Near-wall enstrophy generation in a drag-reduced turbulent channel flow with spanwise wall oscillations

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**Active open-loop technique**

Energy input into system
Pre-determined forcing

**Numerical approach**

Direct numerical simulations of wall turbulence
Fully-developed turbulent channel flow ($Re_\tau = u_\tau h/\nu = 200$)
Compact finite-difference scheme along wall-normal direction
Spectral discretization along streamwise and spanwise directions

**Spanwise wall oscillations**

- New approach: *Turbulent enstrophy*
- *Transient evolution*

**Constant dp/dx**

$\tau_w$ is fixed in fully-developed conditions
**GAIN:** $U_b$ increases
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# Turbulent Drag Reduction

## Active Open-Loop Technique
- Energy input into system
- Pre-determined forcing

## Numerical Approach
- Direct numerical simulations of wall turbulence
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Spanwise Wall Oscillations
Geometry

\[ W_W = A \sin \left( \frac{2\pi}{T} t \right) \]

Mean flow

\[ R = \frac{C_{f,r} - C_{f,o}}{C_{f,r}} = \frac{U_{b,r}^2 - U_{b,o}^2}{U_{b,o}^2} \]

Why does the skin-friction coefficient decrease?

\[ C_f = 2\tau W / (\rho U_b^2) \] decreases → study why \( U_b \) increases

5 December 2012
Spanwise wall oscillations

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$R = \frac{C_{f,r} - C_{f,o}}{C_{f,r}} = \frac{U_{b,o}^2 - U_{b,r}^2}{U_{b,o}^2}$

Why does the skin-friction coefficient decrease?

$C_f = \frac{2\tau_w}{\rho U_b^2}$ decreases $\rightarrow$ study why $U_b$ increases
Averaging Operators

Space: Homogeneous Directions

$$\bar{f}(y, t) = \frac{1}{L_x L_z} \int_0^{L_x} \int_0^{L_z} f(x, y, z, t) \, dz \, dx$$

Phase

$$\hat{f}(y, \tau) = \frac{1}{N} \sum_{n=0}^{N-1} \bar{f}(y, nT + \tau)$$

Time

$$\langle f \rangle (y) = \frac{1}{T} \int_0^{T} f(y, \tau) \, d\tau$$

Global

$$[f]_g = \int_0^{n} \langle f \rangle (y) \, dy$$
AVERAGING OPERATORS

**SPACE: HOMOGENEOUS DIRECTIONS**

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\overline{f}(y, t) = \frac{1}{L_x L_z} \int_0^{L_x} \int_0^{L_z} f(x, y, z, t) dz dx
\]

**PHASE**

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\[ \hat{f}(y, \tau) = \frac{1}{N} \sum_{n=0}^{N-1} \bar{f}(y, nT + \tau) \]

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\[ \langle f \rangle (y) = \frac{1}{T} \int_0^T f(y, \tau) \, d\tau \]

**GLOBAL**

\[ [f]_g = \int_0^h \langle f \rangle (y) \, dy \]
Averaging operators

**Space: Homogeneous Directions**

\[ \tilde{f}(y, t) = \frac{1}{L_x L_z} \int_0^{L_x} \int_0^{L_z} f(x, y, z, t) dz dx \]

**Phase**

\[ \hat{f}(y, \tau) = \frac{1}{N} \sum_{n=0}^{N-1} \tilde{f}(y, nT + \tau) \]

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\[ \langle f \rangle (y) = \frac{1}{T} \int_0^T f(y, \tau) d\tau \]

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\[ [f]_g = \int_0^h \langle f \rangle (y) dy \]
**SPACEx: HOMOGENEOUS DIRECTIONS**

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\bar{f}(y, t) = \frac{1}{L_x L_z} \int_{0}^{L_x} \int_{0}^{L_z} f(x, y, z, t) \, dz \, dx
\]

**PHASE**

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\hat{f}(y, \tau) = \frac{1}{N} \sum_{n=0}^{N-1} \bar{f}(y, nT + \tau)
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\langle f \rangle (y) = \frac{1}{T} \int_{0}^{T} f(y, \tau) \, d\tau
\]

**GLOBAL**

\[
[f]_g = \int_{0}^{h} \langle f \rangle (y) \, dy
\]
Scaling by viscous units
Mean velocity increases in the bulk of the channel
Mean wall-shear stress is unchanged
Optimum period of oscillation $T \approx 75$
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Scaling by viscous units

Mean velocity increases in the bulk of the channel
Mean wall-shear stress is unchanged
Optimum period of oscillation $T \approx 75$
Turbulence kinetic energy decreases.
Streamwise velocity fluctuations are attenuated the most.
New oscillatory Reynolds stress term $\hat{vw}$ is created, $\langle \hat{vw} \rangle = 0$.
Turbulence kinetic energy decreases

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New oscillatory Reynolds stress term $\langle \hat{vw} \rangle$ in created, $\langle \hat{vw} \rangle = 0$
Energy balance: a schematic

Energy is fed through $P_x \rightarrow U_b \tau_w$ and wall motion $\rightarrow \varepsilon_w$

Energy is dissipated through:
- Mean-flow viscous effects $\rightarrow D_U, D_W$
- Turbulent viscous effects $\rightarrow D_T$

![Diagram showing energy balance and dissipation](image-url)
Energy balance: a schematic

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- Mean-flow viscous effects $\rightarrow \mathcal{D}_U, \mathcal{D}_W$
- Turbulent viscous effects $\rightarrow \mathcal{D}_\tau$
Energy is fed through $P_x \rightarrow U_b \tau_w$ and wall motion $\rightarrow E_w$.

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**Global Mean Kinetic Energy Equation**

\[
U_b \tau_w + \left\langle A \frac{\partial \hat{W}}{\partial y} \right|_{y=0} \right\rangle = - \left[ \hat{u} v \frac{\partial \hat{U}}{\partial y} \right]_{g} - \left[ \hat{v} w \frac{\partial \hat{W}}{\partial y} \right]_{g} + \left[ \left( \frac{\partial \hat{U}}{\partial y} \right)^{2} \right]_{g} + \left[ \left( \frac{\partial \hat{W}}{\partial y} \right)^{2} \right]_{g}
\]

**Global Turbulent Kinetic Energy Equation**

\[
\left[ \hat{u} v \frac{\partial \hat{U}}{\partial y} \right]_{g} + \left[ \hat{v} w \frac{\partial \hat{W}}{\partial y} \right]_{g} + \left[ \frac{\partial u_i \partial u_i}{\partial x_j \partial x_j} \right]_{g} = 0
\]

**Total Kinetic Energy Balance**

\[
U_b \tau_w + \varepsilon_w = D_U + D_W + D_T
\]

**Turbulent Dissipation**

\[
D_T = \left[ \hat{\omega}_i \hat{\omega}_i \right]_{g}
\]
**Global mean kinetic energy equation**

\[ U_b \tau_w + \left( A \frac{\partial \hat{W}}{\partial y} \bigg|_{y=0} \right) = - \left[ \hat{u}v \frac{\partial \hat{U}}{\partial y} \right]_{g} - \left[ \hat{w}w \frac{\partial \hat{W}}{\partial y} \right]_{g} + \left[ \left( \frac{\partial \hat{U}}{\partial y} \right)^2 \right]_{g} + \left[ \left( \frac{\partial \hat{W}}{\partial y} \right)^2 \right]_{g} \]

\[ \mathcal{P}_{uv} \]
\[ \mathcal{P}_{ww} \]
\[ \mathcal{D}_{U} \]
\[ \mathcal{D}_{W} \]

**Global turbulent kinetic energy equation**

\[ \left[ \hat{u}v \frac{\partial \hat{U}}{\partial y} \right]_{g} + \left[ \hat{w}w \frac{\partial \hat{W}}{\partial y} \right]_{g} + \left[ \frac{\partial u_i}{\partial x_j} \frac{\partial u_i}{\partial x_j} \right]_{g} = 0 \]

\[ \mathcal{P}_{uv} \]
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**Total kinetic energy balance**

\[ U_b \tau_w + \varepsilon_w = \mathcal{D}_U + \mathcal{D}_W + \mathcal{D}_T \]

**Turbulent dissipation**

\[ \mathcal{D}_T = \left[ \hat{\omega}^i \hat{\omega}^i \right]_{g} \]
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**Global Turbulent Kinetic Energy Equation**

\[ \left[ \hat{u} \frac{\partial \hat{U}}{\partial y} \right]_g + \left[ \hat{w} \frac{\partial \hat{W}}{\partial y} \right]_g + \left[ \frac{\partial u_i \partial u_i}{\partial x_j \partial x_j} \right]_g = 0 \]

**Total Kinetic Energy Balance**

\[ U_b \tau_w + \varepsilon_w = D_U + D_W + D_T \]

**Turbulent Dissipation**

\[ D_T = \left[ \omega_i \omega_i \right]_g \]
**Global Mean Kinetic Energy Equation**

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\]

\( \varepsilon_w \)

\( \mathcal{P}_{uv} \)

\( \mathcal{P}_{vw} \)

\( \mathcal{D}_U \)

\( \mathcal{D}_W \)

**Global Turbulent Kinetic Energy Equation**

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\left[ \hat{u} \frac{\partial \hat{U}}{\partial y} \right]_{g} + \left[ \hat{w} \frac{\partial \hat{W}}{\partial y} \right]_{g} + \left[ \frac{\partial u_i}{\partial x_j} \frac{\partial u_i}{\partial x_j} \right]_{g} = 0
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\( \mathcal{P}_{uv} \)

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\[ U_b \tau_w + \varepsilon_w = \mathcal{D}_U + \mathcal{D}_W + \mathcal{D}_\tau \]

**Turbulent Dissipation**

\[ \mathcal{D}_\tau = \left[ \omega_i \omega_i \right]_{g} \]
**Key Questions**

**Still to be answered**

- Why does TKE decrease?
- Why does $U_b$ increase?

**Three Possibilities**

1. **Stokes layer acts on $D_U$ directly**
   - → excluded because $W$ does not work directly on $(\partial \hat{U}/\partial y)^2$

2. **Stokes layer acts on $P_{uv}$ directly**
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3. **Stokes layer acts on $D_T = [\hat{\omega}_i \hat{\omega}_j]_g$ directly**
   - → $W$ works on turbulent vorticity transport

**Turbulent Enstrophy Transport**

Study the transport of turbulent enstrophy $\hat{\omega}_i \hat{\omega}_j$

The term *enstrophy* was coined by G. Nickel and is from Greek $\sigma \tau \rho \omega \phi \eta$, which means *turn*
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KEY QUESTIONS

STILL TO BE ANSWERED

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Why does $U_b$ increase?

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  $\rightarrow$ $W$ works on turbulent vorticity transport

TURBULENT ENSTROPHY TRANSPORT

Study the transport of turbulent enstrophy $\hat{\omega}_i \hat{\omega}_j$

The term enstrophy was coined by G. Nickel and is from Greek $\sigma \tau \rho \phi \eta$, which means turn
KEY QUESTIONS

STILL TO BE ANSWERED

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TURBULENT ENSTROPHY TRANSPORT

Study the transport of turbulent enstrophy $\hat{\omega}_I \hat{\omega}_I$

The term enstrophy was coined by G. Nickel and is from Greek $\sigma \tau \rho \phi \dot{\eta}$, which means turn
Stokes layer influences dynamics of turbulent enstrophy

Three terms: which is the dominating one?

→ Let’s look at the terms of the equation
Turbulent enstrophy equation

\[
\frac{1}{2} \frac{\partial \omega_j \omega_i}{\partial \tau} = \omega_x \omega_y \frac{\partial \hat{U}}{\partial y} + \omega_z \omega_y \frac{\partial \hat{W}}{\partial y} + \omega_j \frac{\partial u_j}{\partial x} \frac{\partial \hat{W}}{\partial y} - \omega_j \frac{\partial w_j}{\partial x} \frac{\partial \hat{U}}{\partial y} - \frac{\partial \omega_j}{\partial x} \frac{\partial \omega_i}{\partial x}.
\]

Stokes layer influences dynamics of turbulent enstrophy

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- \nu \omega_x \frac{\partial^2 \hat{W}}{\partial y^2} + \nu \omega_z \frac{\partial^2 \hat{U}}{\partial y^2} + \omega_i \omega_j \frac{\partial u_i}{\partial x_j} - \frac{1}{2} \frac{\partial}{\partial y} \left( \nu \omega_i \omega_i \right) \\
+ \frac{1}{2} \frac{\partial^2 \omega_i \omega_i}{\partial y^2} - \frac{\partial \omega_i}{\partial x_j} \frac{\partial \omega_i}{\partial x_j}.
\]

Stokes layer influences dynamics of turbulent enstrophy
Three terms: which is the dominating one?

→ Let's look at the terms of the equation
Term 3, $\omega_z \omega_y \hat{W}/\partial y \rightarrow$ turbulent enstrophy production is dominant.

Other oscillating-wall terms are much smaller.

Turbulent dissipation of turbulent enstrophy increases.
Term 3, $\omega z w_y \partial \widehat{W} / \partial y$ → turbulent enstrophy production is dominant.
Other oscillating-wall terms are much smaller.
Turbulent dissipation of turbulent enstrophy increases.
Term 3, $\tilde{\omega}_y \partial \tilde{W}/\partial y \rightarrow$ turbulent enstrophy production is dominant

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Turbulent dissipation of turbulent enstrophy increases
Term 3, \(\overline{\omega_z \omega_y} \partial \hat{W} / \partial y\) → turbulent enstrophy production is dominant

Other oscillating-wall terms are much smaller

Turbulent dissipation of turbulent enstrophy increases
We have not answered questions on TKE and $U_b$, yet

Key: transient from start-up of wall motion

Useful information
We have not answered questions on TKE and $U_b$, yet

Key: transient from start-up of wall motion

**Useful Information**

**RED:** term 3 increases abruptly, then decreases
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Key: transient from start-up of wall motion

**Useful Information**

**RED:** term 3 increases abruptly, then decreases

**BLACK:** turbulent enstrophy increases, then decreases
Interesting, but...

We have not answered questions on TKE and $U_b$, yet

Key: transient from start-up of wall motion

![Graph](image)

**Useful Information**

**RED:** term 3 increases abruptly, then decreases

**BLACK:** turbulent enstrophy increases, then decreases
We have not answered questions on TKE and $U_b$, yet

Key: transient from start-up of wall motion

**Useful Information**

**RED:** term 3 increases abruptly, then decreases

**BLACK:** turbulent enstrophy increases, then decreases

**BLUE:** TKE decreases monotonically
TRANSIENT: THREE STAGES

**SHORT STAGE**

Turbulent enstrophy increases through $\hat{\omega}_{zy} \partial \hat{W} / \partial y$

**INTERMEDIATE STAGE**

TKE decreases because of enhanced turbulent dissipation

**LONG STAGE**

Bulk velocity increases because of TKE reduction

$\rightarrow$ drag reduction
TRANSIENT: THREE STAGES

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Transient: Three Stages

**Short Stage**
- Turbulent enstrophy increases through $\hat{\omega}_z \hat{w}_y \partial \hat{W} / \partial y$

**Intermediate Stage**
- TKE decreases because of enhanced turbulent dissipation

**Long Stage**
- Bulk velocity increases because of TKE reduction
  - $\rightarrow$ drag reduction
Initial state
Initial state

Short

$t < 50$

$\omega_z \omega_y \frac{\partial \overline{w}}{\partial y} \uparrow \quad \overline{\omega_i \omega_i} \uparrow$
DRAG REDUCTION MECHANISM

Initial state

Short
\( t < 50 \)

\( \omega_z \omega_y \frac{\partial W}{\partial y} \uparrow \quad \omega_i \omega_i \uparrow \)

\( D_T \uparrow \)
Drag reduction mechanism

Initial state

Short
$t < 50$

Intermediate
$50 < t < 400$

$\omega_z \omega_y \frac{\partial \bar{w}}{\partial y} \uparrow \quad \omega_i \omega_i \uparrow$

$D_T \uparrow$

TKE $\downarrow \quad \frac{\partial \bar{u} \bar{v}}{\partial y} \downarrow$
Initial state

Short
$t < 50$

$\omega_z \omega_y \frac{\partial \bar{W}}{\partial y}$ ↑  $\omega_i \omega_i$ ↑

$D_T$ ↑

Intermediate
$50 < t < 400$

TKE ↓  $\frac{\partial \bar{U}}{\partial y}$ ↓

$\frac{\partial \bar{U}}{\partial t} > 0$
Drag reduction mechanism

Initial state

Short
$t < 50$

Intermediate
$50 < t < 400$

Long
$t > 400$

$\omega_z \omega_y \frac{\partial \bar{w}}{\partial y} \uparrow$  \hspace{5mm} $\omega_i \omega_i \uparrow$

$\bar{D}_T \uparrow$

$TKE \downarrow$  \hspace{5mm} $\frac{\partial u \bar{v}}{\partial y} \downarrow$

$\frac{\partial \bar{U}}{\partial t} > 0$

$\int_0^h \bar{U} dy \uparrow$
\( \hat{\omega}_{z\omega y} \partial \hat{W} / \partial y \) is key term leading to drag reduction

\( \hat{\omega}_{z\omega y} \partial \hat{W} / \partial y \rightarrow \partial \hat{W} / \partial y \) acts on \( \hat{\omega}_{z\omega y} \)

\( \hat{\omega}_{z\omega y} \approx \frac{\partial u}{\partial y} \frac{\partial u}{\partial z} \)

\( \frac{\partial u}{\partial y} \rightarrow \) upward eruption of near-wall low-speed fluid

\( \frac{\partial u}{\partial z} \rightarrow \) lateral flanks of the low-speed streaks

\( \frac{\partial u}{\partial y} \frac{\partial u}{\partial z} \) located at the sides of high-speed streaks
MODELLING TURBULENT ENSTROPHY PRODUCTION

SIMPLIFIED TURBULENT ENSTROPHY EQUATION

\[
\frac{1}{2} \frac{\partial}{\partial t} \left( \omega_y^2 + \omega_z^2 \right) = \omega_z \omega_y G - \left( \frac{\partial \omega_y}{\partial y} \right)^2 - \left( \frac{\partial \omega_z}{\partial y} \right)^2
\]

Rotation of axis

\[
\frac{1}{2} \frac{\partial \omega_n^2}{\partial t} = S_{nn} \omega_n^2 - \left( \frac{\partial \omega_n}{\partial y} \right)^2
\]

Integration by Charpit's method

\[
\omega_n = \omega_{n,0} e^{\sin \alpha \cos \alpha G t} e^{-\beta^2 t} e^{-\beta y}, \quad \beta = \frac{\partial \omega_n}{\partial t} \frac{\partial \omega_n}{\partial y} \approx \frac{\lambda_y}{\lambda_t}
\]

stretching
dissipation
Drag reduction grows monotonically with global production term
This happens up to optimum period
THANK YOU!

REFERENCE

Ricco, P. Ottonelli, C. Hasegawa, Y. Quadrio, M.
Changes in turbulent dissipation in a channel flow with oscillating walls