Direct Numerical Simulation of Turbulent Wall Flows at Constant Power Input

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Flow Condition in Numerical Simulation

Example: Channel flow

Pressure drop: \( \Delta P = \frac{\tau_w L}{\delta} \)

Conventional approaches

- **Constant Flow Rate (CFR):** pressure drop (wall friction) fluctuates in time
  
  **Successful Control**  Reduction of pressure drop

- **Constant Pressure Gradient (CPG):** The flow rate fluctuates in time
  
  **Successful Control**  Increase of flow rate

Are they the only available options?  No!
Flow control problem
compromise between convenience and energy consumption

Money versus Time (Frohnapfel, Hasegawa & Quadrio, JFM 2012)

\[ E_p \propto (U_b)^{7/4} \]

\[ C_f \propto U_b^{-1} : \text{laminar} \]
\[ C_f \propto U_b^{-1/4} : \text{turbulent} \]
Practical Problems

Unsteady flow in piping system

Stenosis of arteries

Most flow conditions in real systems should be neither CFR nor CPG !
laminar flow in pipe w/wo orifice

Flow rate: $Q$

Πρεσσυρε γραδιεντ: $\Delta p/\Delta x$

Power input: $P_p$

CFR
$Q=\text{const}$

CPI
$P_p=\text{const}$

CPG
$\Delta p/\Delta x=\text{const}$

color code corresponds to pressure gradient
Comparison between Different Flow Conditions

<table>
<thead>
<tr>
<th></th>
<th>$Ub$</th>
<th>$\Delta P \propto \tau_w$</th>
<th>Pumping power $\propto Ub \Delta P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CFR</td>
<td>Const.</td>
<td>$\downarrow$</td>
<td>$\downarrow$</td>
</tr>
<tr>
<td>CPG</td>
<td>$\uparrow$</td>
<td>Const.</td>
<td>$\uparrow$</td>
</tr>
</tbody>
</table>

Successful control
Comparison between Different Flow Conditions

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<tr>
<th></th>
<th>$Ub$</th>
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<tbody>
<tr>
<td>CFR</td>
<td>Const.</td>
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</tr>
<tr>
<td>CPG</td>
<td>↑</td>
<td>Const.</td>
<td>↑</td>
</tr>
<tr>
<td>CPI</td>
<td>↑</td>
<td>↓</td>
<td>Const.</td>
</tr>
</tbody>
</table>

**Successful control**

**Advantage of CPI**

- Close to real operational condition (mechanical pump, heart, ……)
- Constant power input = constant dissipation = constant energy transfer rate
- Optimal ratio of total power $P_{total}$ and control power input $P_c$

$$\gamma = \frac{\text{control power input}}{\text{total power input}} = \frac{P_c}{P_{total}} = \frac{P_c}{P_p + P_c}$$
Introduction to CPI concept
Problem Setting

Channel flow

Control power input $P_c$

Pumping power $P_p$

Depth: $2\delta$

\[ P_{\text{total}} = P_p + P_c = \text{const.} \]

Prescribed quantities

- Channel half depth $\delta$
- Fluid physical properties (kinetic viscosity: $\nu$)
- Total power input: $P_{\text{total}} = P_p + P_c = \text{const.}$
Velocity Scale based on Power Input

“The lower-limit of power consumption under CFR is achieved in the Stokes flow” Bewley (JFM, 2009), Fukagata et al. (Physica D, 2009)

The flow rate becomes maximum under CPI in the Stokes flow.

- Pumping power per unit wetted area

\[ P_p = \left( - \frac{dp}{dx} \right) \delta \cdot U_b \]

- Bulk velocity in the Stokes flow

\[ U_b = \frac{1}{3\mu} \left( - \frac{dp}{dx} \right) \delta^2 = \sqrt{\frac{P_p \delta}{3\mu}} \]

- The upper-limit of the bulk mean velocity under CPI

\[ U_p = \sqrt{\frac{P_t \delta}{3\mu}} \]

Velocity scale based on the total power consumption.
Non-dimensionalization

Channel flow

Total power input: $P_{total}$

Depth: $2\delta$

All quantities are normalized by

- $U_p = (P_{total}\delta/3\mu)^{1/2}$
- $\delta$

Navier-Stokes & Continuity Equations:

$$\frac{\partial u_i}{\partial t} + \frac{\partial (u_i u_j)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{1}{Re_p} \frac{\partial^2 u_i}{\partial x_j \partial x_j}, \quad \frac{\partial u_i}{\partial x_i} = 0$$

Total power input:

$$P_{total} = \frac{3}{Re_p} (= \text{const.})$$

Power-based Reynolds number

$$Re_p = \frac{U_p \delta}{\nu} \approx 6500$$

(Re$_{\tau,0}$ = 200)

Evaluation of control performance

Gain in flow rate

$$U_b / U_p \leq 1$$
Uncontrolled flow under CPI
## Relationship between Different Reynolds Numbers in Uncontrolled Flow

<table>
<thead>
<tr>
<th>$\text{Re}_\tau$</th>
<th>$\text{Re}_b$</th>
<th>$\text{Re}_p$</th>
<th>$U_b/\nu_\tau$</th>
<th>$U_p/\nu_\tau$</th>
<th>$U_p/U_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>1440</td>
<td>2191</td>
<td>14.4</td>
<td>21.9</td>
<td>1.52</td>
</tr>
<tr>
<td>150</td>
<td>2289</td>
<td>4143</td>
<td>15.3</td>
<td>27.6</td>
<td>1.81</td>
</tr>
<tr>
<td><strong>200</strong></td>
<td><strong>3179</strong></td>
<td><strong>6511</strong></td>
<td><strong>15.9</strong></td>
<td><strong>32.6</strong></td>
<td><strong>2.05</strong></td>
</tr>
<tr>
<td>300</td>
<td>5054</td>
<td>12310</td>
<td>16.9</td>
<td>41.0</td>
<td>2.44</td>
</tr>
<tr>
<td>450</td>
<td>8032</td>
<td>23280</td>
<td>17.9</td>
<td>51.7</td>
<td>2.90</td>
</tr>
<tr>
<td>650</td>
<td>12230</td>
<td>41500</td>
<td>18.8</td>
<td>63.8</td>
<td>3.39</td>
</tr>
</tbody>
</table>
Time Trace of $U_b$ & $dp/dx$
Fundamental Flow Statistics

Mean Velocity

Velocity fluctuation

Results in CFR, CPG & CPI converge to the identical flow state in uncontrolled flow if $Re_b$, $Re_\tau$, $Re_p$ are adjusted properly.
Controlled flow under CPI

(Spanwise wall oscillation)

\[ P_{\text{total}} = P_{\text{pump}} + P_{\text{control}} = \text{const}. \]
Optimal Power Input

\[ \gamma = \frac{P_c}{P_{total}} \sim 0.1 \] leads to the maximum bulk mean velocity.
Conclusions

- Constant power input (CPI) condition is proposed as a flow condition alternative to conventional CFR and CPG
  ✓ close to real operational condition
  ✓ power input (= energy transfer rate = dissipation) is kept constant
  ✓ optimal ratio of total power input and control power input

- CPI condition is first implemented in DNS of wall turbulence
  ✓ Power-based velocity scale: Up
  ✓ dimensionless total power input: 3/Rep

- CPI simulation successfully run for the uncontrolled and controlled flows.
  ✓ Uncontrolled flow under CPI is essentially same as those under CFR and CPG.
  ✓ In the controlled flow, the maximum Ub is obtained when $\gamma$ is around 10%.
Turbulent Intensity in Spanwise Wall Oscillation Control

$u_{rms}$

$w_{rms}$

$v_{rms}$
Turbulent Intensity in Spanwise Wall Oscillation Control

$U^+_j$ rms

- uncontrolled
- CFR normalized by reference $u \tau,0$
Turbulent Intensity in Spanwise Wall Oscillation Control

\[ U^+_\text{rms} \]

- \text{uncontrolled}
- CFR normalized by reference \( u \tau,0 \)
- CFR normalized by actual \( u \tau \)

\( y^+ \)
Turbulent Intensity in Spanwise Wall Oscillation Control

Interpretation can be changed depending on flow conditions and normalization!
Fundamental Flow Statistics

Mean Velocity

Velocity fluctuation
Numerical Implementation

**Advance Flow Simulation**

- Calculate control Power Input $P_c$
- Calculate bulk mean velocity
- Calculate pressure gradient

Time step: $n \rightarrow n + 1$

$$P_{pump}^{n+1} = P_{total} - P_c^n$$

$$\left(- \frac{dp}{dx}\right)^{n+1} = \frac{P_{pump}^{n+1}}{U_b}$$
Energy Box

Ricco, Ottonelli, Hasegawa & Quadrio. (JFM, 2012)