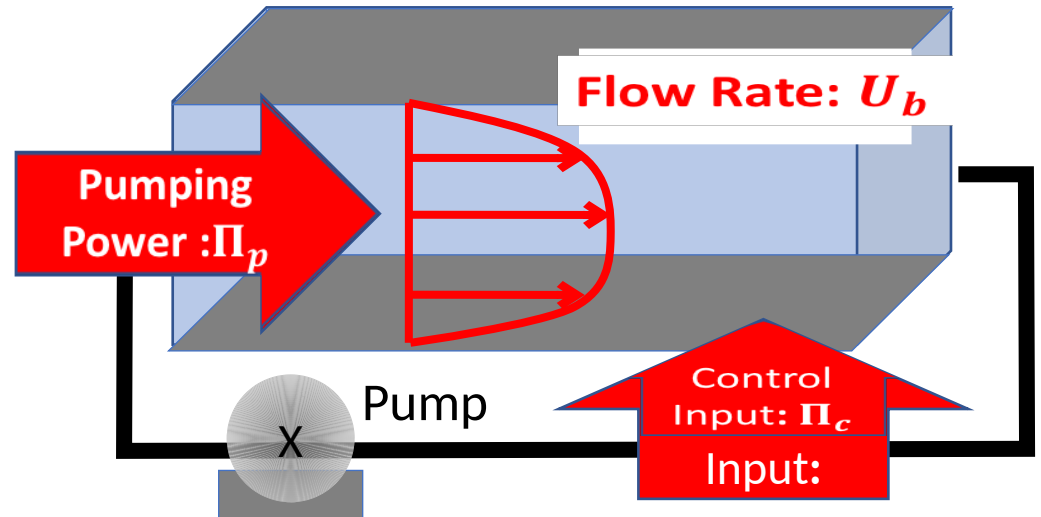


- Control Schemes
 - Opposition Control (Choi et al. JFM 1998)
 - Spanwise Wall Oscillation (Jung et al. PoF 1992)

	$U_b = U_b^*/U_{\Pi}^*$	$\gamma = \Pi_c^*/\Pi_t^*$	$U_b/U_{b,0}$		$U_b = U_b^*/U_{\Pi}^*$	$\gamma = \Pi_c^*/\Pi_t^*$	$U_b/U_{b,0}$		$U_b = U_b^*/U_{\Pi}^*$	$\gamma = \Pi_c^*/\Pi_t^*$	$U_b/U_{b,0}$
Reference (NC)	0.4887	0	1.0	Reference (NC)	0.4887	0	1.0	Reference (NC)	0.4887	0	1.0
Reference (NC)	0.4887				0				1.0		

Example

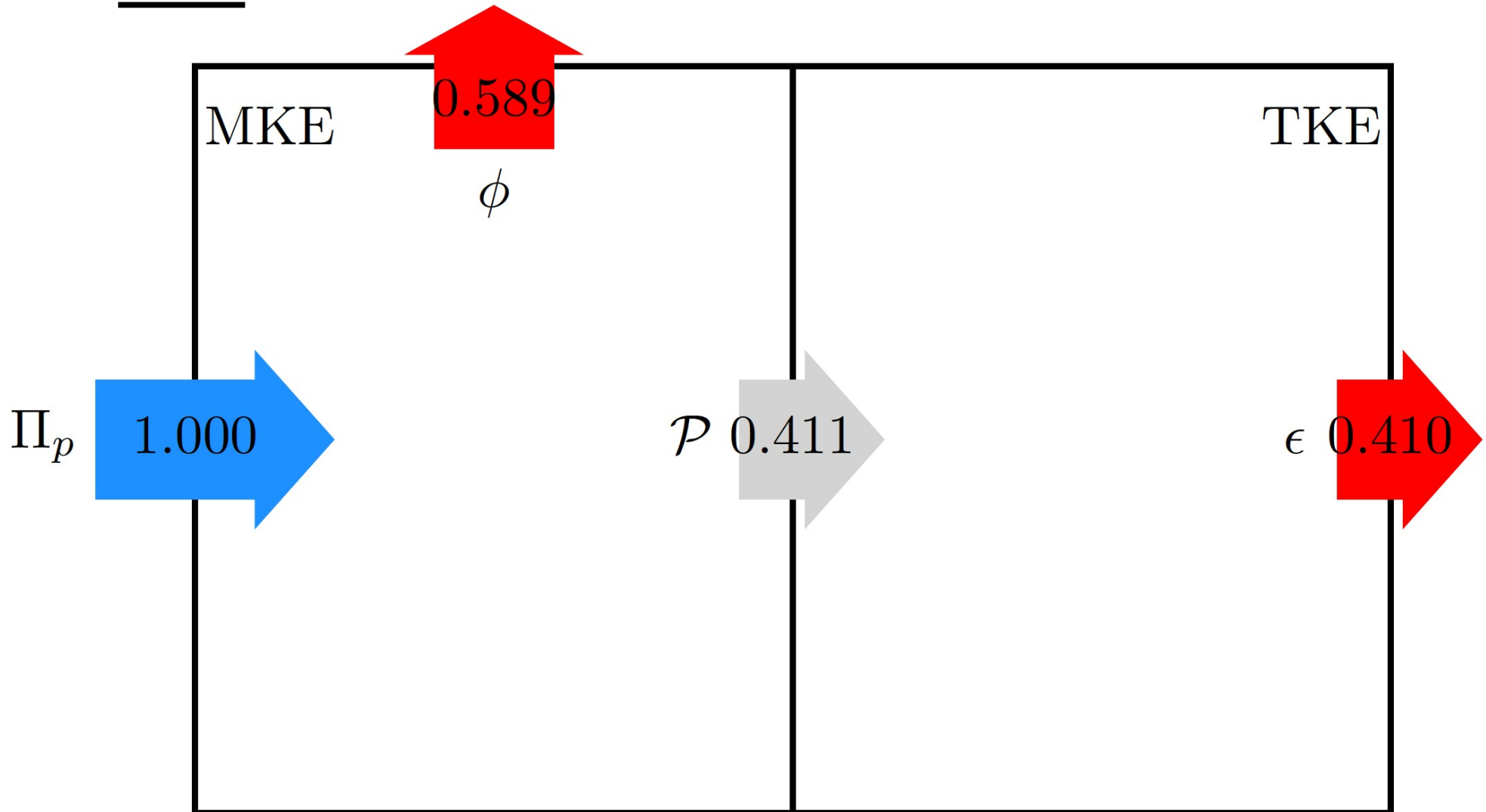
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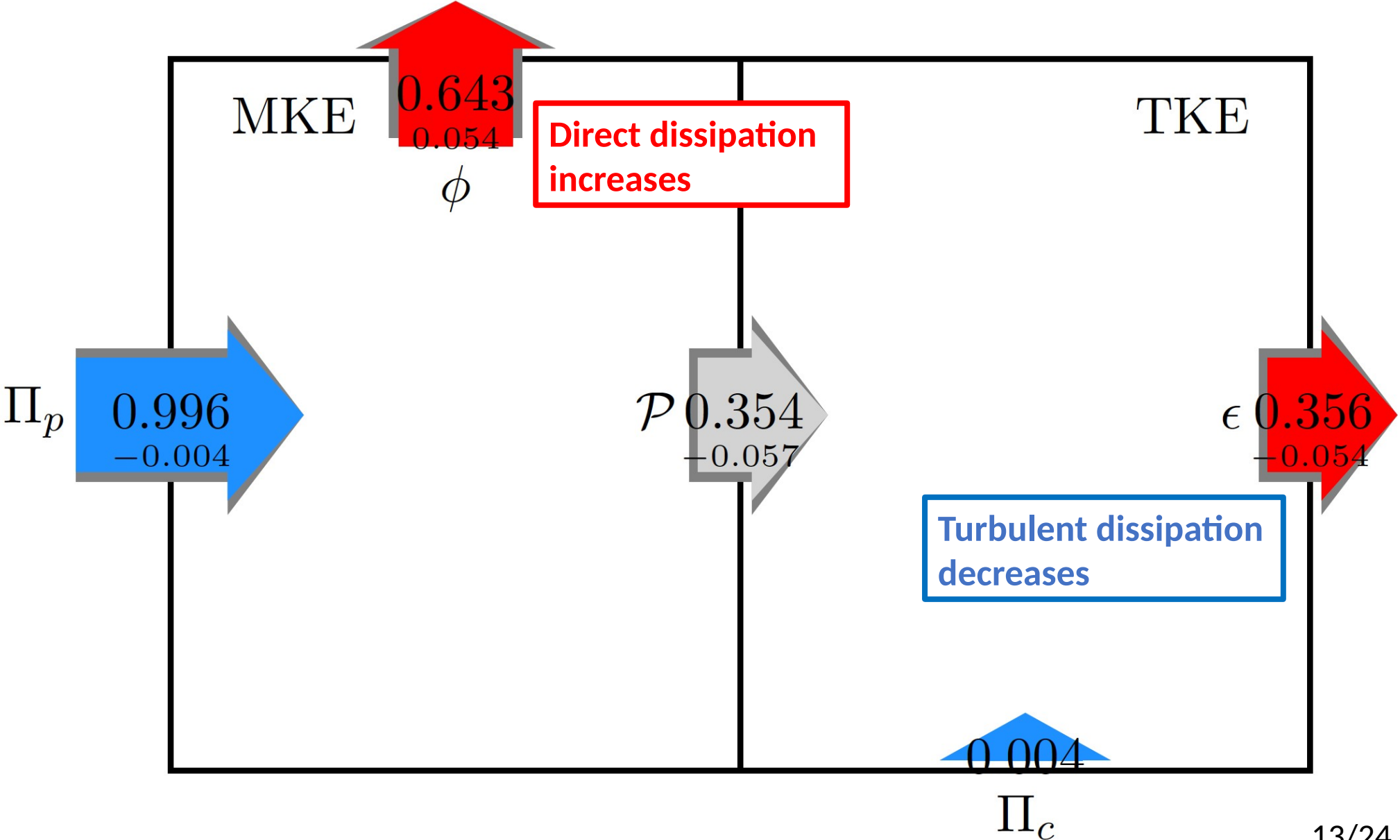
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	$U_b = U_b^*/U_{b,0}^*$	$\gamma = \Pi_c^*/\Pi_c^*$	$U_b/U_{b,0}$		$U_b = U_b^*/U_{b,0}^*$	$\gamma = \Pi_c^*/\Pi_c^*$	$U_b/U_{b,0}$		$U_b = U_b^*/U_{b,0}^*$	$\gamma = \Pi_c^*/\Pi_c^*$	$U_b/U_{b,0}$
Reference (NC)	0.4887	0	1.0	Reference (NC)	0.4887	0	1.0	Reference (NC)	0.4887	0	1.0
Wall Oscillation	0.5026	0.0978	1.028	Wall Oscillation	0.5026	0.0978	1.028	Wall Oscillation	0.5026	0.0978	1.028
Opposition Control	0.5345	0.0035	1.094	Opposition Control	0.5345	0.0035	1.094	Opposition Control	0.5345	0.0035	1.094
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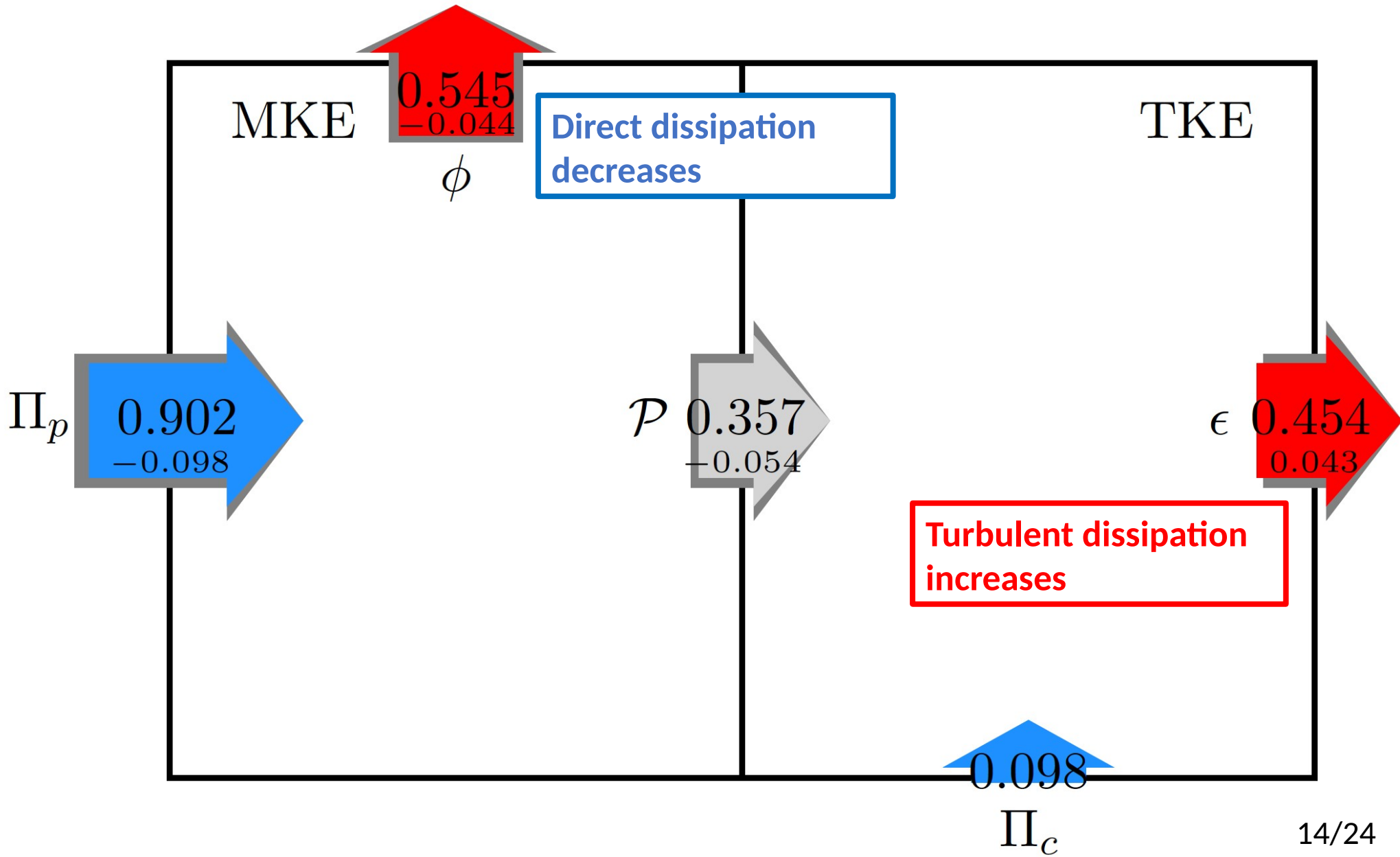
Energy Box of Uncontrolled Flow



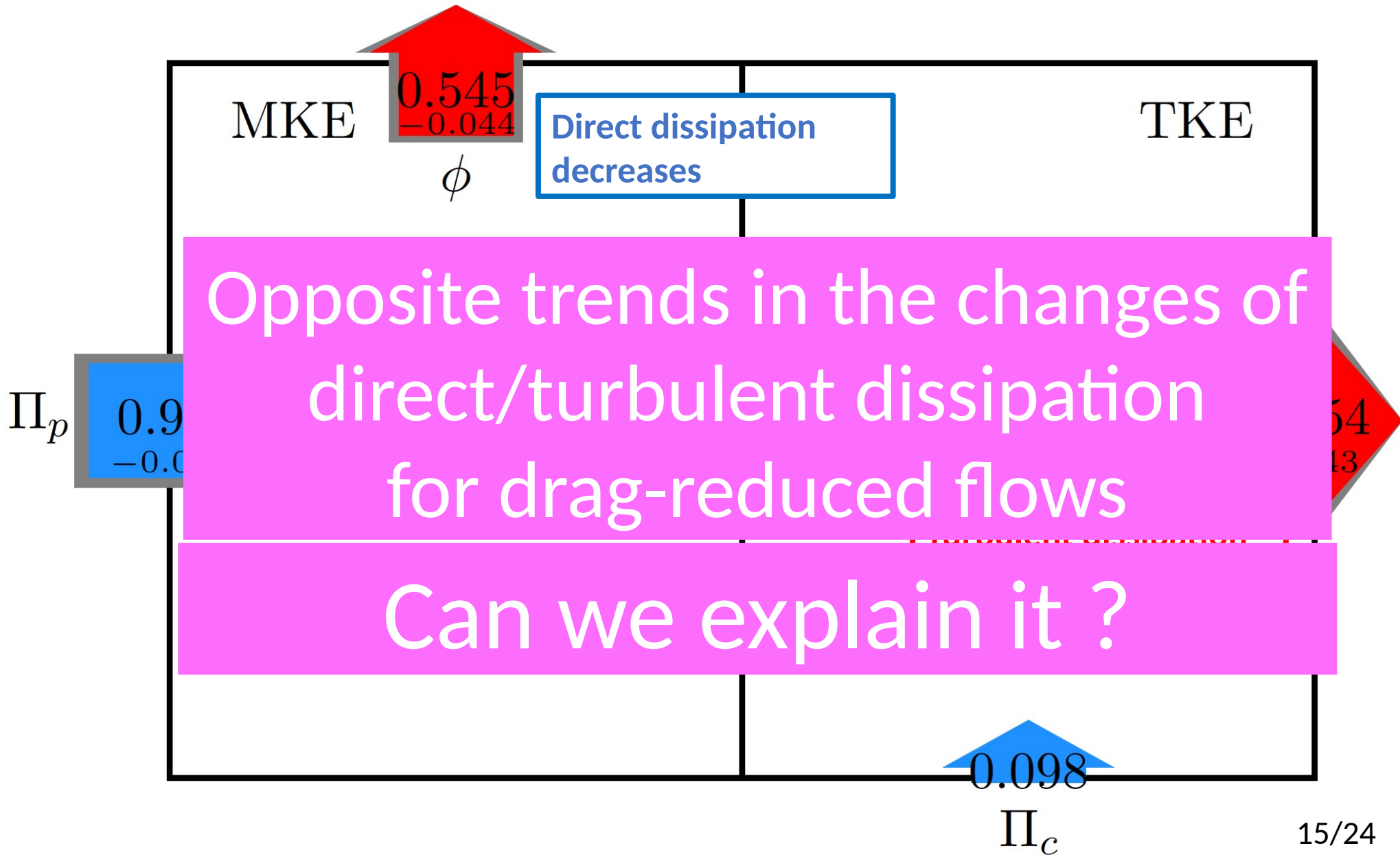
Energy Box under Opposition Control



Energy Box under Wall Oscillation Control



Energy Box under Wall Oscillation Control



Flow Rate Under CPI

- FIK Identity (Fukagata et al. PoF 2002)

$$\overline{U}_b = \frac{\alpha Re_{III}}{2} \left(-1 + \sqrt{1 + \frac{4(1-\gamma)}{(\alpha Re_{II})^2}} \right) \quad (\text{Hasegawa et al. JFM 2014})$$

The flow rate is determined by and only.

- Fraction of Control Power Input

$$\gamma = \overline{M}_c^* / \overline{M}_t^*$$

- Weighted Reynolds Shear Stress

$$\alpha = \int_0^1 (1-\gamma y) (-\overline{u'v''}) dy$$

Two Limiting Cases

$$\begin{cases} \alpha \rightarrow 0 \\ \gamma \rightarrow 0 \end{cases} \overline{U}_b \rightarrow 1$$

$$\alpha \rightarrow \infty \quad \overline{U}_b \rightarrow 0$$

Energy Flux Under CPI

- Triple Decomposition

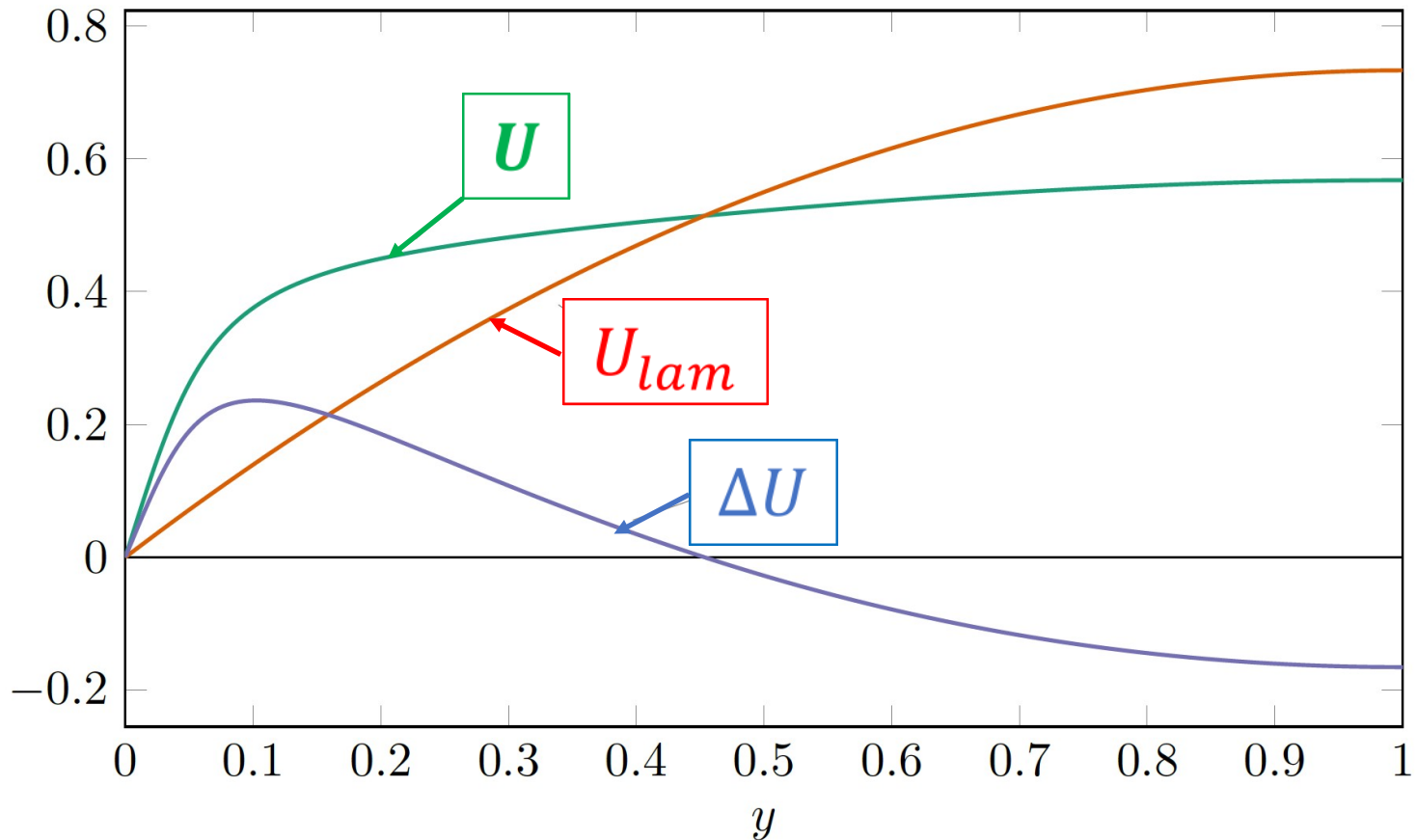
$$u = U + u'$$

Energy Flux Under CPI

U_{lam} : parabolic profile with the same flow rate
 ΔU : deviation from parabolic
 : deviation from parabola

- Triple Decomposition

$$w = U + u' = U_{lam} + \Delta U + u' \quad (\text{Echhardt et al., JFM 2007})$$



Energy Flux Under CPI

- Production

$$P = P_{lam} + P_{\Delta} = \int_0^1 u'v' \frac{dU_{lam}}{dy} dy + \int_0^1 u'v' \frac{d\Delta U}{dy} dy$$

- Direct Dissipation

$$\phi = \frac{1}{Re_{III}} \int_0^1 \left(\frac{dU}{dy} \right)^2 dy = \frac{1}{Re_{III}} \int_0^1 \left\{ \frac{d}{dy} (U_{lam} + \Delta U) \right\}^2 dy$$

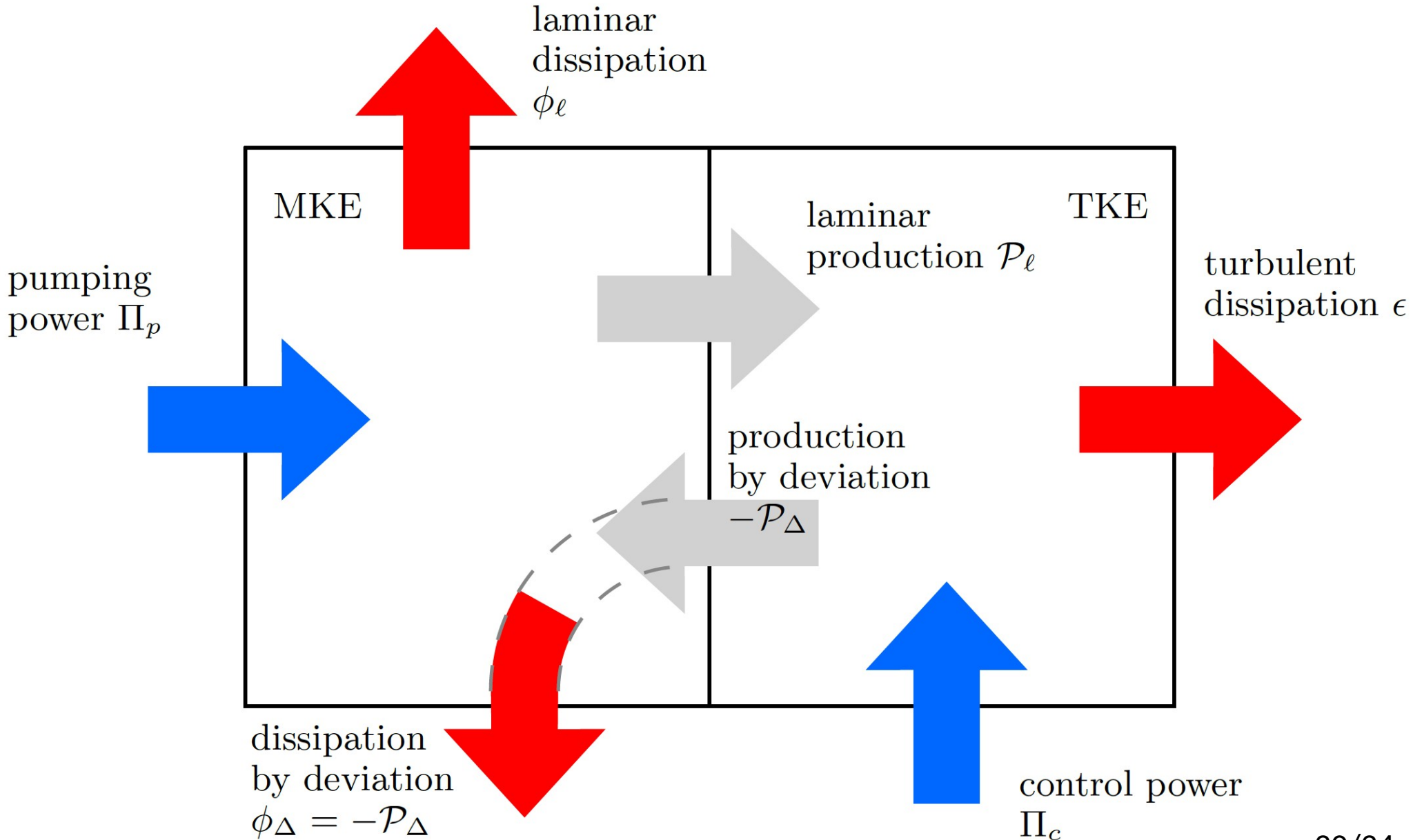
$$\approx \frac{1}{Re_{III}} \int_0^1 \left\{ \left(\frac{dU_{lam}}{dy} \right)^2 + \left(\frac{d\Delta U}{dy} \right)^2 + 2 \frac{dU_{lam}}{dy} \frac{d\Delta U}{dy} \right\} dy$$

$$\approx \phi_{lam} + \phi_{\Delta}$$

Only true in the present triple decomposition

In addition, it can be shown that $P_{\Delta} > 0$

Modified Energy Box



Modified Energy Box

- Direct Dissipation ($\phi = \phi_{lam} + \phi_{\Delta}$)

$$\phi_{lam} = \frac{3}{Re_{IH}} \left\{ \frac{(\alpha Re_{IH}^2)^2}{22} \left(1 - \sqrt{1 + \frac{4(1-\gamma)}{\alpha Re_{IH}^2}} \right) - (1-\gamma) \right\}$$

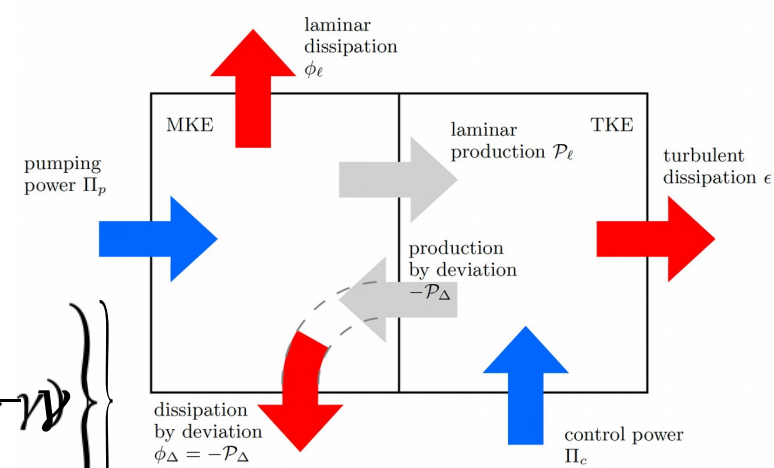
$$\phi_{\Delta} = Re_{IH} (\beta = 3\alpha^2)$$

- Turbulent Dissipation

$$\varepsilon = \frac{3}{Re_{IH}} \left\{ \frac{(\alpha Re_{IH}^2)^2}{22} \left(1 + \sqrt{1 + \frac{4(1-\gamma)}{\alpha Re_{IH}^2}} \right) - \frac{\beta Re_{IH}^2}{3} + \gamma \right\}$$

The following two quantities dictates all the fluxes in Energy Box !!!

$$\alpha \equiv \int_0^1 \int_0^1 (1-\gamma) (-\overline{u'w''}) dy \quad \beta \equiv \int_0^1 \int_0^1 -(\overline{u'v'})^2 dy$$



Conclusions

- Constant Power Input (CPI) is beneficial to analyze global energy budgets of turbulent flows w/o control
- There exists no unique relationship between the changes in direct/turbulent dissipation and DR effect
- DR effect under CPI is determined only by α integral of weighted RSS
- Triple decomposition of the velocity field reveals that global energy fluxes expressed by two quantities: α and β .

$$\alpha \equiv \int_0^1 \int_0^1 (1-y) \overline{(\mathbf{u}\mathbf{w}'')} d\mathbf{y} \quad \beta \equiv \int_0^1 \int_0^1 \overline{(\mathbf{u}\mathbf{v}')^2} d\mathbf{y}$$

- Total dissipation: $\phi + \varepsilon = \phi_{lam} + \phi_{\Delta} + \varepsilon$
- “Wind” dissipation ($\phi + \varepsilon$) can be considered to be a loss from energetic viewpoint.
 - A target quantity to be minimized

Thank you for your kind attention



Questions ?