Non-sinusoidal wall oscillation for drag reduction

A. Cimarelli[1], B. Frohnapfel[2], Y. Hasegawa[2,3], E. De Angelis[1] and M. Quadrio[4]

[1]DIEM, Università di Bologna
[2]Center of Smart Interfaces, TU Darmstadt
[3]University of Tokyo
[4]Politecnico di Milano

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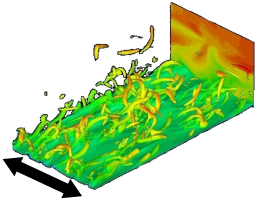
Wall oscillations for drag reduction

It is well known that drag reduction in wall turbulent flows can be induced by the cyclic movement of the wall in the spanwise direction. The amount of drag reduction obviously depends on

- Period of the oscillations T;
- Amplitude of the oscillations W_m;
- Shape of the oscillations;



All the attemps have been carried out by using sinusoidal wall oscillations and the effects of the oscillating function are still not determined!

- How do deviations from the sinusoid - which might occur when trying to realize an oscillating wall in practice - influence the control performance?
- Is it possible to obtain flow control performance superior to the sinusoidal wall oscillations with other oscillations shapes?

Numerical experiments

The problem we are addressing takes this form

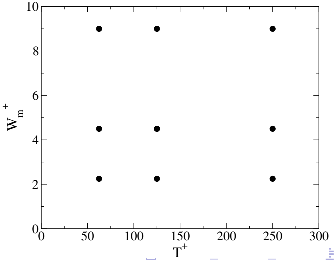
$$\textit{CONTROL PERFORMANCES} = f_i(W_m, T; Re)$$

The control performances depend on the oscillating parameter W_m and T and this dependency, $f_i()$, varyies changing the oscillating function, i-pedix.

$$\Downarrow$$

For each shape, several simulations to cover the space of parameter (W_m, T)

- Channel flow;
- Pseudo-spectral code;
- $Re_\tau = 200$, $\Delta x^+ = 10$, $\Delta z^+ = 5$



Selected wall oscillations

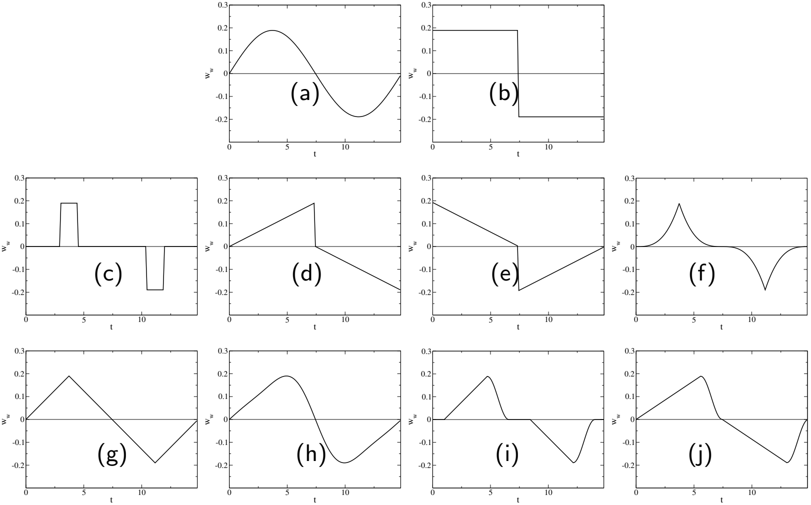


Figure: Oscillating wall function used in the present work.

Numerical results

The overall behavior in the (W_m, T)-space appears unaffected by the wave-form

Power Input,

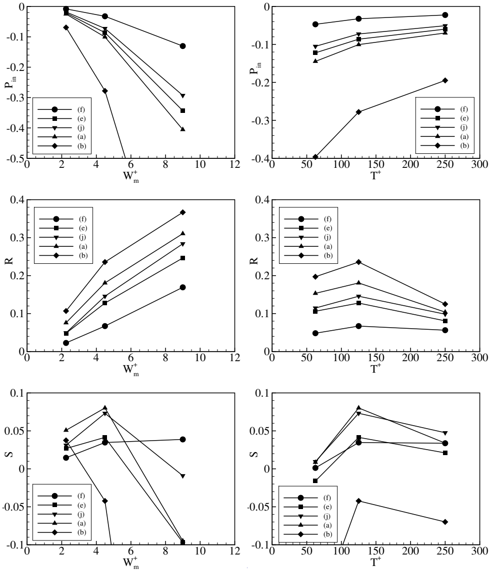
$$P_{in} = \frac{1}{t_f - t_i} \int_{t_i}^{t_f} w_w(t) \tau_z dt / P_0$$

Power Saved,

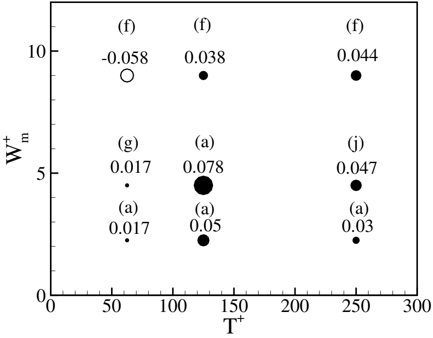
$$R = \frac{U_b}{t_f - t_i} \int_{t_i}^{t_f} (\tau_x^0 - \tau_x) dt / P_0$$

Net Energy Saving,

$$S = R - P_{in}$$



Overall performances



- No energy saving data larger than the optimal one, $S = 0.073$, for the sinusoidal case;
- There are regions of the (W_m, T)-space where the sinusoidal case is not the best;
- Local increase of the performances;

Generalized Stokes layer

To understand the effects of non-sinusoidal oscillations and to generalize the present results we consider the analytycal solutions of the Stokes problem

$$\frac{\partial \langle w(\mathbf{x},t)\rangle}{\partial t}(y,t) = \frac{1}{Re}\frac{\partial^2 \langle w(\mathbf{x},t)\rangle}{\partial y^2} \; ,$$

found to describe the spanwise turbulent flow above sinusoidal oscillations.

Linear diffusive problem $\Rightarrow$ Linear superposition of solutions

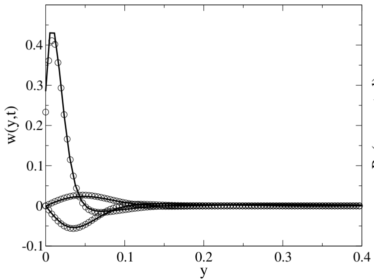
Harmonic decomposition:

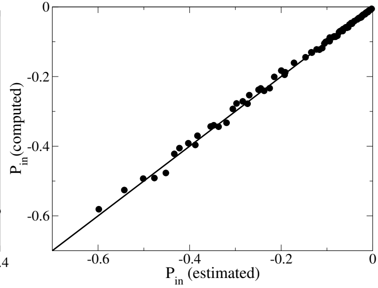
$$w_w(t) = W_m \sum_{k=0}^{\infty} W_k e^{j(2\pi k/T)t} + \text{c.c.}$$

Solution: linear superposition of spanwise layers due to all the harmonics composing the wall oscillation

$$w(y,t) = W_m \sum_{k=0}^{\infty} W_k e^{-y\sqrt{Re\pi k/T}} e^{j\left[(2\pi k/T)t - \sqrt{Re\pi k/T}y\right]} + \text{c.c.}\,.$$

Validation





- The averaged spanwise turbulent flow collapse with the Stokes prediction;
- The resulting power input, $P^{\star}_{in}$, can be directly predicted a priori as

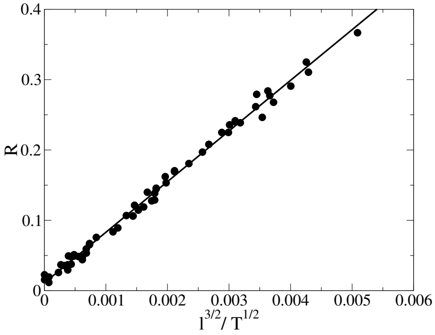
$$P^{\star}_{in} = W_m^2 \sum_{k=-\infty}^{\infty} W_k W_k^* \sqrt{\frac{\pi k}{ReT}}$$

Drag reduction prediction

The drag reduction is more difficult to determine since results from the complex interaction between spanwise oscillations and turbulence. Nonetheless it is possible to link the Stokes parameters to drag reduction for $T^+ < 150$.

Stokes thickness l: max wall distance y where the induced spanwise velocity variance is higher than a threshold W_{th}

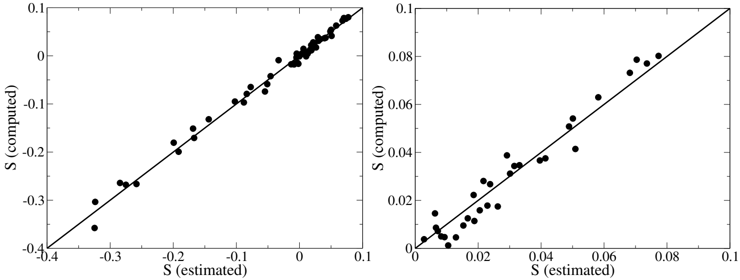
$$\begin{aligned} \langle w(y,t)^2\rangle(l) &= \\ W_m^2 \sum_{k=-\infty}^{\infty} W_k^2 e^{-2l\sqrt{Re\pi k/T}} &= W_{th} \end{aligned}$$



$$R \propto l^{3/2}/\sqrt{T}$$

Net energy saving prediction

From the Stokes prediction of P_{in} and the drag reduction scaling with l we are able to predict a priori the control performances of any kind wall oscillations for $T^+ < 150$.



$\Longrightarrow$ Predictive tool based on the generalized Stokes layer

Implications: sinusoid is the best?

Given the validity of the tool we are proposing we have that no better performances can be reached by using non-sinusoidal oscillations.

Power input:

$$P_{in} = W_m^2 \sum_{k=-\infty}^{\infty} W_k W_k^* \sqrt{\frac{\pi k}{Re T}}$$

Power saved:

$$R = f(l) \Longrightarrow \langle w(l,t)^2 \rangle (l) = W_m^2 \sum_{k=-\infty}^{\infty} W_k^2 e^{-2l\sqrt{Re \pi k / T}}$$

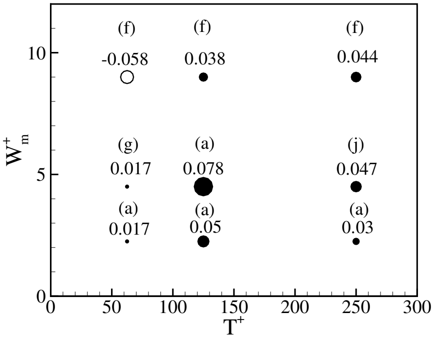
Each armonic W_k contributes to P_{in} with a weighting factor $\sqrt{k}$ while to the spanwise fluctuation with $\exp(-\sqrt{k})$.

$\Downarrow$

Given the base sinusoid, W_1, the superimposed armonics, W_k ($k > 1$), contribute more to power input than to the heigh of the Stokes layer!

But...

- Given a technological constrain in the maximum wall velocity W_m or period T it is possible to increase the performances by using non-sinusoidal oscillations with *ad-hoc* spectral decomposition



- What about large oscillating period, $T^+ > 150$?

Summary

The numerical analysis of non-sinusoidal wall oscillations for drag reduction highlighted:

- The oscillating shape actually matters for the control performances;
- No better control performances have been reached;
- Generalization of the Stokes solution to non-sinusoidal oscillations;
- Predictive tool based on the Stokes layer (important for the applications);
- The sinusoid is the best for drag reducing techniques;
- Given technological constraints the spectral behavior of non-sinusoidal oscillations could be helpfull;
- $T^+ > 150$?