NEAR-WALL ENSTROPHY GENERATION IN A DRAG-REDUCED TURBULENT CHANNEL FLOW WITH SPANWISE WALL OSCILLATIONS

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ERCOFTAC/PLASMAERO Workshop, Toulouse, 10 December 2012

ACTIVE OPEN-LOOP TECHNIQUE

Energy input into system

Pre-determined forcing

NUMERICAL APPROACH

Direct numerical simulations of wall turbulence

Fully-developed turbulent channel flow ($Re_\tau = u_\tau h/\nu = 200$)

Compact finite-difference scheme along wall-normal direction

Spectral discretization along streamwise and spanwise directions

SPANWISE WALL OSCILLATIONS

- New approach: *Turbulent enstrophy*
- *Transient evolution*

CONSTANT DP/DX

τ_w is fixed in fully-developed conditions

GAIN: U_b increases

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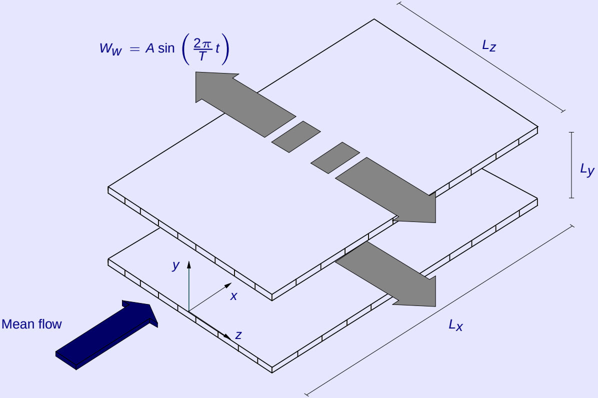
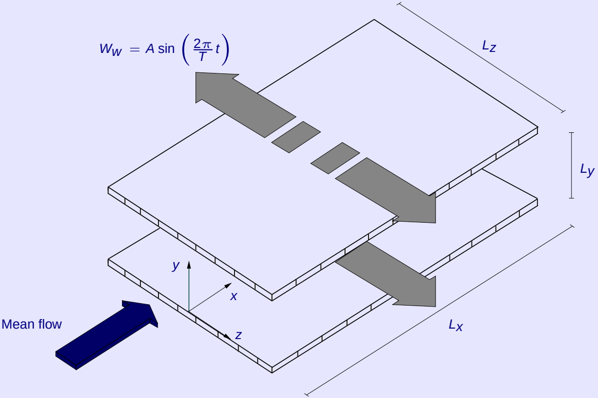


$$R = \frac{C_{f,r} - C_{f,o}}{C_{f,r}} = \frac{U_{b,o}^2 - U_{b,r}^2}{U_{b,o}^2}$$

Why does the skin-friction coefficent decrease?

$C_f = 2\tau_w/(\rho U_b^2)$ decreases $\rightarrow$ study why U_b increases



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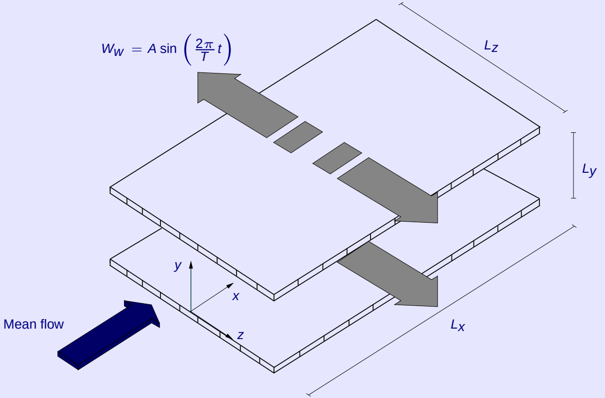

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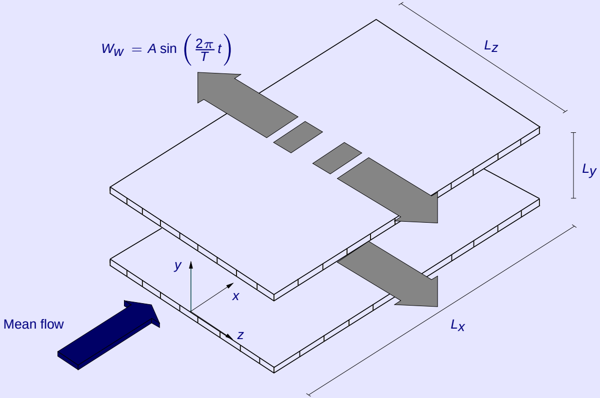

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SPACE: HOMOGENEOUS DIRECTIONS

$$\bar{f}(y,t) = \frac{1}{L_x L_z} \int_0^{L_x} \int_0^{L_z} f(x,y,z,t) \mathrm{d}z \mathrm{d}x$$

PHASE

$$\widehat{f}(y,\tau) = \frac{1}{N} \sum_{n=0}^{N-1} \bar{f}(y, nT + \tau)$$

TIME

$$\langle f \rangle (y) = \frac{1}{T} \int_0^T f(y,\tau) \mathrm{d}\tau$$

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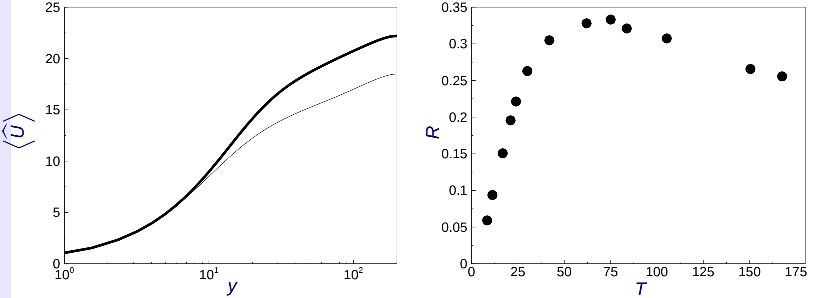
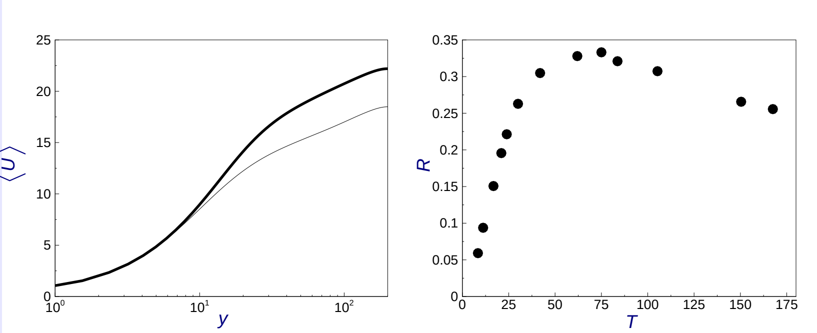


Scaling by viscous units

Mean velocity increases in the bulk of the channel

Mean wall-shear stress is unchanged

Optimum period of oscillation $T \approx 75$



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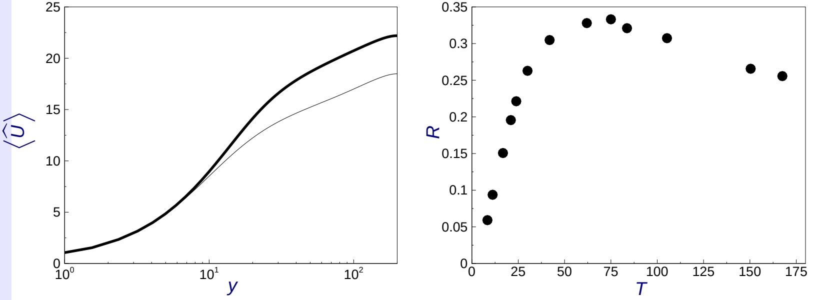

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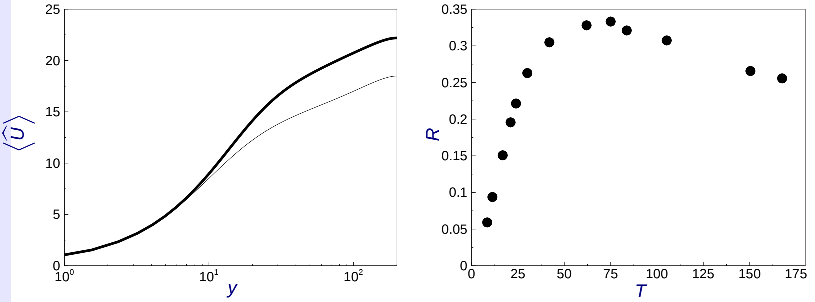

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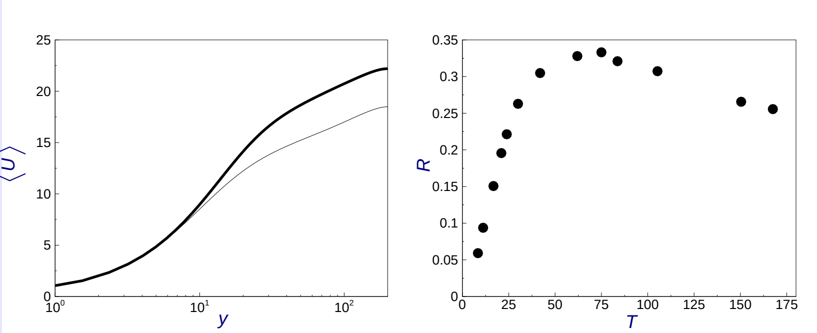

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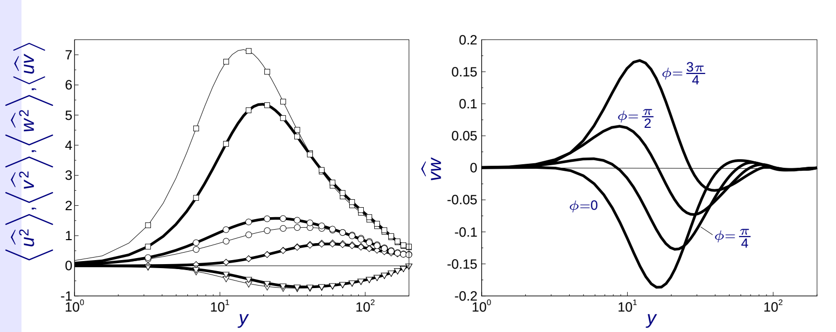
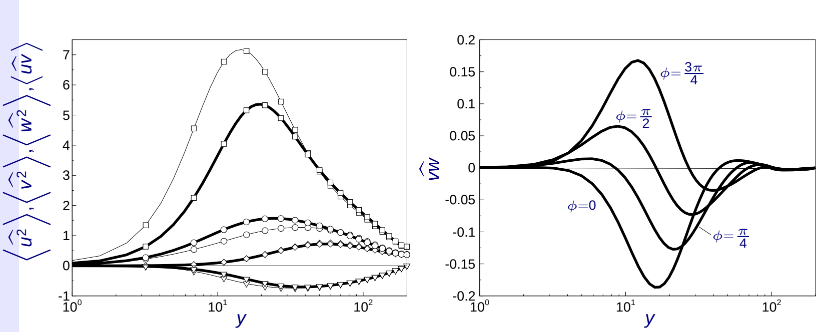


- Turbulence kinetic energy decreases
- Streamwise velocity fluctuations are attenuated the most
- New oscillatory Reynolds stress term $\widehat{vw}$ in created, $\langle \widehat{vw} \rangle = 0$



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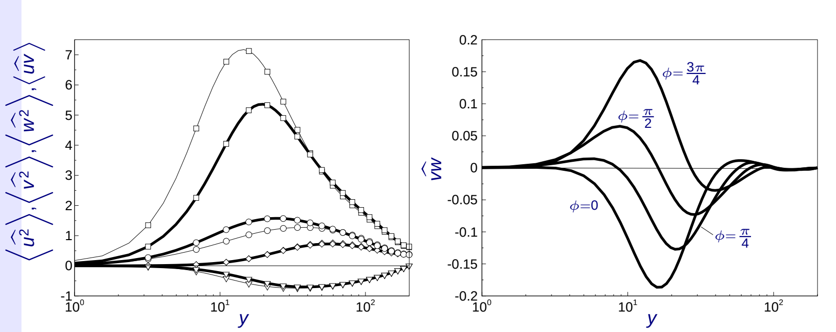

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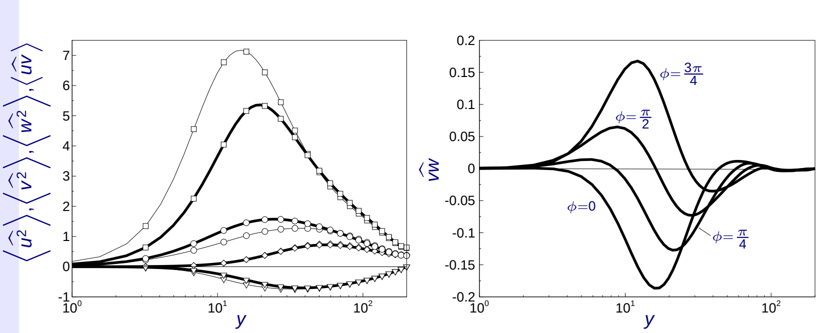

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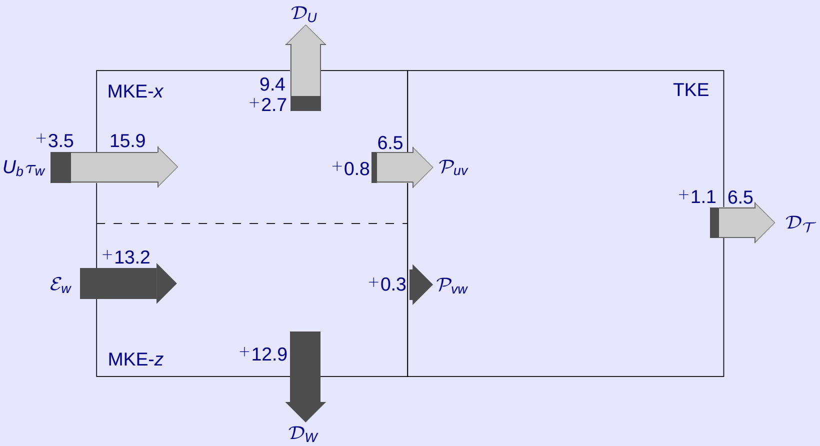
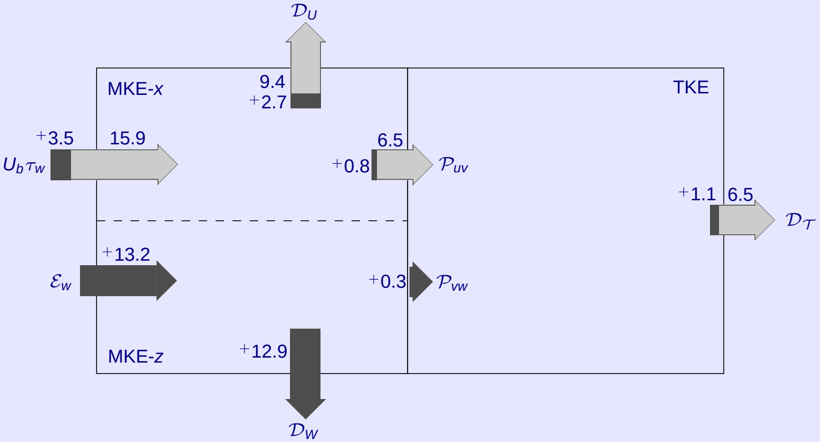
ENERGY BALANCE: A SCHEMATIC



- Energy is fed through P_x ($\rightarrow U_b\tau_W$) and wall motion ($\rightarrow \mathcal{E}_W$)
- Energy is dissipated through:
  - Mean-flow viscous effects ($\rightarrow \mathcal{D}_U, \mathcal{D}_W$)
  - Turbulent viscous effects ($\rightarrow \mathcal{D}_T$)



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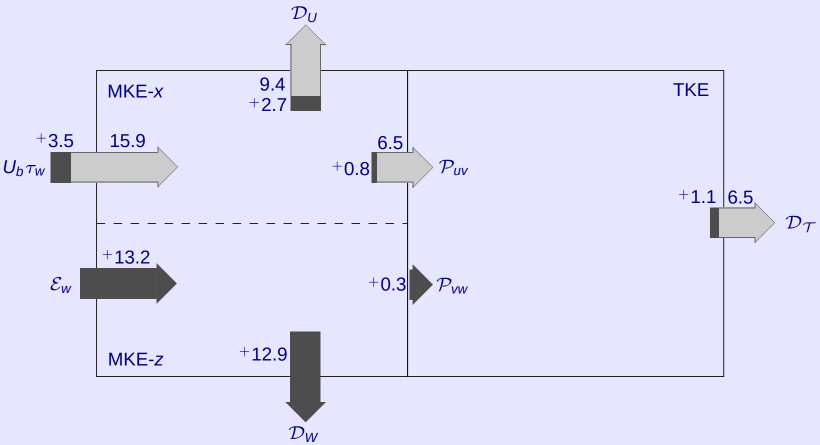

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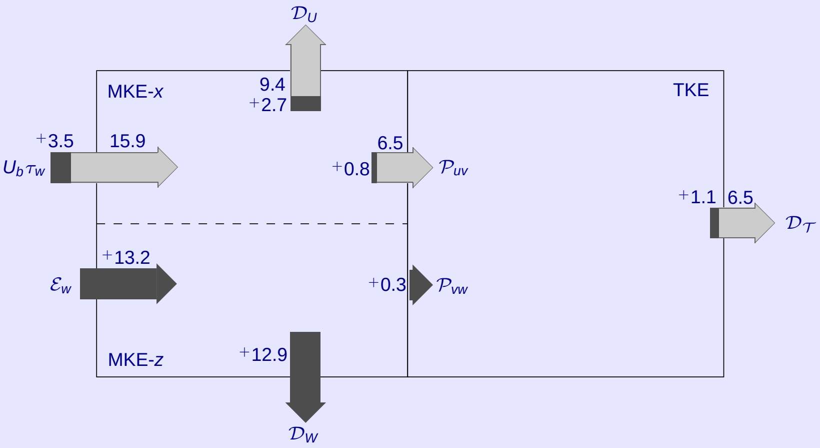

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GLOBAL MEAN KINETIC ENERGY EQUATION

$$\underbrace{U_b \tau_w + \left\langle A \left. \frac{\partial \widehat{W}}{\partial y} \right|_{y=0} \right\rangle}_{\mathcal{E}_W} = - \underbrace{\left[\widehat{uv} \frac{\partial \widehat{U}}{\partial y} \right]_g}_{\mathcal{P}_{uv}} - \underbrace{\left[\widehat{vw} \frac{\partial \widehat{W}}{\partial y} \right]_g}_{\mathcal{P}_{vw}} + \underbrace{\left[\left(\frac{\partial \widehat{U}}{\partial y} \right)^2 \right]_g}_{\mathcal{D}_U} + \underbrace{\left[\left(\frac{\partial \widehat{W}}{\partial y} \right)^2 \right]_g}_{\mathcal{D}_W}$$

GLOBAL TURBULENT KINETIC ENERGY EQUATION

$$\underbrace{\left[\widehat{uv} \frac{\partial \widehat{U}}{\partial y} \right]_g}_{\mathcal{P}_{uv}} + \underbrace{\left[\widehat{vw} \frac{\partial \widehat{W}}{\partial y} \right]_g}_{\mathcal{P}_{vw}} + \left[\widehat{\frac{\partial u_i}{\partial x_j} \frac{\partial u_i}{\partial x_j}} \right]_g = 0$$

TOTAL KINETIC ENERGY BALANCE

$$U_b \tau_w + \mathcal{E}_W = \mathcal{D}_U + \mathcal{D}_W + \mathcal{D}_T$$

TURBULENT DISSIPATION

$$\mathcal{D}_T = \left[\widehat{\omega_i \omega_i} \right]_g$$

Global mean kinetic energy equation

$$\underbrace{U_b \tau_w + \left\langle A \left. \frac{\partial \widehat{W}}{\partial y} \right|_{y=0} \right\rangle}_{\mathcal{E}_w} = -\underbrace{\left[\widehat{uv} \frac{\partial \widehat{U}}{\partial y} \right]_g}_{\mathcal{P}_{uv}} - \underbrace{\left[\widehat{vw} \frac{\partial \widehat{W}}{\partial y} \right]_g}_{\mathcal{P}_{vw}} + \underbrace{\left[\left(\frac{\partial \widehat{U}}{\partial y} \right)^2 \right]_g}_{\mathcal{D}_U} + \underbrace{\left[\left(\frac{\partial \widehat{W}}{\partial y} \right)^2 \right]_g}_{\mathcal{D}_W}$$

Global turbulent kinetic energy equation

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Total kinetic energy balance

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Turbulent dissipation

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Still to be answered

Why does TKE decrease?

Why does U_b increase?

Three possibilities

1. Stokes layer acts on $\mathcal{D}_U$ directly

   → excluded because W does not work directly on $\left(\partial\widehat{U}/\partial y\right)^2$

2. Stokes layer acts on $\mathcal{P}_{uv}$ directly

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3. Stokes layer acts on $\mathcal{D}_{\mathcal{T}} = \left[\widehat{\omega_i\omega_i}\right]_g$ directly

   → W works on turbulent vorticity transport

Turbulent enstrophy transport

Study the transport of turbulent enstrophy $\widehat{\omega_i\omega_i}$

The term *enstrophy* was coined by G. Nickel and is from Greek $\sigma\tau\rho o\phi\acute{\eta}$, which means *turn*

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$$\underbrace{\frac{1}{2}\frac{\partial \widehat{\omega_i \omega_i}}{\partial \tau}}_{1} = \underbrace{\widehat{\omega_x \omega_y}\frac{\partial \widehat{U}}{\partial y}}_{2} + \underbrace{\boxed{\widehat{\omega_z \omega_y}\frac{\partial \widehat{W}}{\partial y}}}_{3} + \underbrace{\boxed{\widehat{\omega_j \frac{\partial u}{\partial x_j}}\frac{\partial \widehat{W}}{\partial y}}}_{4} - \underbrace{\widehat{\omega_j \frac{\partial w}{\partial x_j}}\frac{\partial \widehat{U}}{\partial y}}_{5}$$

$$- \underbrace{\boxed{\widehat{v\omega_x}\frac{\partial^2 \widehat{W}}{\partial y^2}}}_{6} + \underbrace{\widehat{v\omega_z}\frac{\partial^2 \widehat{U}}{\partial y^2}}_{7} + \underbrace{\widehat{\omega_i \omega_j \frac{\partial u_i}{\partial x_j}}}_{8} - \underbrace{\frac{1}{2}\frac{\partial}{\partial y}\left(\widehat{v\omega_i \omega_i}\right)}_{9}$$

$$+ \underbrace{\frac{1}{2}\frac{\partial^2 \widehat{\omega_i \omega_i}}{\partial y^2}}_{10} - \underbrace{\widehat{\frac{\partial \omega_i}{\partial x_j}\frac{\partial \omega_i}{\partial x_j}}}_{11}.$$

Stokes layer influences dynamics of turbulent enstrophy

Three terms: which is the dominating one?

$\rightarrow$ Let's look at the terms of the equation

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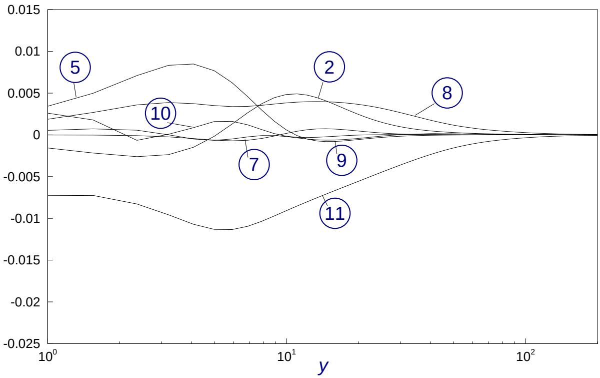
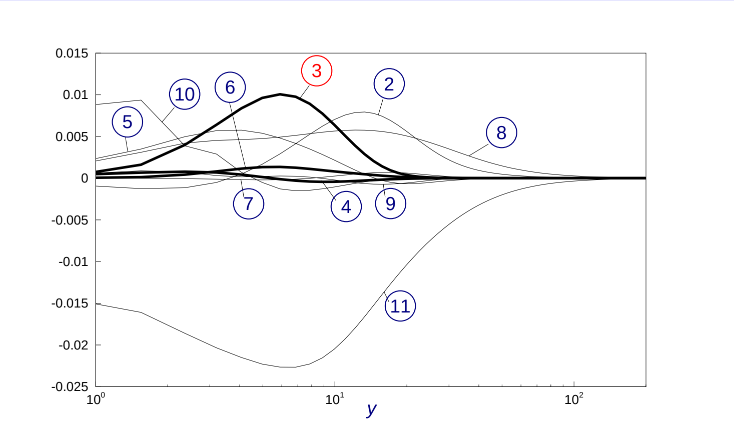
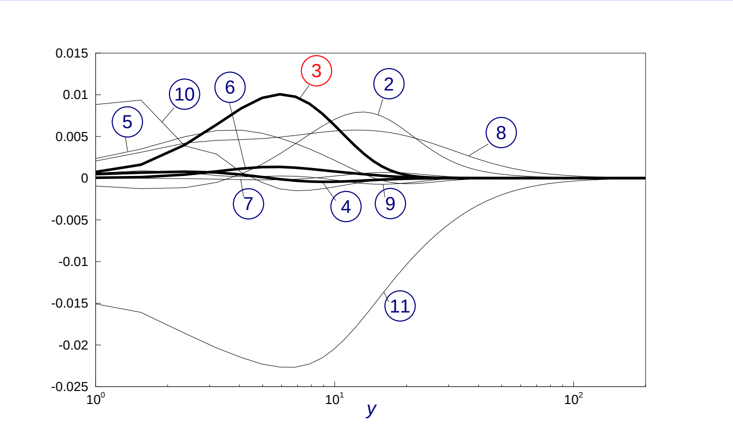

0.015
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02
-0.025
10^0
10^1
10^2
y
5
2
8
10
7
9
11



- Term 3, $\widehat{\omega_z \omega_y} \partial \widehat{W} / \partial y \rightarrow$ turbulent enstrophy production is dominant
- Other oscillating-wall terms are much smaller
- Turbulent dissipation of turbulent enstrophy increases



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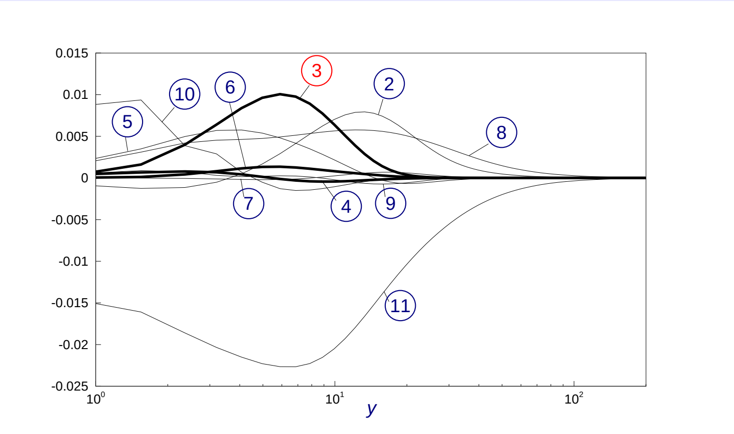

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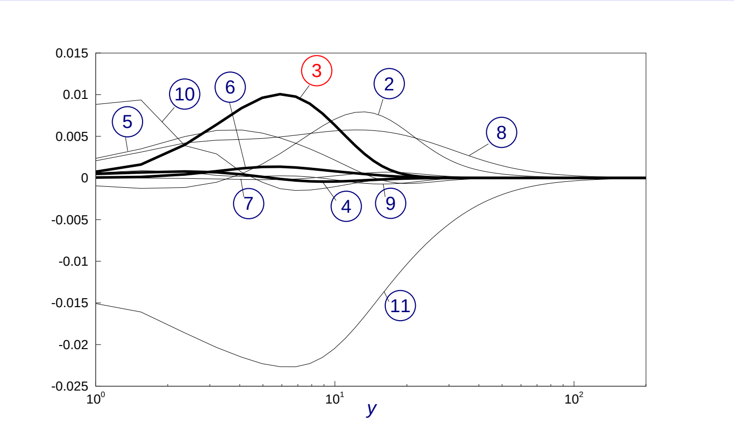

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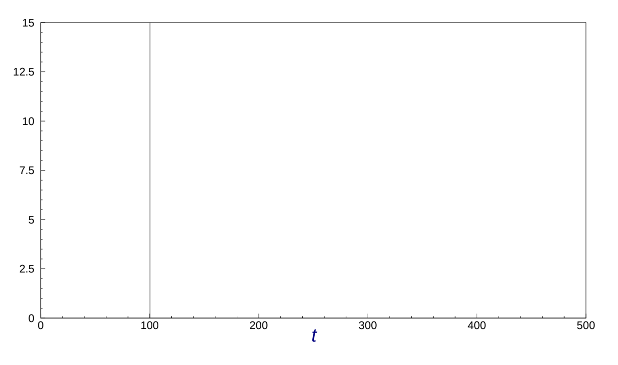
Key: transient from start-up of wall motion



USEFUL INFORMATION

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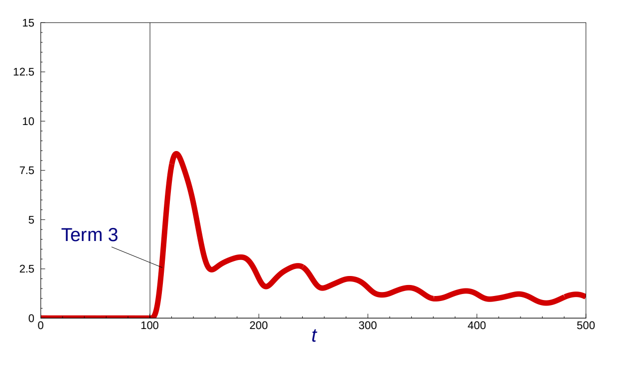

USEFUL INFORMATION

RED: term 3 increases abruptly, then decreases

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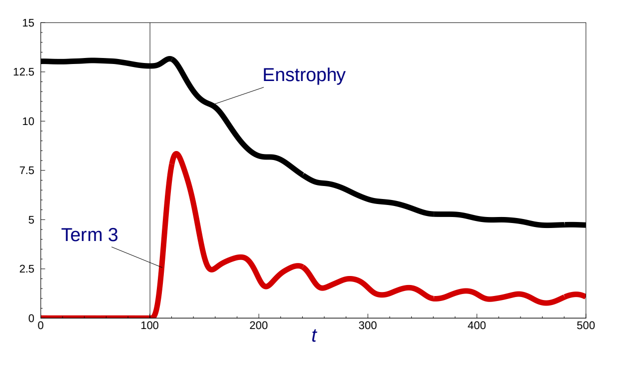

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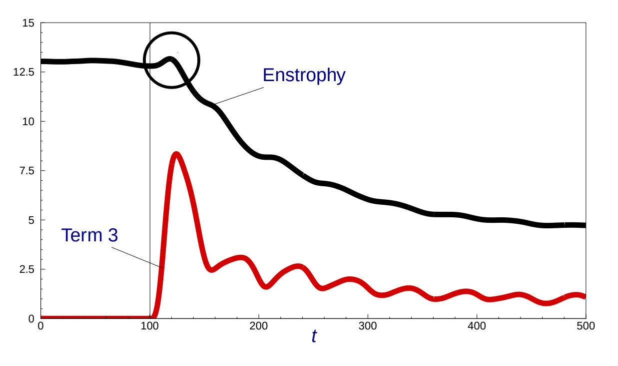

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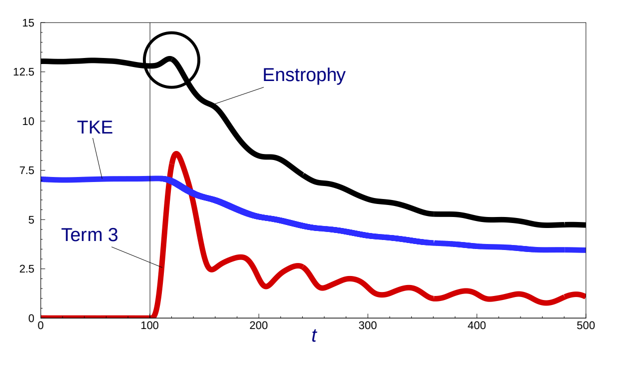

USEFUL INFORMATION

RED: term 3 increases abruptly, then decreases

BLACK: turbulent enstrophy increases, then decreases

BLUE: TKE decreases monotonically

Short stage

Turbulent enstrophy increases through $\widehat{\omega_z \omega_y} \partial \widehat{W} / \partial y$

Intermediate stage

TKE decreases because of enhanced turbulent dissipation

Long stage

Bulk velocity increases because of TKE reduction

$\longrightarrow$ drag reduction

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Initial state

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$t < 50$

$\overline{\omega_z \omega_y} \frac{\partial \overline{W}}{\partial y} \uparrow$ $\overline{\omega_i \omega_i} \uparrow$

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$\boxed{\mathcal{D}_T \uparrow}$

DRAG REDUCTION MECHANISM

Initial state

Short

$t < 50$

$\overline{\omega_z \omega_y} \frac{\partial \overline{W}}{\partial y} \uparrow$ $\overline{\omega_i \omega_i} \uparrow$

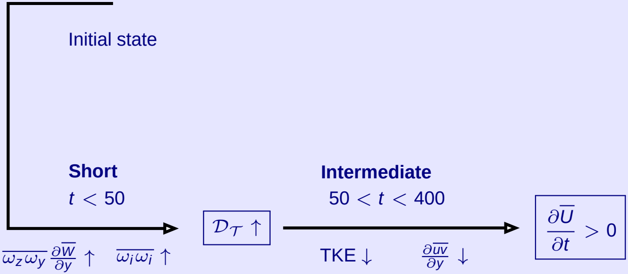
$\boxed{\mathcal{D}_T \uparrow}$

Intermediate

$50 < t < 400$

TKE $\downarrow$ $\frac{\partial \overline{uv}}{\partial y} \downarrow$

Drag reduction mechanism



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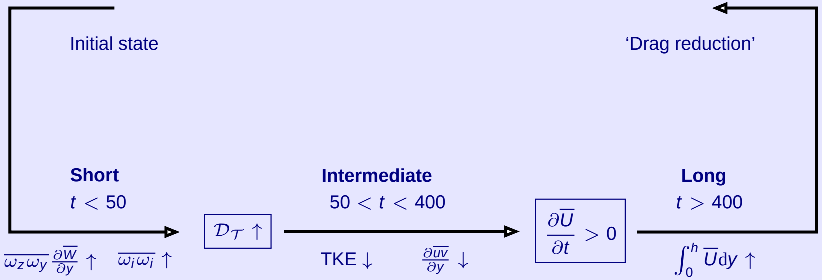
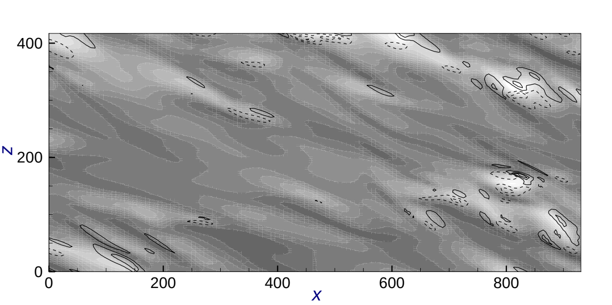


Physical interpretation of $\widehat{\omega_z \omega_y}\partial \widehat{W}/\partial y$

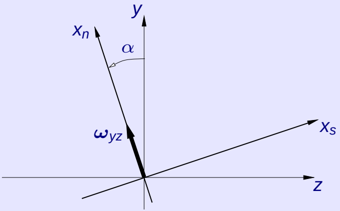
- $\widehat{\omega_z \omega_y}\partial \widehat{W}/\partial y$ is key term leading to drag reduction
- $\widehat{\omega_z \omega_y}\partial \widehat{W}/\partial y \rightarrow \partial \widehat{W}/\partial y$ acts on $\widehat{\omega_z \omega_y}$
- $\widehat{\omega_z \omega_y} \approx \widehat{\frac{\partial u}{\partial y}\frac{\partial u}{\partial z}}$

  $\frac{\partial u}{\partial y} \rightarrow$ upward eruption of near-wall low-speed fluid

  $\frac{\partial u}{\partial z} \rightarrow$ lateral flanks of the low-speed streaks



$\frac{\partial u}{\partial y}\frac{\partial u}{\partial z}$ located at the sides of high-speed streaks



SIMPLIFIED TURBULENT ENSTROPHY EQUATION

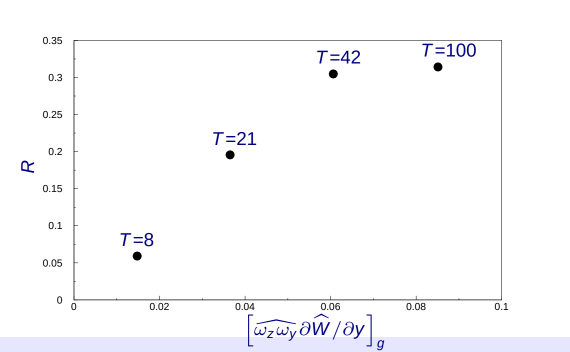
$$\frac{1}{2}\frac{\partial}{\partial t}\left(\omega_y^2+\omega_z^2\right)=\omega_z\omega_y G-\left(\frac{\partial\omega_y}{\partial y}\right)^2-\left(\frac{\partial\omega_z}{\partial y}\right)^2$$

Rotation of axis

$$\frac{1}{2}\frac{\partial\omega_n^2}{\partial t}=S_{nn}\omega_n^2-\left(\frac{\partial\omega_n}{\partial y}\right)^2$$

Integration by Charpit's method

$$\omega_n=\omega_{n,0}\underbrace{\mathrm{e}^{\sin\alpha\cos\alpha G t}}_{\text{stretching}}\underbrace{\mathrm{e}^{-\beta^2 t}\mathrm{e}^{-\beta y}}_{\text{dissipation}},\ \beta=\frac{\partial\omega_n/\partial t}{\partial\omega_n/\partial y}\sim\frac{\lambda_y}{\lambda_t}$$



Drag reduction grows monotonically with global production term

This happens up to optimum period

THANK YOU!

Reference

Ricco, P. Ottonelli, C. Hasegawa, Y. Quadrio, M.
Changes in turbulent dissipation in a channel flow with oscillating walls
J. Fluid Mech., 700, 77-104, 2012.