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Turbulent drag reduction at moderate Reynolds numbers via spanwise velocity waves

Davide Gatti[1,2,*] and **Maurizio Quadrio**[2,**]

[1] Center of Smart Interfaces, Petersenstr. 32, 64287 Darmstadt
[2] Politecnico di Milano, Via La Masa 34, 20156 Milano

The question whether large turbulent drag reduction can be achieved at the high values of Re typical of applications is addressed. Answering such question, either by experiments or DNS, is obviously challenging. For DNS, the problem lies in the tremendous increase of the computational cost with Re, that has to be appreciated in view of the need of carrying out an entire parametric study at every Re, owing to the unknown location of the optimal forcing parameters.

In this paper we limit ourselves to considering an open-loop technique based on spanwise forcing, the streamwise-traveling waves introduced by [1], and explore via Direct Numerical Simulations (DNS) how the drag reduction varies when the friction Reynolds number is increased from $Re_\tau = 200$ to $Re_\tau = 2000$. To achieve high Re while keeping the computational cost affordable, computational domains of reduced size are employed. We adopted special care to interpret results that are indeed still box-size dependent, as well as strategies to compute the random errors and give the results an error bar.

Our results indicate that still $R = 0.29$ can be obtained at $Re_\tau = 2000$ in the partial region of the parameter space studied. The maximum R is found to decrease as $R \sim Re_\tau^{-0.22}$ in the Reynolds range investigated. As most important outcome, we find that the sensitivity of R to Re becomes smaller when far from the low-Re optimum parameters: in this region, we suggest $R \sim Re_\tau^{-0.08}$.

1 Introduction

In the last decades a large research effort has been devoted to conceive, understand and develop techniques for the reduction of the turbulent skin-friction drag, that is of paramount importance in applications, due to the potentially high economical savings. Among these techniques open-loop active techniques are a good compromise between energy gain and energy expenditure to yield sizable net energy savings with the advantage of moderate complexity, as they do not require the deployment of micron-sized sensors and only require relatively large-scale actuators. All the active techniques lack a throurough evaluation in flow characterized by an high value of the Reynolds number, due to the rapidly increasing computational costs and difficulties in setting up experiments. Unfortunately, most of the anticipated applications are at high Re. The low-Re data available show that the drag reduction (DR) decay with Re, even though it remains to be understood whether such a decrease is a low-Re effect, and performances are deemed to stabilize as soon as Re becomes large enough, or is expected to continue indefinitely, so that no drag reduction can be achieved at values of Re typical of the applications. In this paper, we focus on open-loop spanwise wall forcing, and in particular the streamwise traveling waves of Quadrio *et al.* [1]:

$$W(x,t) = A \cos\left(\frac{2\pi}{\lambda_x} x - \frac{2\pi}{T} t\right)$$

where $W(x,t)$ is the spanwise velocity forcing at the wall, of amplitude A. This forcing reduces to the spanwise-oscillating wall for $\lambda_x \to \infty$. This technique has been studied intensively only at $Re_\tau = 200$ and with few DNS at $Re_\tau = 400$ ([1]). For the sole case of oscillating wall, the Re-effect on the point of maximum drag reduction has been studied by J.I.Choi *et al.* [2] with DNS up to $Re_\tau = 400$ and through LES up to $Re_\tau = 1000$ by Touber & Leschziner [3]. All the studies suggest that the drag reduction decay as $R \sim Re_\tau^{-0.2}$.

In this paper, we use DNS to obtain information on the performance of the streamwise-traveling waves up to $Re_\tau = 2000$. The aim is to be able to carry out a parametric study at higher Re, avoiding the usual hypothesis of constancy and wall-units scaling of the optimal forcing parameters. To keep the computational study manageable, we adjust the size of the computational domain while carefully controlling the effect of this essential discretization parameters on the reliability of the obtained drag reduction.

2 Method

The DNS channel-flow code used is a mixed-discretization parallel solver of the incompressible Navier–Stokes equations described by [4], based on Fourier expansions in the homogeneous directions and high-order explicit compact finite-difference

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[*] Corresponding author: e-mail gatti@csi.tu-darmstadt.de, phone +49 6151 16 75281
[**] Corresponding author: e-mail maurizio.quadrio@polimi.it, phone +49 6151 16 75281

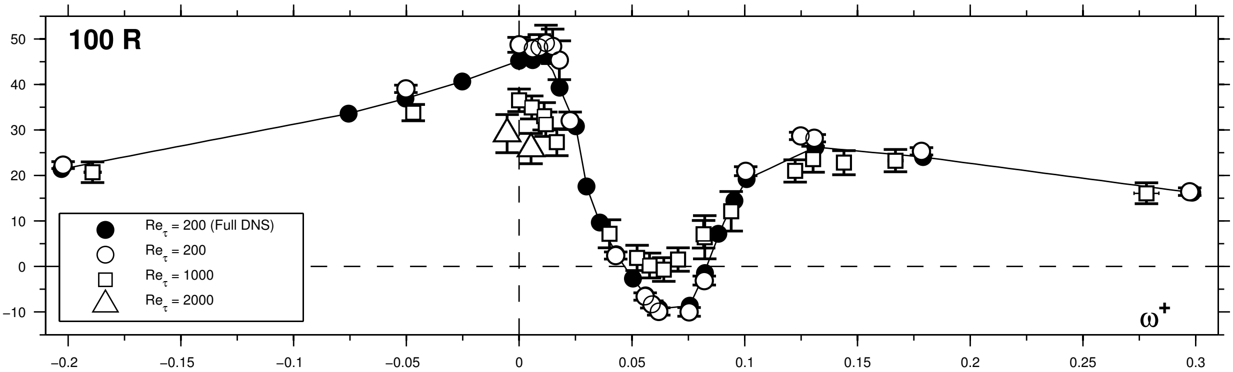

Fig. 1: Drag reduction ratio R against the forcing frequency $\omega = 2\pi/T$ for waves at $\lambda_x^+ = 1256$ and $A^+ = 12$. Error bars at 95% confidence level.

schemes in the wall-normal direction. Constant flow-rate simulations are run from a properly generated initial condition. Parametric DNS studies have been carried out at $Re_\tau = u_\tau h/\nu = 200$ and 1000 respectively, based on the friction velocity u_τ of the uncontrolled flow. A more limited dataset is produced at $Re_\tau = 2000$, the highest Reynolds number at which the technique has been tested. The resolution has been kept constant in wall units and is $\Delta x^+ = 10$ and $\Delta z^+ = 5$ respectively in the stream- and spanwise directions, while in the wall-normal direction varies from $\Delta y^+_{min} = 1$ at the wall to $\Delta y^+_{max} = 6.9$ at the centerline. The domain size was slightly smaller than a usual channel flow, being the $L_x^+ = 1250$ and $L_z^+ = 625$ the stream- and spanwise lengths. The domain size has been increased to $L_x^+ = 2700$ and $L_z^+ = 1350$ at $Re_\tau = 2000$ to improve the reliability and control the uncertainty of the results. Particular care have been taken in choosing the domain size, founding on criteria given by Flores and Jiménez [5] and on several checks. Due to the high fluctuation of the skin-friction coefficient and the finite averaging time, the results have been provided an error bar. Only waves at $A^+ = 12$ and $\lambda^+ = 1256$ have been simulated because in this region of the parameter space we have more literature data to compare with and because at this λ the maximum DR occurs. As measure of drag reduction, the ratio $R = (C_{f,0} - C_f)/C_{f,0}$ is used, that is the relative reduction of the skin-friction coefficient C_f with respect to the uncontrolled flow.

3 Result

The main result of the work is presented in the figure 1, where the drag reduction ratio is plotted against the forcing frequency ω. The first comparison should be made between full DNS results (black circles) from Quadrio *et al.* [1] and present results (circles) at $Re_\tau = 200$: DR is well predicted when a slightly smaller domain is used. When Re increases, the drag reduction diminishes. Unexpectedly, the reduction is heterogeneous in this part of the parameter space. The two relative drag reduction and increase maxima are very sensitive to a change in Re while far away from this region, the drag reduction is nearly unaffected and keeps almost the same. The only two points at $Re_\tau = 2000$ confirm the trend and show that $R = 0.29$ can still be achieved.

4 Conclusion

In this work a DNS parametric study of the streamwise-traveling waves has been carried on to Re_τ up to 2000 and the influence of Re on the drag reduction ave been evaluated. When optimal control parameters are used, the empirical law $R \sim Re_\tau^{-0.2}$ is found, in agreement with the studies on the sole oscillating wall by J.I.Choi *et al.* [2] and Touber & Leschziner [3]. The sensitivity of R to Re is found to be heterogeneous in the parameter space and when non-optimal parameters are used, the law $R \sim Re_\tau^{-0.08}$ is suggested, being the DR nearly unaffected by a change in Re. As most important outcome we claim the need for extensive parametric studies at higher Re, since focusing on the optimal parameters only, a common approach in literature, does not give complete information on the real complexity of the mechanisms that lead to the drag reduction.

References

[1] M. Quadrio, P. Ricco, and C. Viotti, J. Fluid Mech. **627**, 161–178 (2009).
[2] J. I. Choi, C. X. Xu, and H. J. Sung, AIAA J. **40**(5), 842–850 (2002).
[3] E. Touber and M. Leschziner, J. Fluid Mech. **693**, 150–200 (2012).
[4] P. Luchini and M. Quadrio, J. Comp. Phys. **211**(2), 551–571 (2006).
[5] O. Flores and J. Jiménez, Physics of Fluids **22**(7), 071704 (2010).