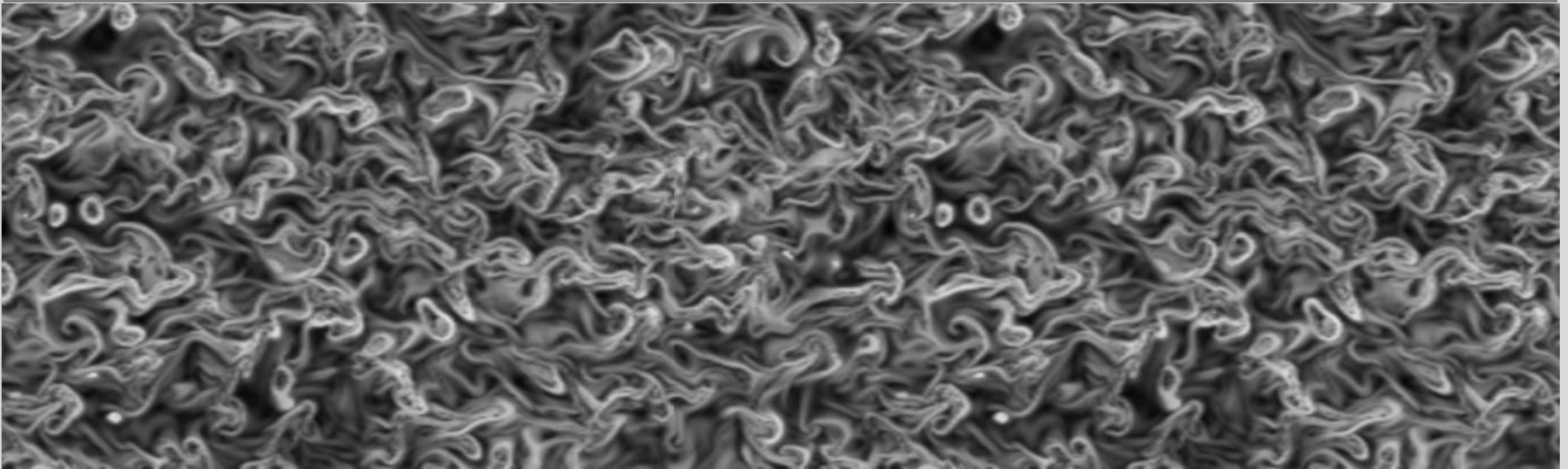


# Turbulent skin-friction drag reduction in the Constant Power Input framework

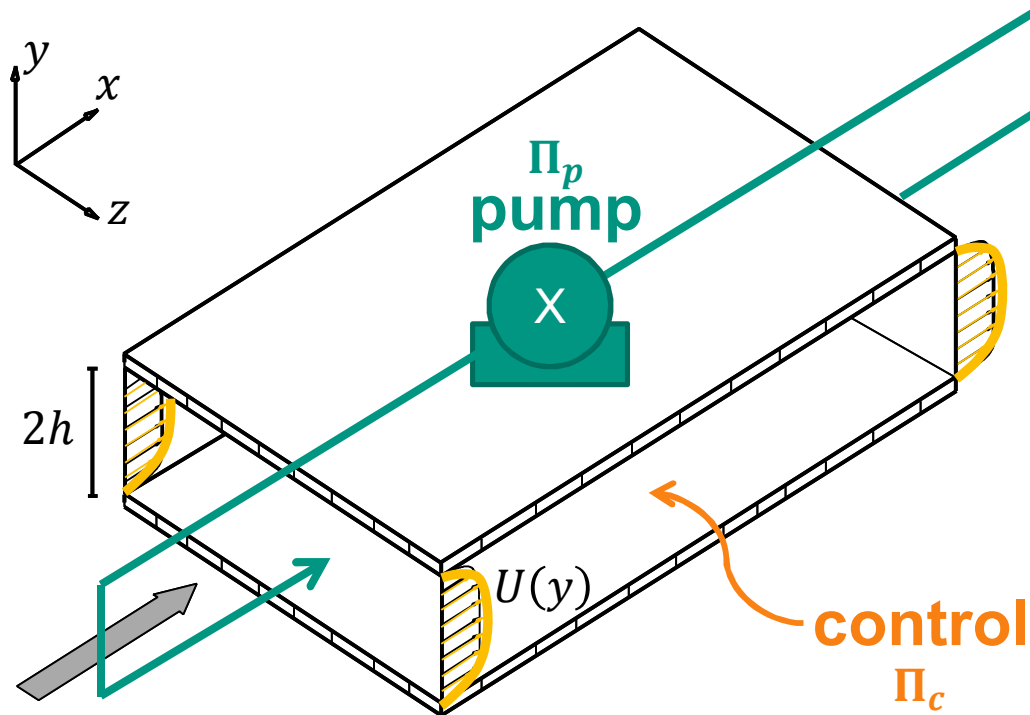
Daive Gatti, B. Frohnäpfel,  
A. Cimarelli, M. Quadrio, Y. Hasegawa

9.3.2016

JOINT ANNUAL MEETING OF GAMM AND DMV, Braunschweig, Germany



# The Drag Reduction Experiment



drag reduction rate:  $R = 1 - \frac{C_f}{C_{f,0}}$

bulk velocity:  $U_b$

pressure gradient:

$$-\frac{dp}{dx} = \frac{\tau_w}{h}$$

skin-friction coefficient:

$$C_f = \frac{2\tau_w}{\rho U_b^2}$$

pumping power  
(per unit area):

$$\Pi_p = -\frac{dp}{dx} h U_b$$

# The choice of flow condition (1)

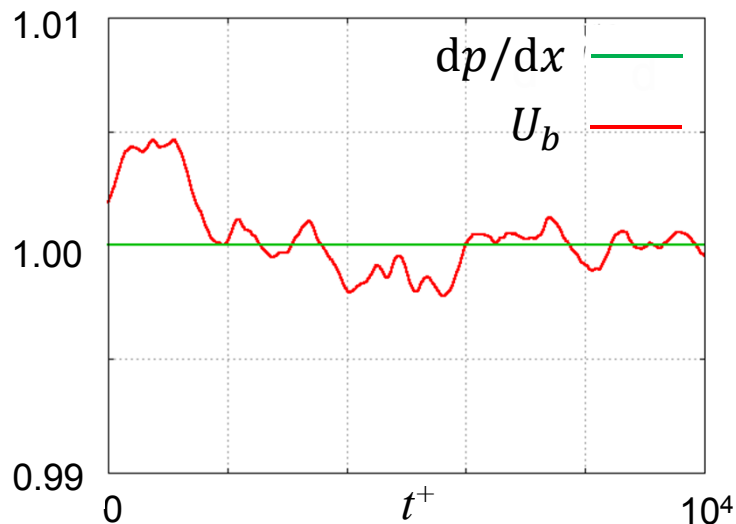
- Navier-Stokes equations alone do not pump fluid through the duct
- **Forcing term needed** to mimic pump
  
- Many **arbitrary choices** possible
- Often equivalent on physical grounds
- Different on practical grounds
- Different realizations, same statistics

# The choice of flow condition (2)

Constant Pressure Gradient  
(CPG)



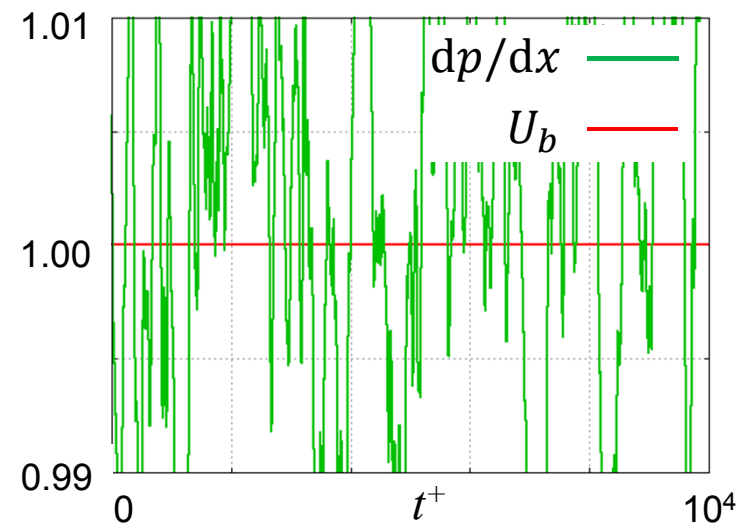
Flow rate fluctuates in time



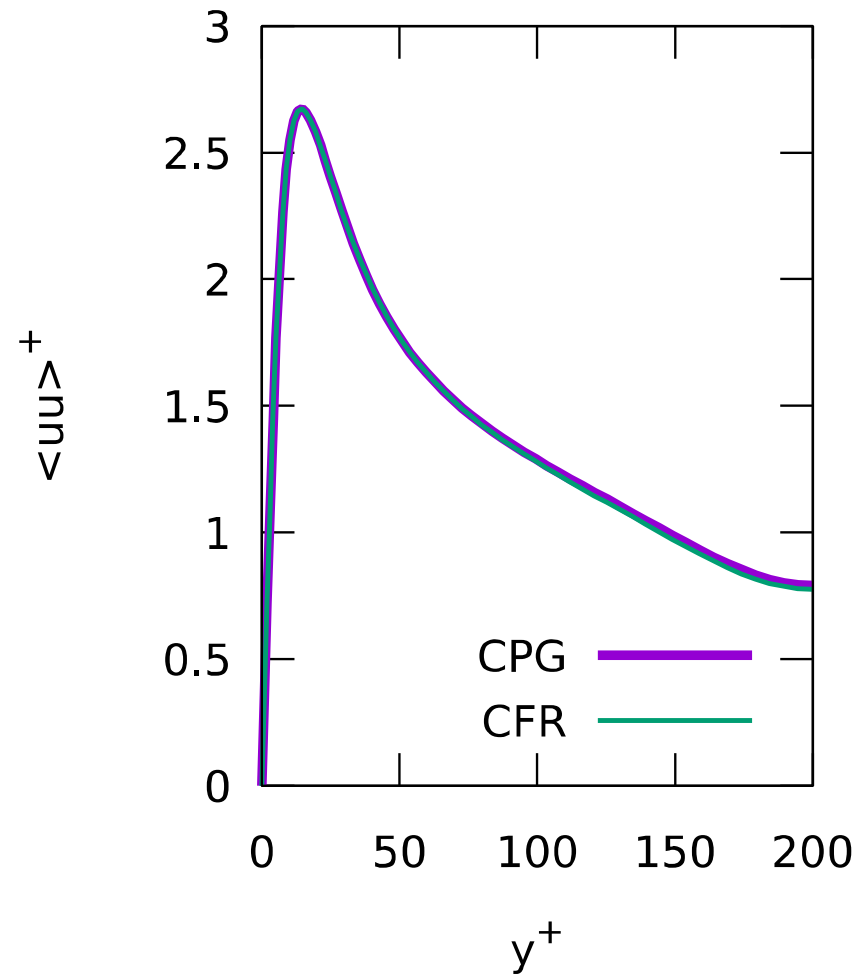
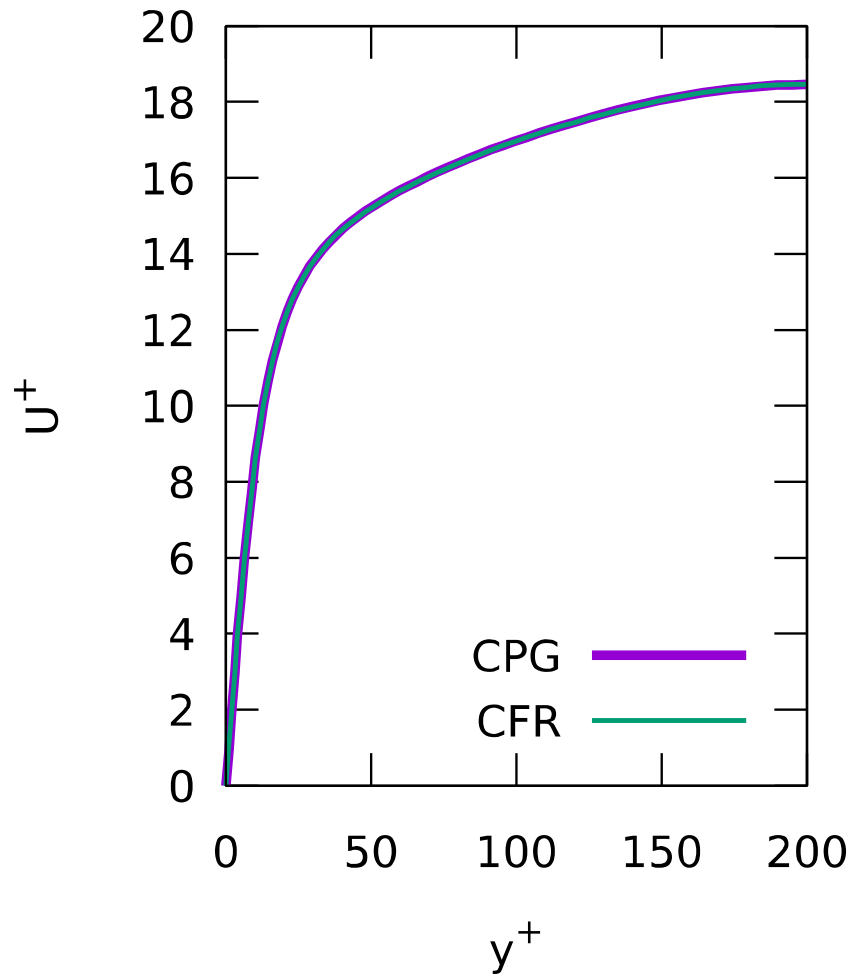
Constant Flow Rate  
(CFR)



Pressure gradient fluctuates in time

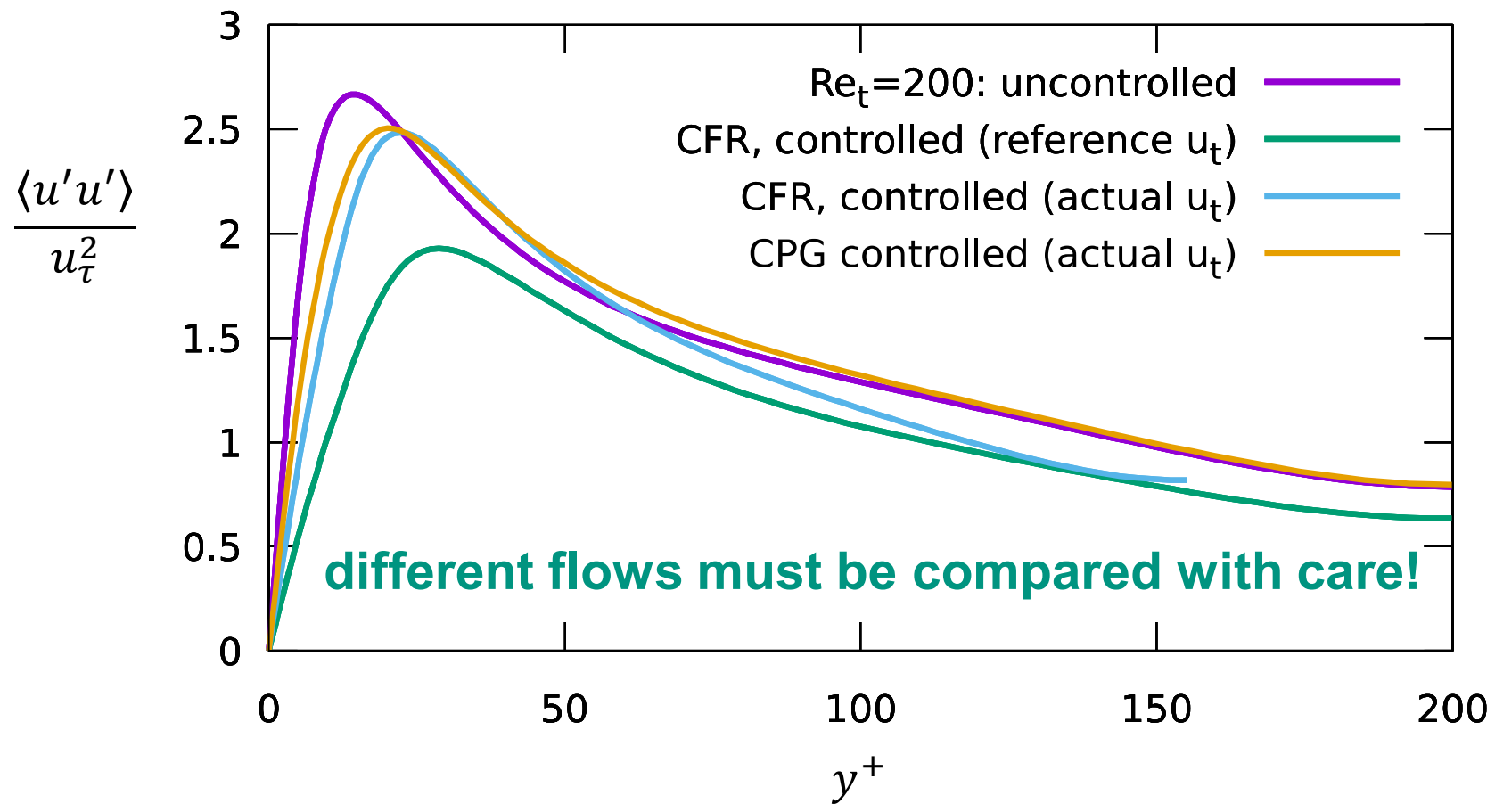


# Unimportant choice for uncontrolled flows



# Important choice in flow control!

“Turbulent fluctuations are destroyed” ?



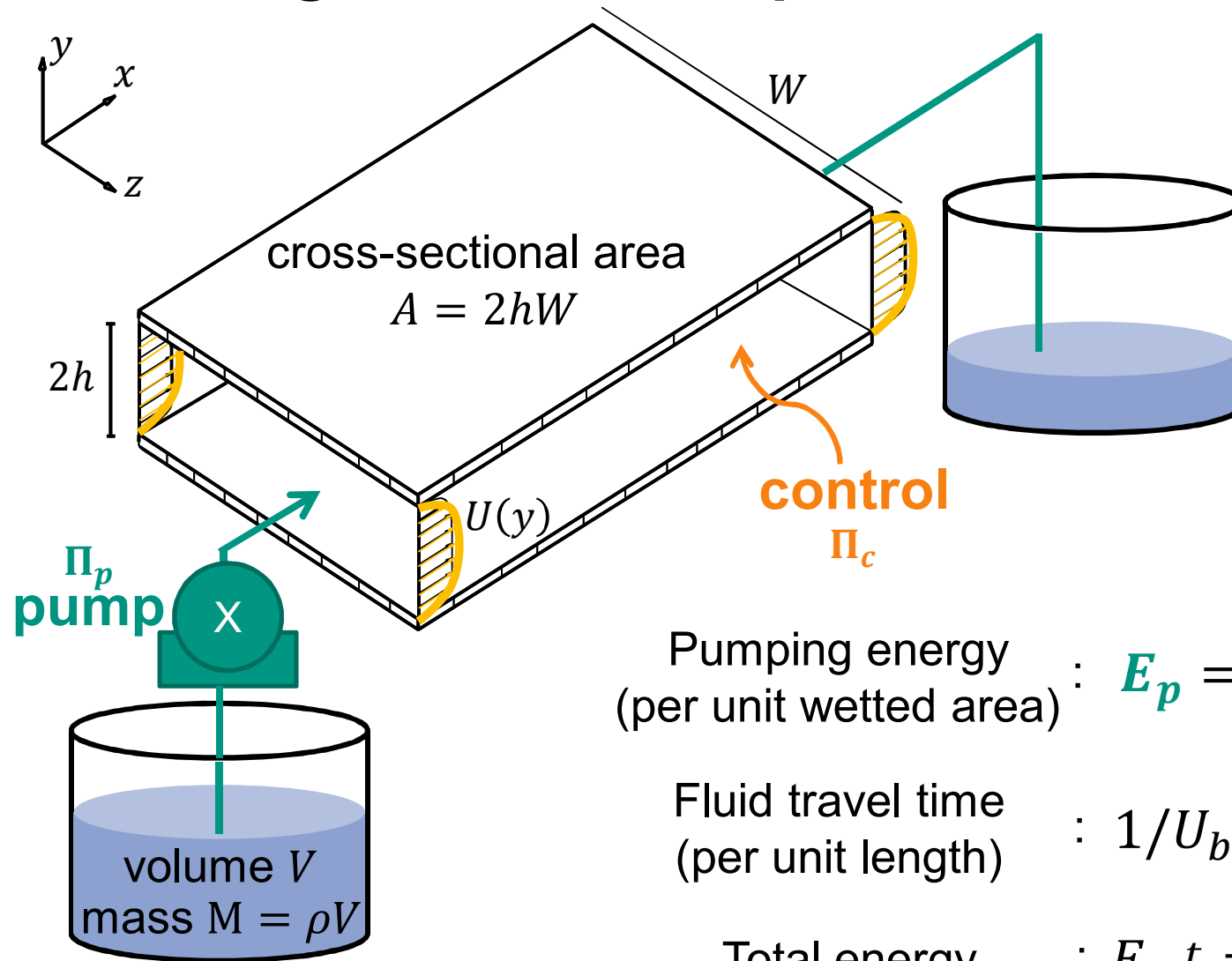
# Important choice in flow control!

successful control  $R = 1 - \frac{C_f}{C_{f,0}} > 0$  manifests differently

	$U_b$	$-\frac{dp}{dx}$	$\Pi_p = -\frac{dp}{dx} h U_b$	$C_f$
CPG	↑	=	↑	↓
CFR	=	↓	↓	↓

- With control: either different  $Re_\tau$  or different  $Re_B$
- $C_f \neq \Pi_p$  : successful control can increase pumping power!

# The Drag Reduction Experiment



Pumping energy  
 (per unit wetted area) :  $E_p = \frac{\tau_w V}{A} = \frac{M U_b^2 C_f}{2A}$

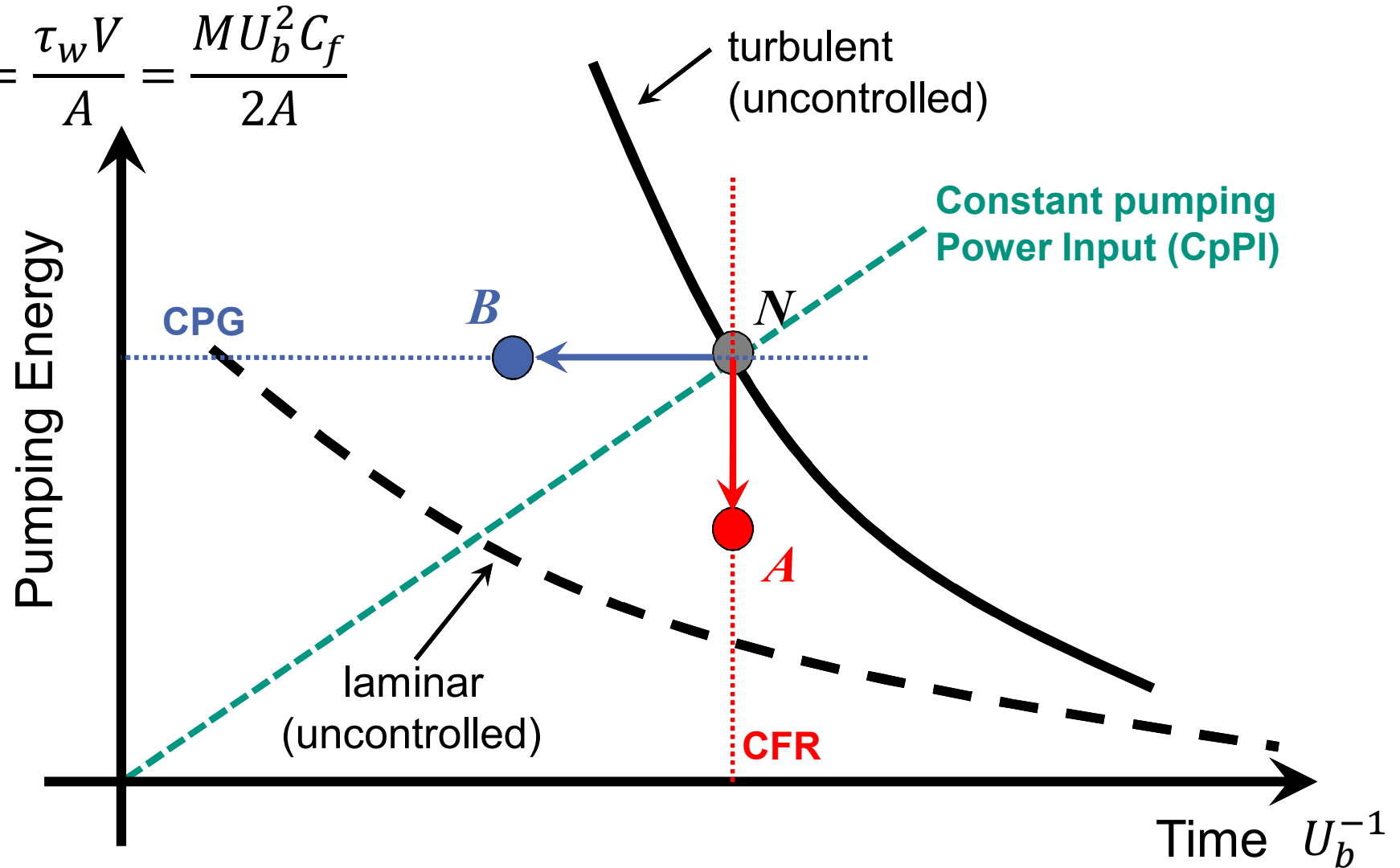
Fluid travel time  
 (per unit length) :  $1/U_b$

Total energy :  $E_{to}^t = E_p + E_c$



# Energy (cost) vs. Time

$$E_p = \frac{\tau_w V}{A} = \frac{M U_b^2 C_f}{2A}$$



Frohnäpfel, Hasegawa, Quadrio, JFM 2012



# Comparison of different flow condition

successful control  $R = 1 - \frac{c_f}{c_{f,0}} > 0$  manifests differently

	$U_b$	$-\frac{dp}{dx}$	$\Pi_p = -\frac{dp}{dx} h U_b$	$c_f$
CPG	↑	=	↑	↓
CFR	=	↓	↓	↓
CPI	↑	↓	=	↓

# Constant Power Input framework

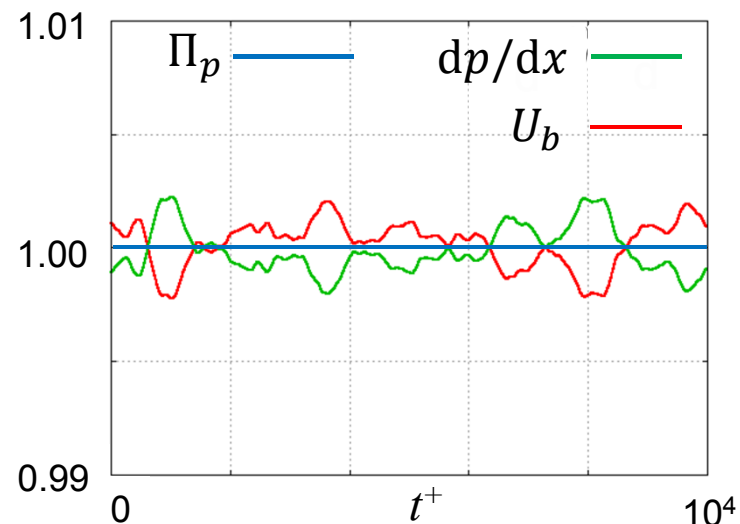
$$Re_{\Pi} = \frac{U_{\Pi} \delta}{\nu} \quad U_{\Pi} = \sqrt{\frac{\Pi_t h}{3\mu}}$$

the velocity of a Poiseuille flow with the same total power  $\Pi_t$

a Poiseuille flow maximizes the flow rate for a given total power  $\Pi_t$

$$\Pi_t = \frac{3}{Re_{\Pi}} \quad \gamma = \frac{\Pi_c}{\Pi_t}$$

$$\Pi_c = \gamma \Pi_t \quad \Pi_p = (1 - \gamma) \Pi_t$$



# Checkpoint: what you should not forget

How to drive the flow (CFR, CPG, CPI)?

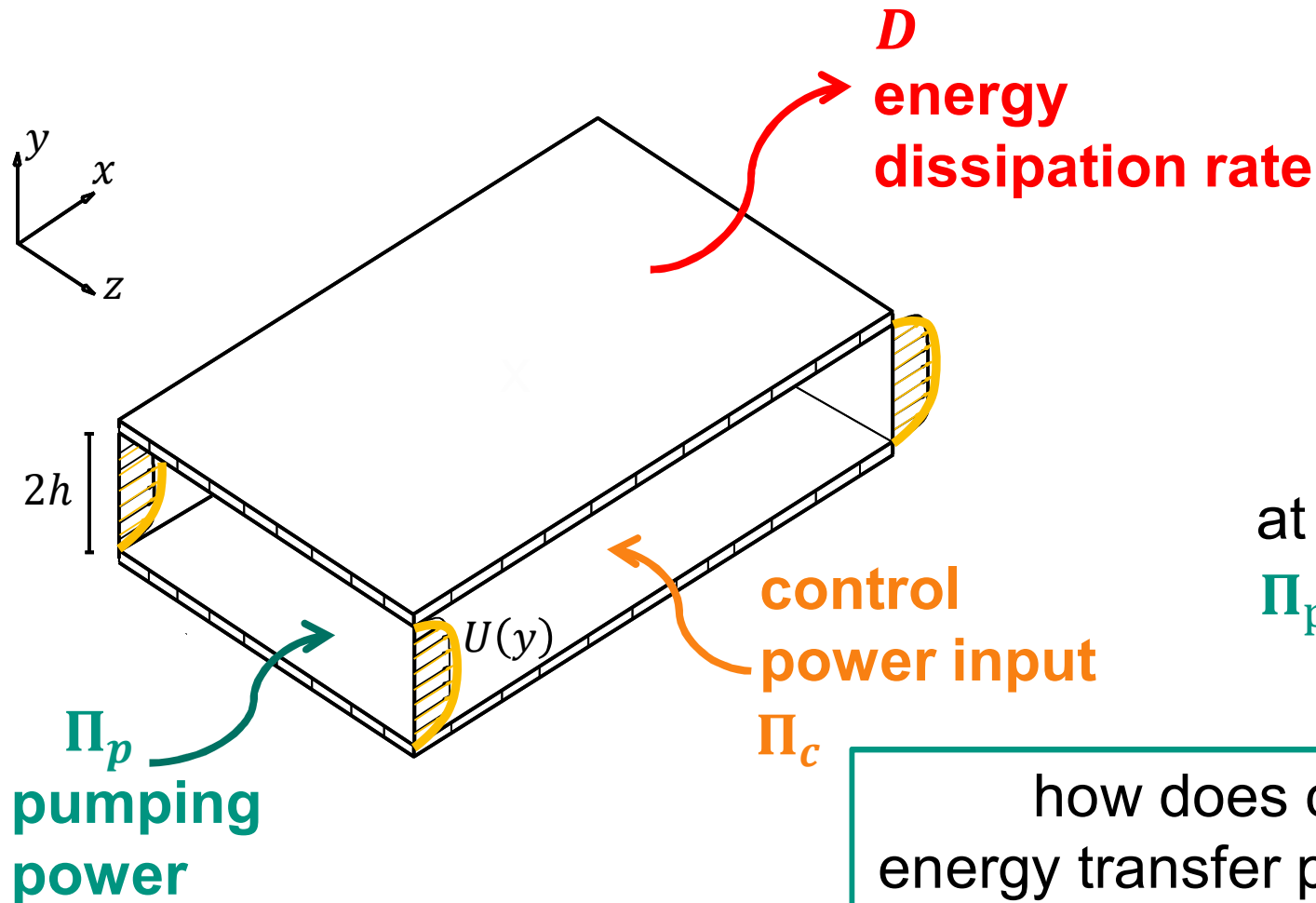
- necessary and **important choice**
- **affects the results** and their interpretations
- different manifestations of “drag reduction”

Constant Power Input

- possible choice close to real conditions (pump)
- **power input** (energy transfer rate) is kept **constant**

# Control from the energetic viewpoint

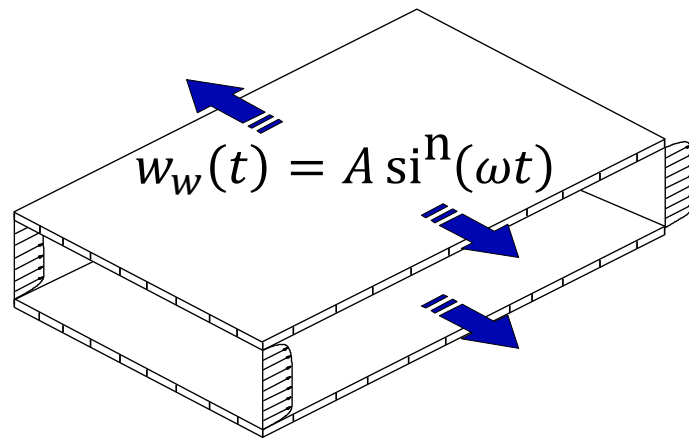
CPI ideal framework to study energy transfer rates



# Model control strategies

$Re_\pi = 6500$        $Re_{\tau,ref} = 20^0$       Constant total Power Input (CtPI)

Spanwise wall oscillations

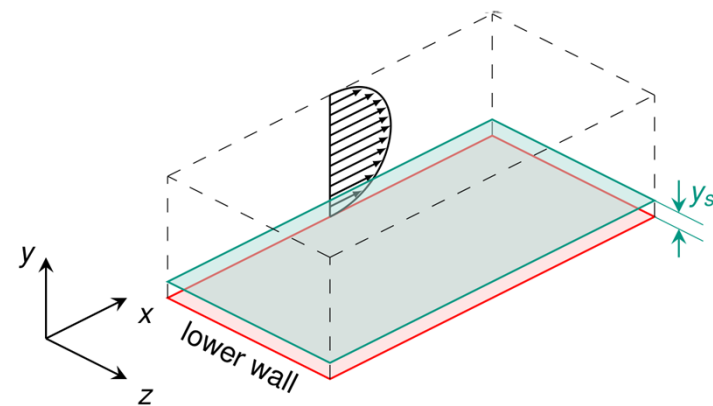


drag reduction  $R = 17.2\%$

control power fraction  $\gamma = 0.11$

$$\frac{U_b}{U_{b,ref}} = 1.02^5$$

Opposition control



$R = 23.0\%$

$\gamma = 0.01$

$$\frac{U_b}{U_{b,ref}} = 1.08^2$$

# One-point kinetic energy budgets (MKE)

Mean Kinetic Energy  $\left(\frac{1}{2} U_i^2\right)$  budget:

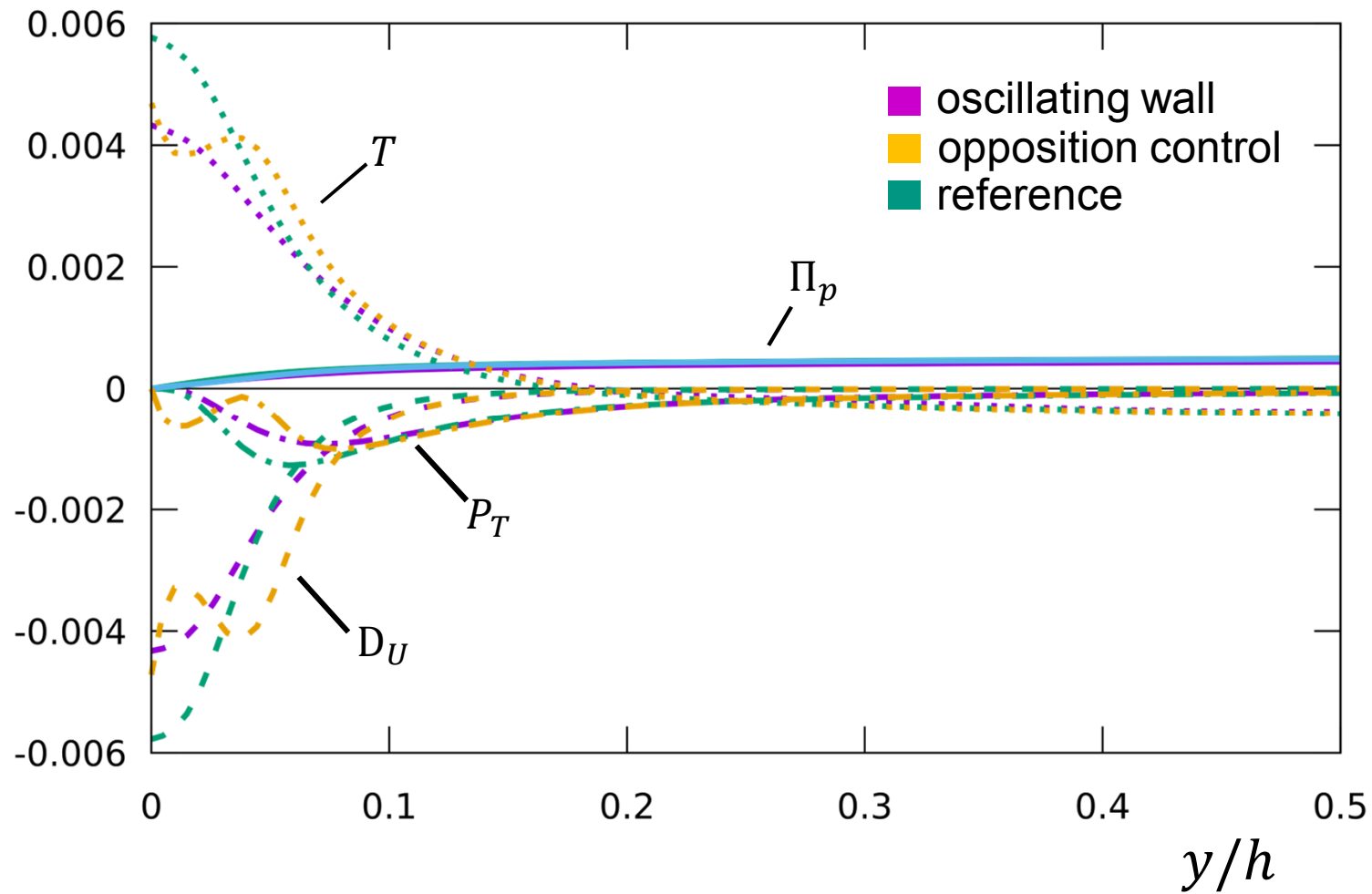
$$\underbrace{U \frac{dp}{dx}}_{\text{pumping power}} = \underbrace{\langle u'v' \rangle \frac{\partial U}{\partial y}}_{\text{turbulent production}} - \underbrace{\frac{\partial(\langle u'v' \rangle U)}{\partial y} + \nu \frac{\partial}{\partial y} \left( U \frac{\partial U}{\partial y} \right)}_{\text{turbulent and laminar transport}} - \underbrace{\nu \left( \frac{\partial U}{\partial y} \right)^2}_{\text{MKE dissipation rate}}$$

integrated in the whole channel:

$$\Pi_p = P_T - D_U$$



# One-point kinetic energy budgets (MKE)



# One-point kinetic energy budgets (TKE)

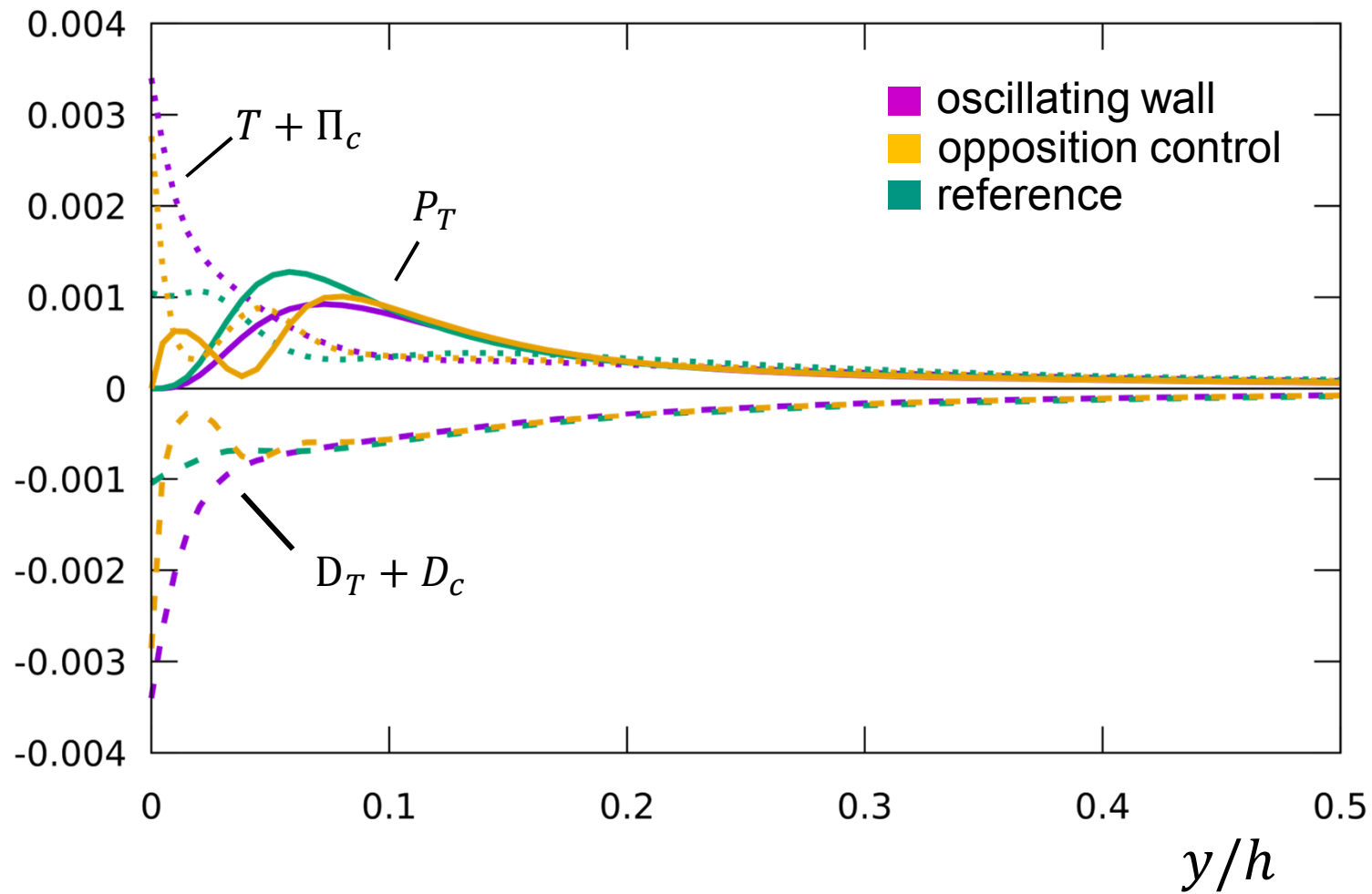
Turbulent Kinetic Energy  $\left(k = \frac{1}{2} u_i'^2\right)$  budget:

$$\underbrace{\langle u'v' \rangle \frac{\partial U}{\partial y}}_{\text{turbulent production}} = -\nu \underbrace{\left\langle \frac{\partial u_i'}{\partial x_j} \frac{\partial u_i'}{\partial x_j} \right\rangle}_{\text{TKE dissipation rate}} + \underbrace{\frac{\partial}{\partial y} \left( \frac{\nu}{2} \frac{\partial \langle u_i'^2 \rangle}{\partial y} - \langle v'p' \rangle - \frac{1}{2} \langle v'u_i'^2 \rangle \right)}_{\text{transport (space fluxes)}}$$

integrated in the whole channel: with control

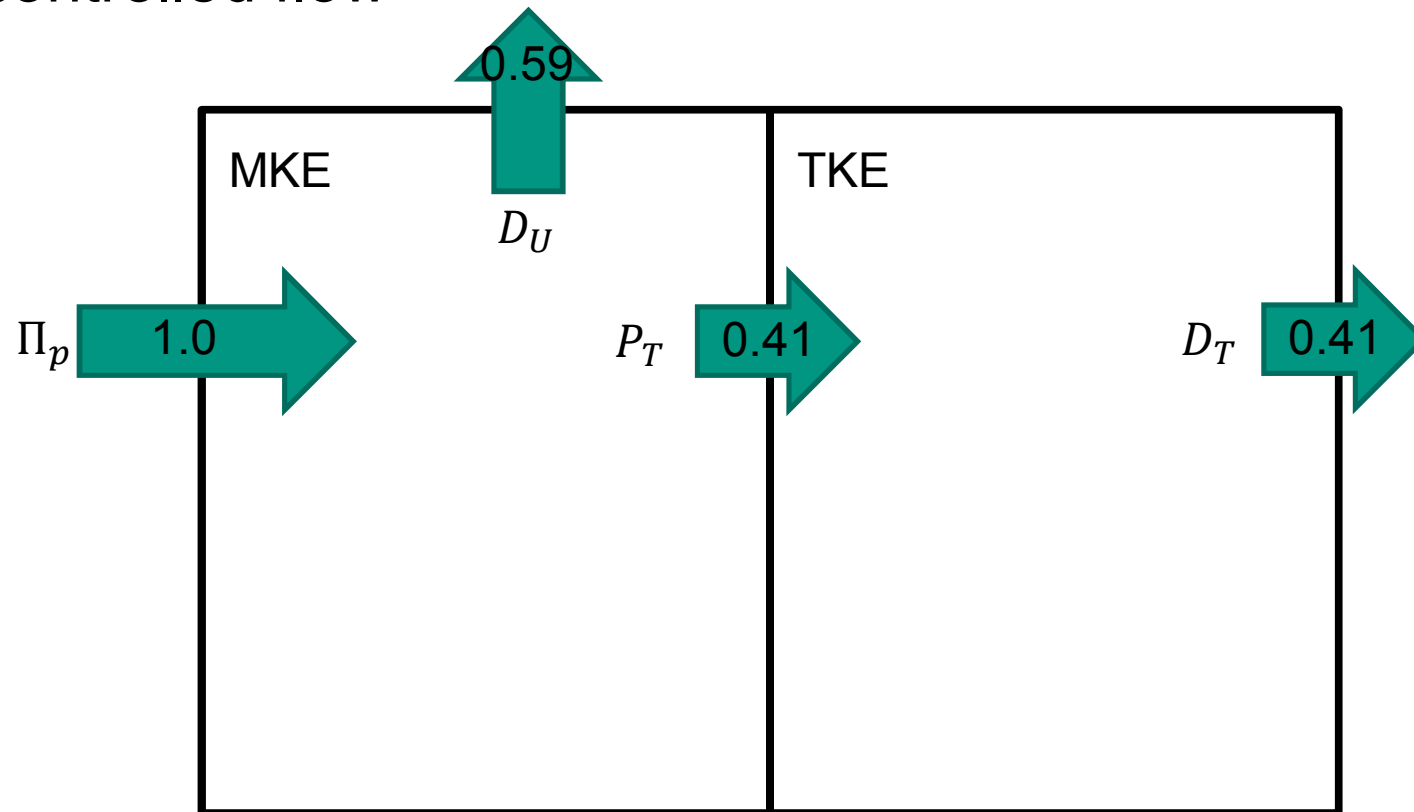
$$\Pi_c + P_T = D_T + D_C$$

# One-point kinetic energy budgets (TKE)



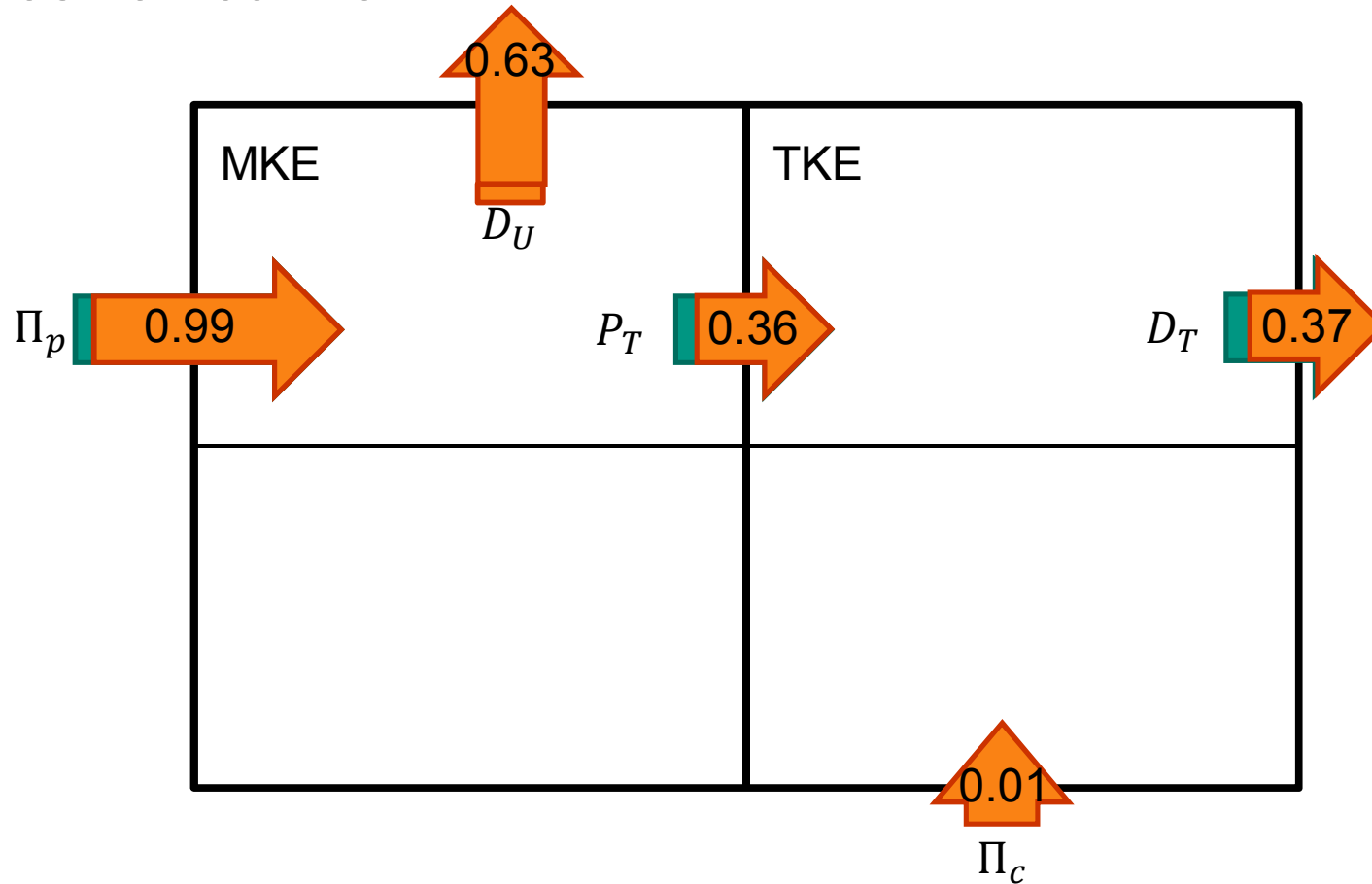
# The Energy Box

uncontrolled flow



# The Energy Box

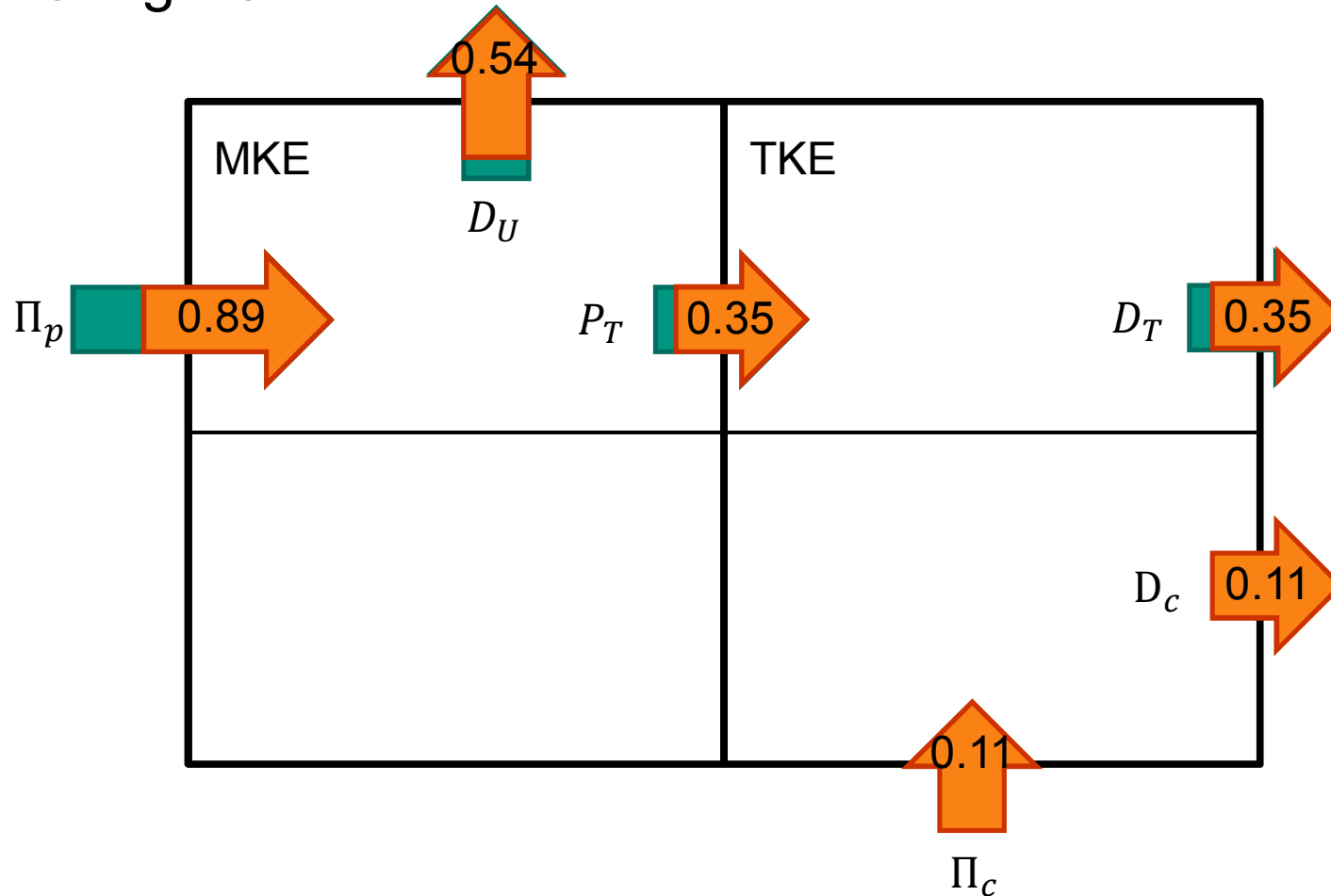
opposition control



MKE dissipation rate increases  
 TKE dissipation rate decreases

# The Energy Box

oscillating wall



MKE and TKE dissipation rate decrease:  $U_b$  increases though!

# The Energy Box: lesson

Drag reduction  $\Leftrightarrow$  reduction of TKE dissipation rate

Drag reduction  $\neq$  increase of MKE dissipation rate

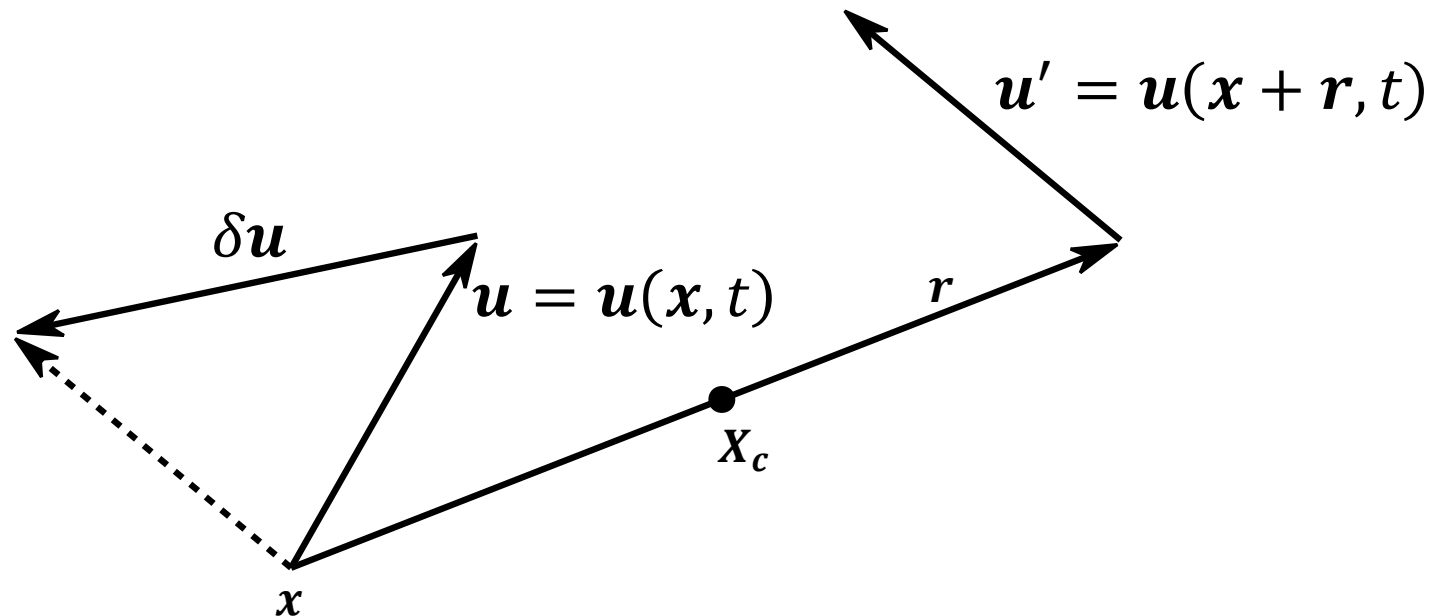
Effect of control on energy transfer rates unveiled!!

Sometimes  $\Pi_c$  is a good alternative to  $\Pi_p$

## How do we analyse the physics?

## Second-order structure function

$$\langle \delta u^2 \rangle(\mathbf{r}, \mathbf{X}_c) = \langle [u(x + \mathbf{r}) - u(x)]_i^2 \rangle$$



loosely speaking, amount of fluctuation **energy** at **scale**  $\|\mathbf{r}\|$



# Scale-energy budget equation (1)

$$\nabla_{\mathbf{r}} \cdot \mathbf{\Phi}_{\mathbf{r}}(\mathbf{r}, Y_c) + \frac{d\Phi_c(\mathbf{r}, Y_c)}{dY_c} = s(\mathbf{r}, Y_c)$$

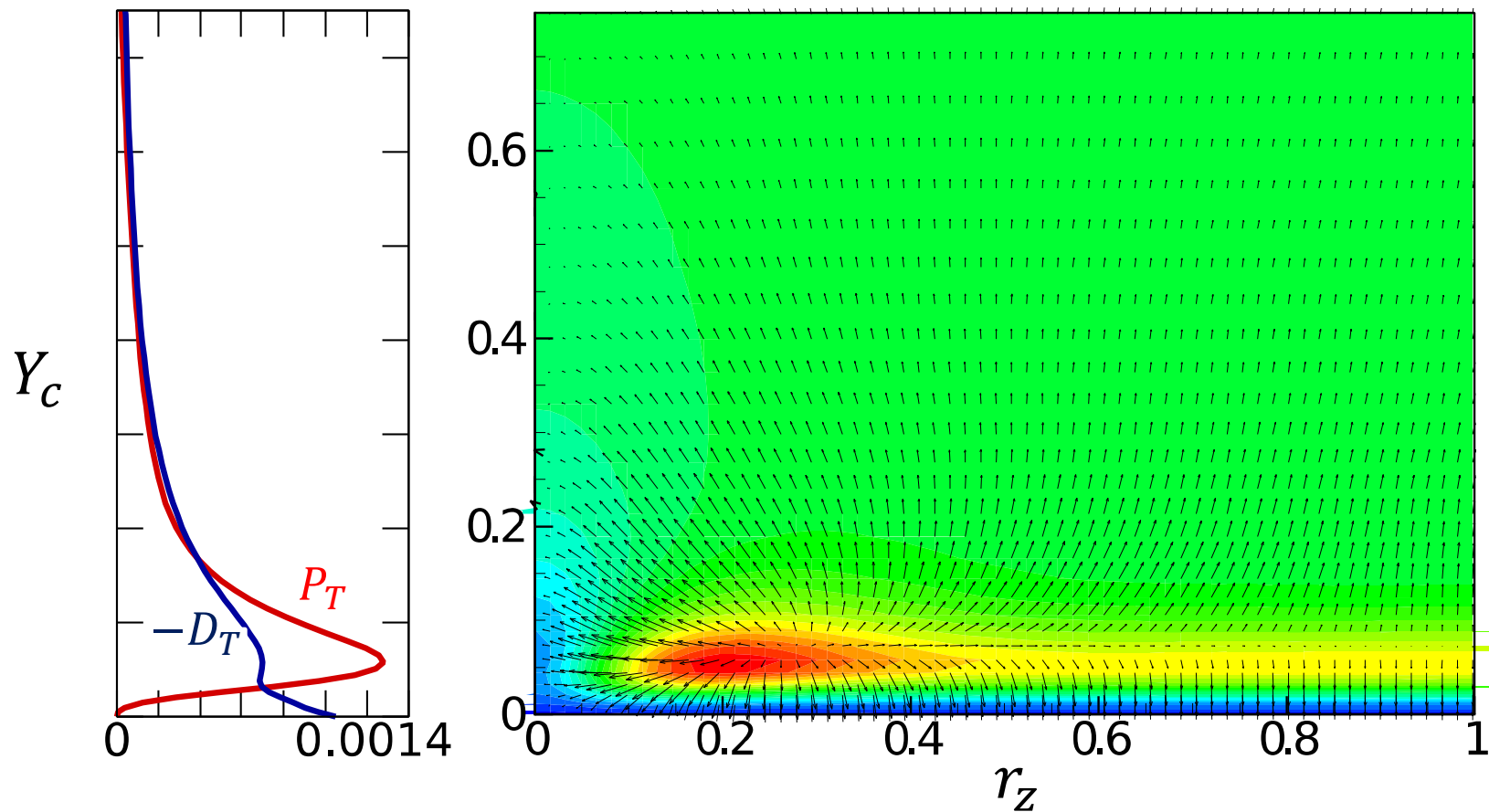
source term

$$\underbrace{-2\langle \delta u \delta v \rangle (dU/dy)^*}_{\text{production}} - \underbrace{4\langle \varepsilon \rangle^*}_{\text{dissipation}} = s(\mathbf{r}, Y_c)$$

budget among **production** and **dissipation** of scale energy

# Scale-energy budget equation (1)

$$-2\langle \delta u \delta v \rangle (dU/dy)^* - 4\langle \varepsilon \rangle^* = s(\mathbf{r}, Y_c)$$



# Scale-energy budget equation (2)

$$\nabla_r \cdot \mathbf{\Phi}_r(\mathbf{r}, Y_c) + \frac{d\Phi_c(\mathbf{r}, Y_c)}{dY_c} = s(\mathbf{r}, Y_c)$$

space flux

$$\underbrace{\langle \delta u^2 \delta v^* \rangle}_{\text{turbulent}} + \underbrace{\frac{2}{\rho} \langle \delta p \delta v \rangle}_{\text{pressure}} - \underbrace{\frac{\nu}{2} \frac{d\langle \delta u^2 \rangle}{dY_c}}_{\text{viscous}} = \Phi_c(\mathbf{r}, Y_c)$$

transport of scale energy in geometric space

in a channel flow, transfer of energy at scale  $r$  in  $y$ -direction

# Scale-energy budget equation (3)

$$\nabla_r \cdot \Phi_r(\mathbf{r}, Y_c) + \frac{d\Phi_c(\mathbf{r}, Y_c)}{dY_c} = s(\mathbf{r}, Y_c)$$

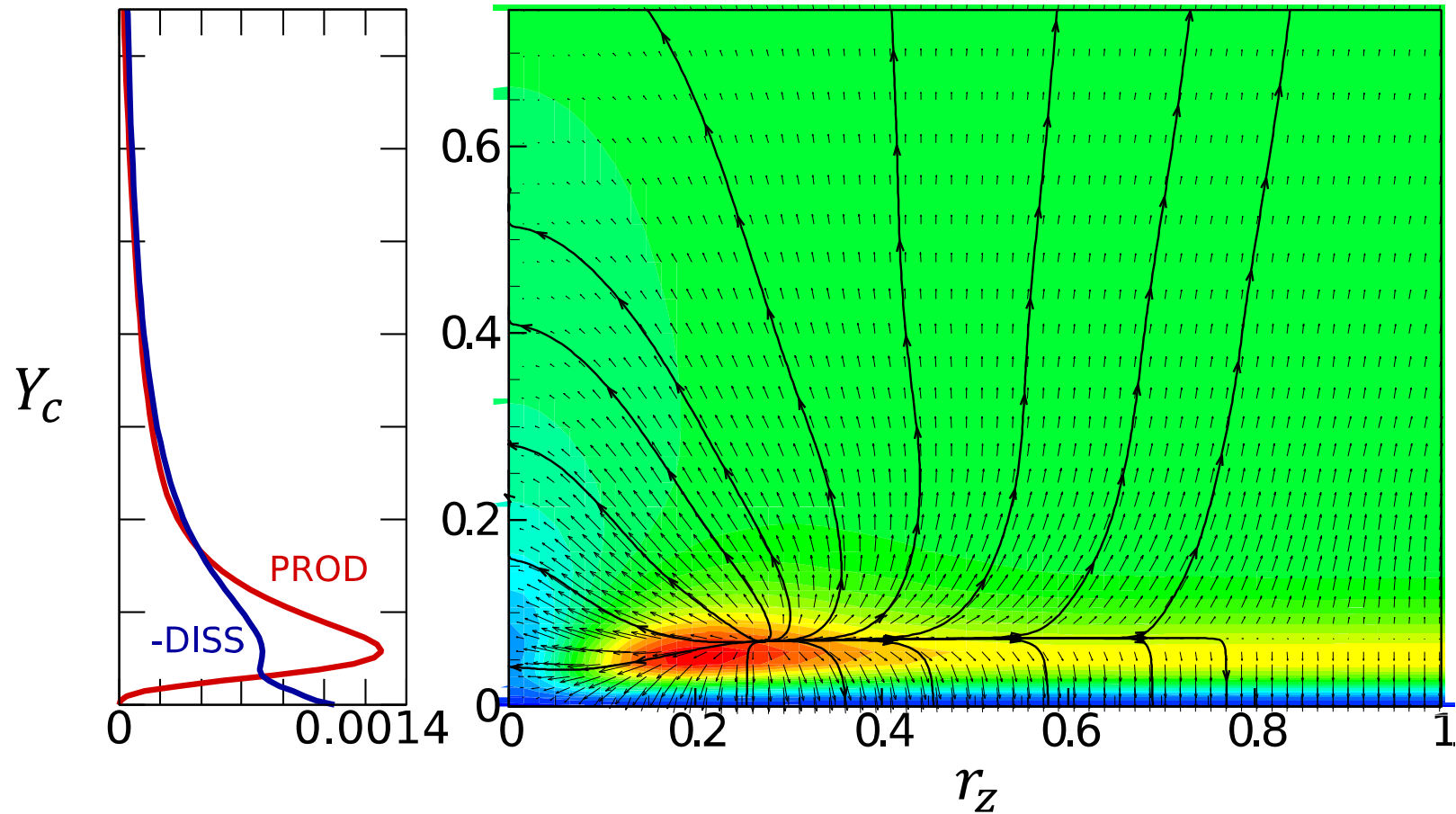
↓  
**scale flux**

$$\Phi_r(\mathbf{r}, Y_c) = \overbrace{\langle \delta u^2 \delta \mathbf{u} \rangle}^{\text{turbulent}} + \overbrace{\langle \delta u^2 \delta \mathbf{U} \rangle}^{\text{mean}} - \overbrace{2\nu \nabla_r (\delta u^2)}^{\text{viscous}}$$

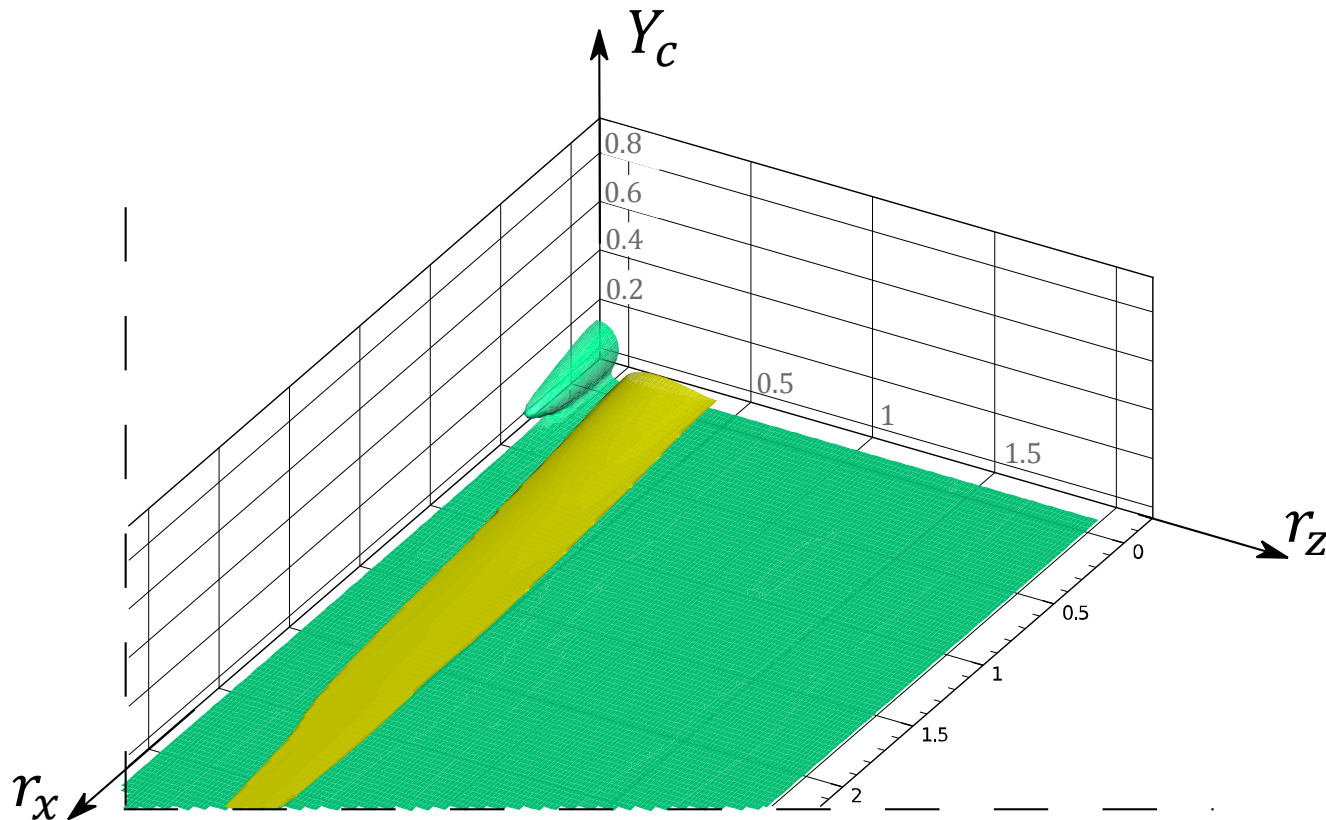
transport of scale energy across scales

not visible in the TKE budget!

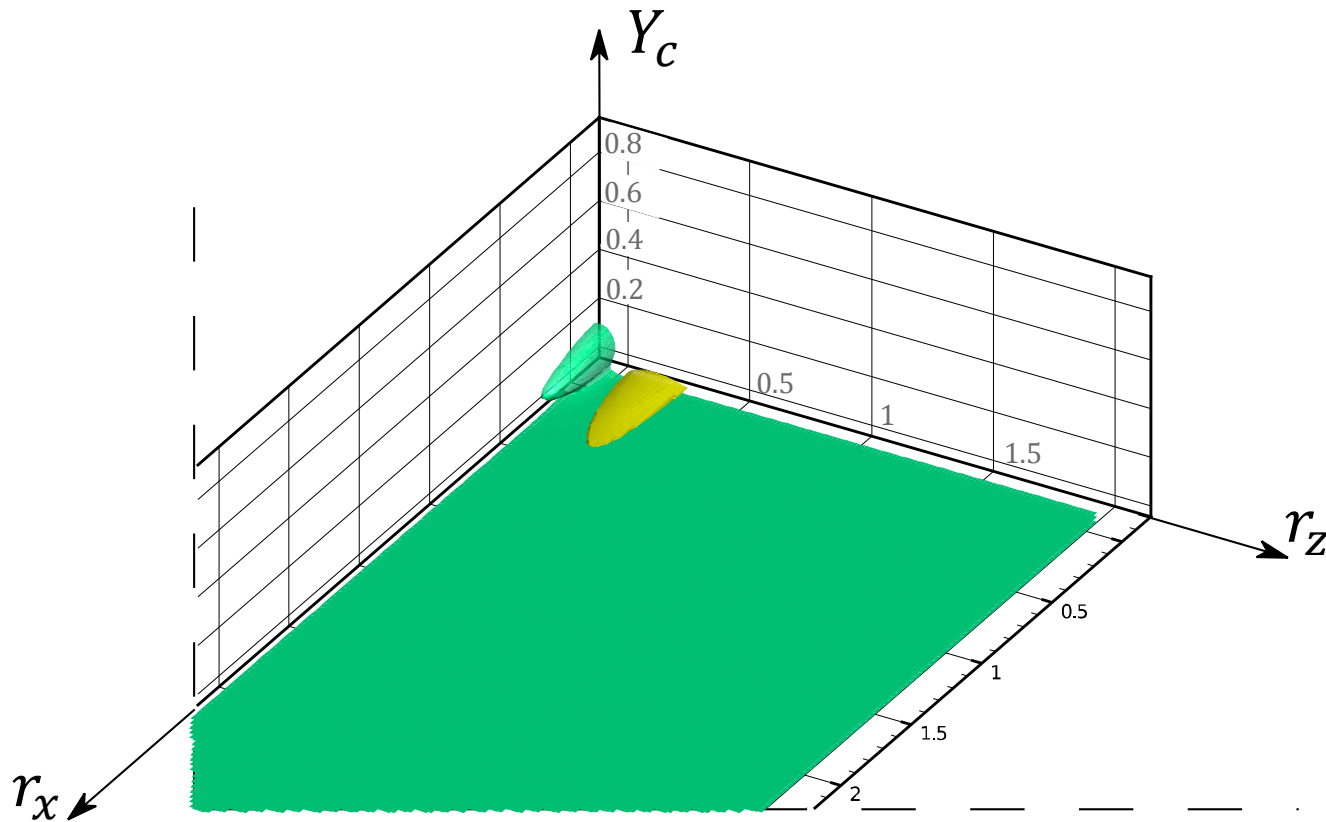
# Paths of energy (uncontrolled)



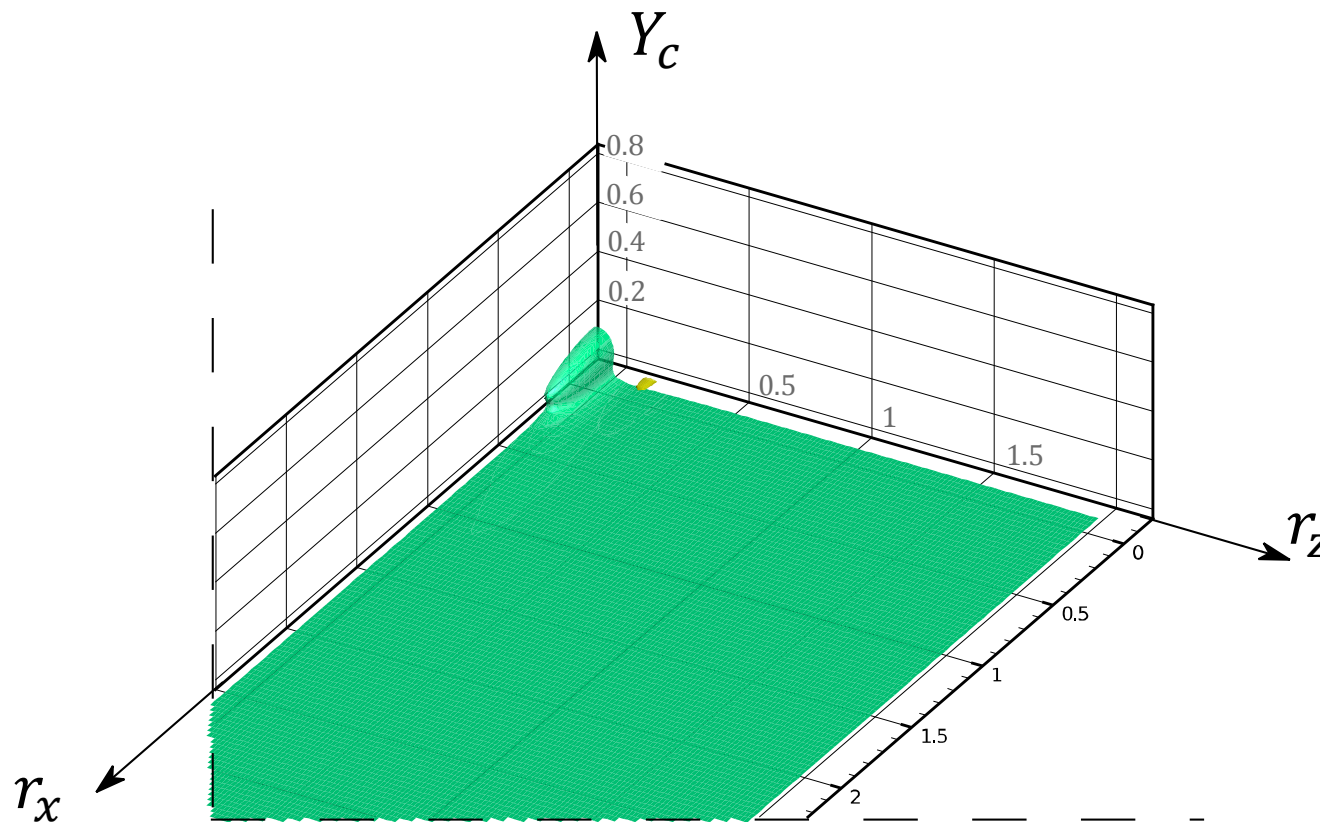
# Source term (uncontrolled)



# Source term (opposition control)

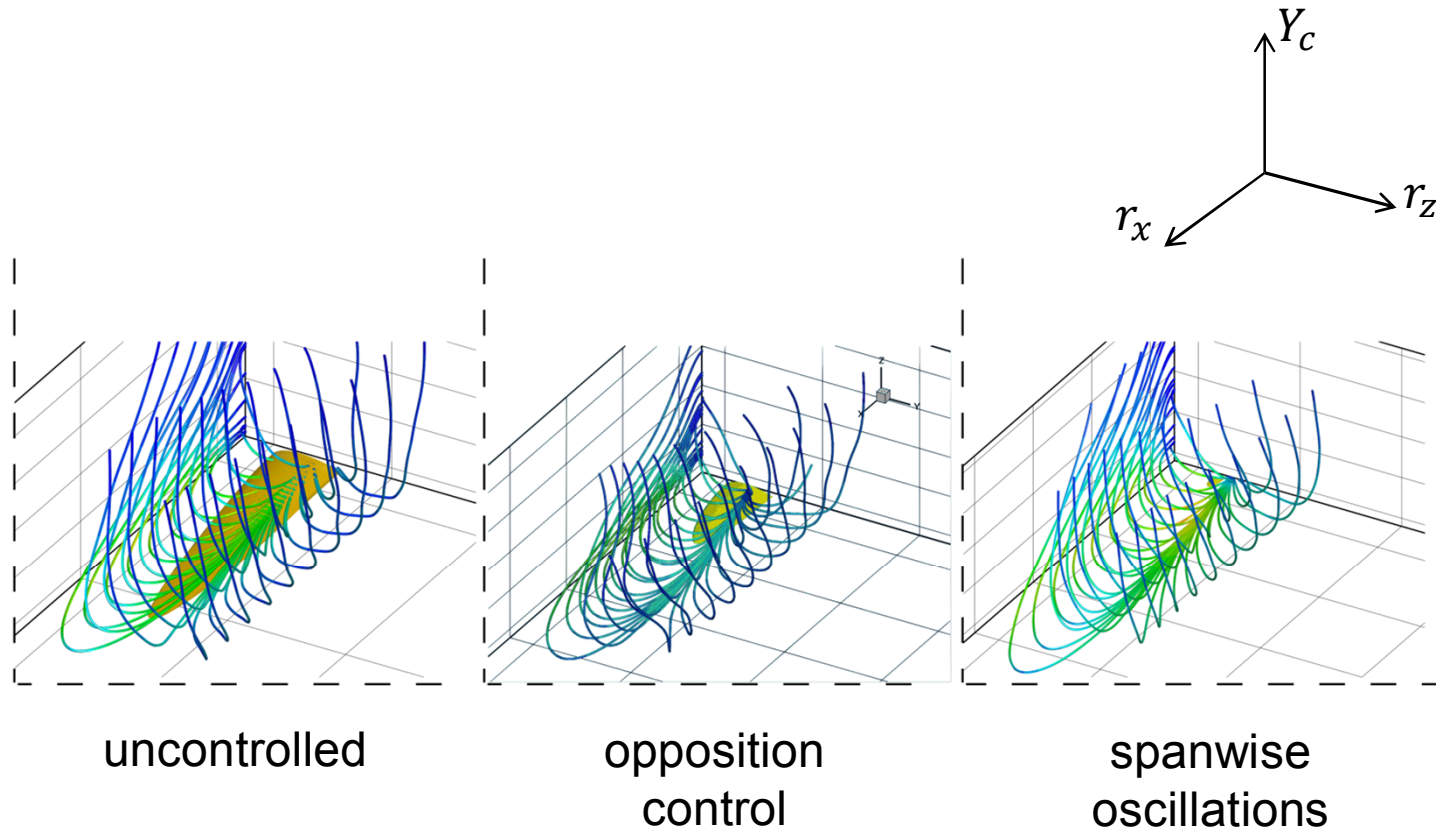


# Source term (wall oscillations)



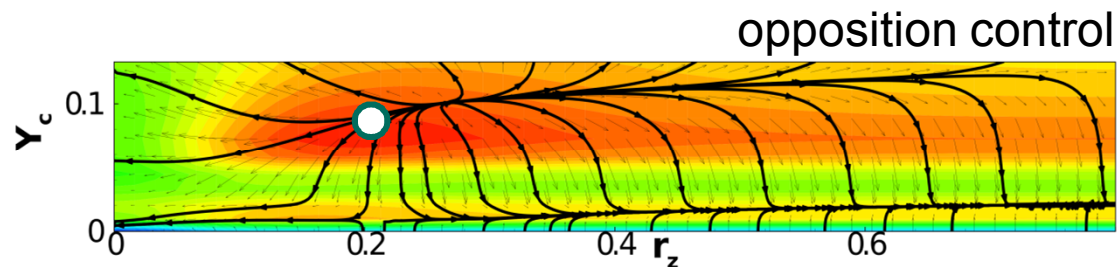
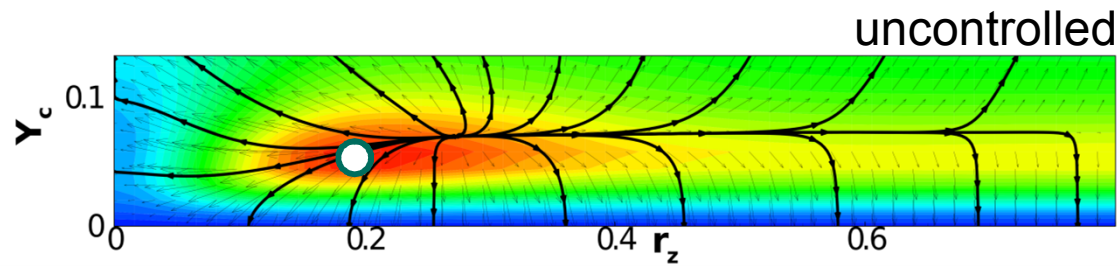


# Paths of energy (outer topology)

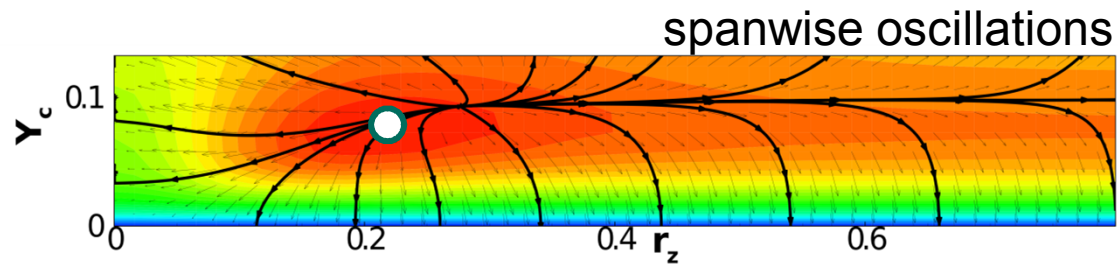


No significant changes in the outer topology  
of the scale energy fluxes

# Paths of energy (inner topology) $r_x = 0$



intermediate  
source region,  
shield wall dissipation



much lower production  
(and dissipation) of  
scale energy

# Conclusions

- CPI natural framework to study control-induced changes of energy transfer rates
- Energy-box shows integral changes at a glance:  
**drag reduction**  $\Leftrightarrow$  **reduction of TKE dissipation** rate
- Scale-energy budgets show the changes in space at different scales  
**drag reduction**  $\Leftrightarrow$  upward **shift of peak production**  
reduction of wall prod. and diss.

# Outlook

- Honestly... these are preliminary results!!!

from qualitative to quantitative

- Find universal phenomena related to drag reduction

which mechanisms should  
successful control address?

- Large-scale energy production at high  $Re$

is near-wall control affected  
by large-scale energy production?

# THANKS

for your kind attention!

for questions, complaints, ideas:

**[davide.gatti@kit.edu](mailto:davide.gatti@kit.edu)**