STABILITY OF PLANAR SHEAR FLOW IN THE PRESENCE OF ELECTROCONVECTION

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OUTLINE

1. INTRODUCTION
2. EHD WITHOUT CROSS-FLOW
   - Modal
   - Non-modal
3. EHD WITH CROSS-FLOW
   - Modal
   - Non-modal
4. CONCLUSIONS

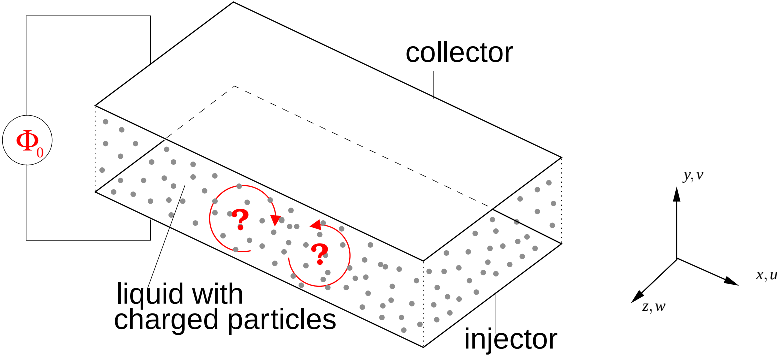
OUTLINE

WHAT IS EHD?

- Dielectric fluid
- Negligible magnetic effects
- Charge injection at the boundary
- Fully coupled problem owing to Coulomb force

What is electroconvection?

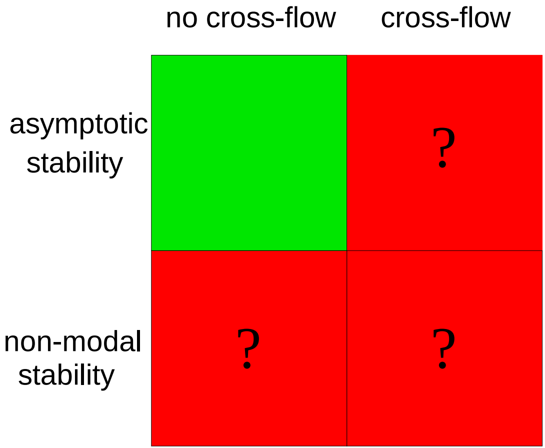
Review by P.Atten, IEEE Trans., 1996



- Planar indefinite geometry (periodic box)
- Unipolar autonomous injection
- "Analogous" to Rayleigh-Bénard thermal convection

WHAT IS KNOWN ABOUT ELECTROCONVECTION?

RESULTS FOR LINEAR STABILITY DATE BACK TO ’70-’80



	no cross-flow	cross-flow
asymptotic stability	🟩	🟥 ?
non-modal stability	🟥 ?	🟥 ?

EQUATIONS

TWO-WAY COUPLING BETWEEN KINETIC AND ELECTRIC FIELD

$$\nabla^2 \Phi = -\frac{q}{\varepsilon}$$

Quasi-electrostatic limit of Maxwell equations

EQUATIONS

TWO-WAY COUPLING BETWEEN KINETIC AND ELECTRIC FIELD

$$\nabla^2 \Phi = -\frac{q}{\varepsilon}$$

$$\frac{\partial q}{\partial t} + \nabla \cdot (q\mathbf{V} + qK\mathbf{E} - D\nabla q) = 0$$

Conservation of charge density q

EQUATIONS

TWO-WAY COUPLING BETWEEN KINETIC AND ELECTRIC FIELD

$$\nabla^2\Phi = -\frac{q}{\varepsilon}$$

$$\frac{\partial q}{\partial t} + \nabla\cdot(q\mathbf{V} + qK\mathbf{E} - D\nabla q) = 0$$

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\cdot\nabla)\mathbf{V} = -\frac{1}{\rho}\nabla P + \nu\nabla^2\mathbf{V} + \mathbf{F}_e$$

Electric force is $\mathbf{F}_e = q\mathbf{E}$ (no dielectric force since ε is uniform)

EQUATIONS

TWO-WAY COUPLING BETWEEN KINETIC AND ELECTRIC FIELD

$$\nabla^2 \Phi = -\frac{q}{\varepsilon}$$

$$\frac{\partial q}{\partial t} + \nabla \cdot (q\mathbf{V} + qK\mathbf{E} - D\nabla q) = 0$$

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla)\mathbf{V} = -\frac{1}{\rho}\nabla P + \nu \nabla^2 \mathbf{V} + \mathbf{F}_e$$

$$\nabla \cdot \mathbf{V} = 0$$

Incompressibility

DIMENSIONLESS PARAMETERS

Reference length, potential and velocity are h, Φ_0 and $K\Phi_0/h$

- Taylor number T (forcing par., fluid properties + Φ_0)
- Ionic mobility M (fluid properties)
- Charge diffusivity Fe (fluid properties + Φ_0)

Moreover:

- Charge injection coefficient C (boundary condition only)
- Reynolds number Re (in base flow)

FORMULATION, NUMERICS

- v-η-Φ formulation
- Fourier transform in x,z directions
- Small perturbations, linearization
- y discretization with N Chebyshev polynomials

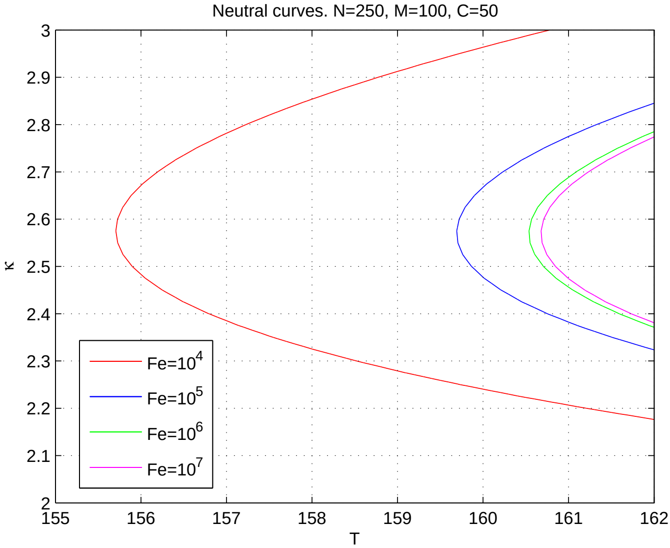
OUTLINE

STATE OF THE ART

P.ATTEN 1996

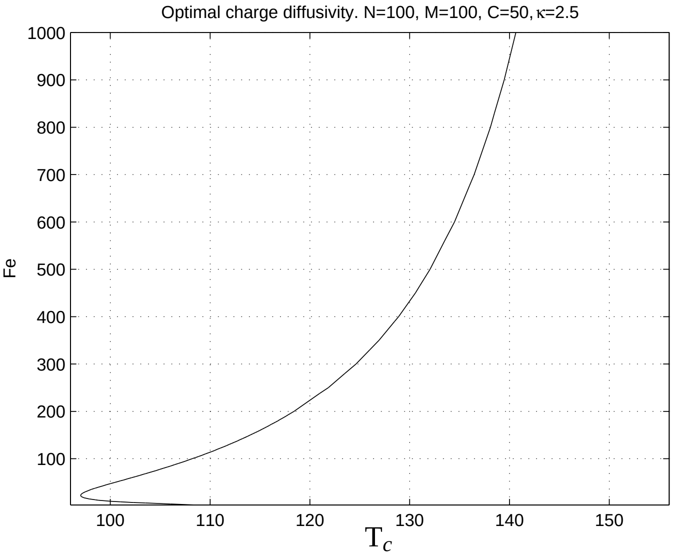
- Charge diffusion assumed to be negligible, $Fe \to \infty$
- Instability for $\kappa \approx 2.5$ and $T = T_c \approx 161$
- Discrepancy between numerical T_c and experimental $T_c \approx 100$

NEUTRAL CURVE
DIFFUSION MATTERS!



"OPTIMAL" Fe

EXPLAINS DIFFERENCE BETWEEN EXPERIMENTAL AND NUMERICAL T_c?



DEFINITION OF ENERGY

- Total energy of the system split into mechanical and electric contributions

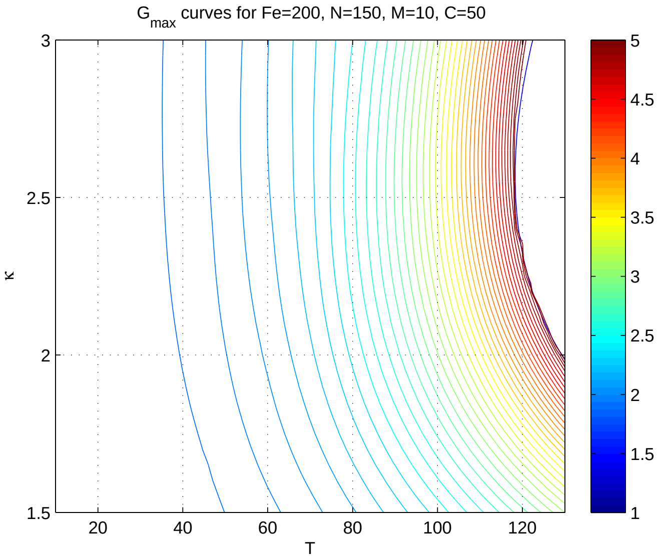
$$\mathscr{E} = \mathscr{E}_m + \mathscr{E}_e = \frac{1}{2}(u^2 + v^2 + w^2) + \frac{1}{2}\varepsilon \mathbf{E} \cdot \mathbf{E}$$

- Transient growth function defined as

$$G(t) = \max \frac{\mathscr{E}(t)}{\mathscr{E}(0)} = \max_{\mathbf{x}_0 \neq 0} \frac{\| \mathbf{x}(t)^2 \|_E}{\| \mathbf{x}_0^2 \|_E}$$

MAP OF G_{max}

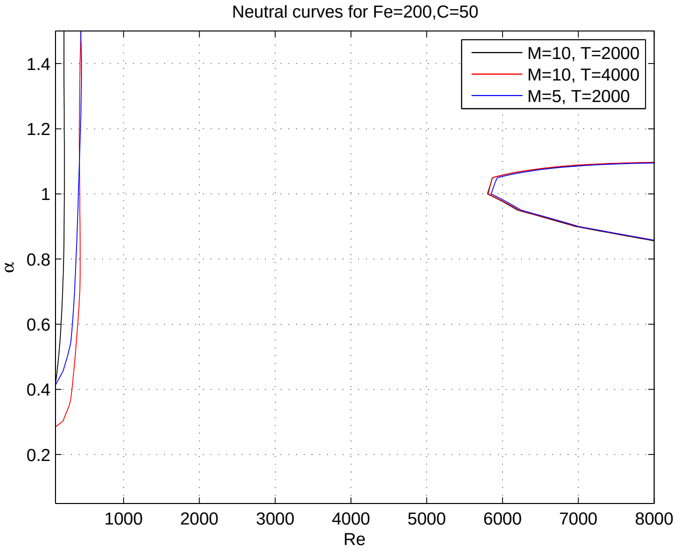
MILD TRANSIENT GROWTH



OUTLINE

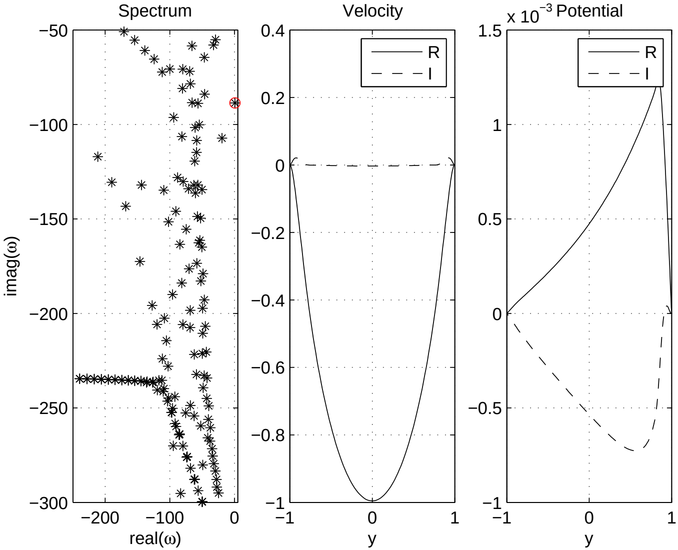
NEUTRAL CURVE

SQUIRE THEOREM STILL APPLIES: $\beta = 0$



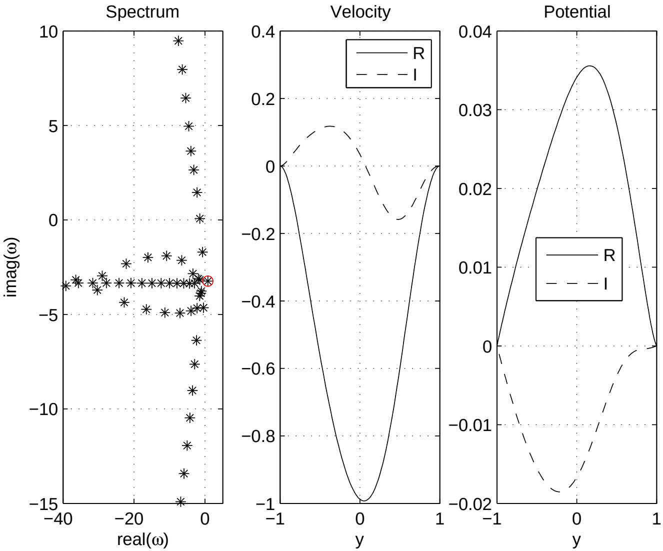
MOST UNSTABLE HYDRODYNAMIC MODE

$Re = 7000,\ \alpha = 1$

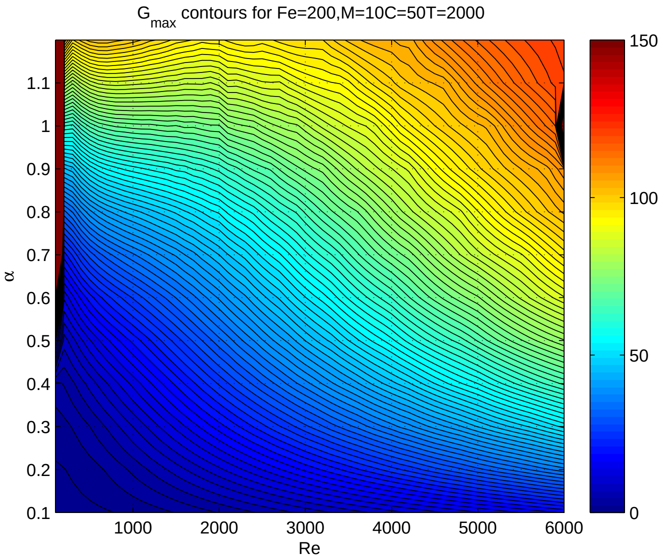


MOST UNSTABLE ELECTRIC MODE

$Re = 100,\ \alpha = 1$

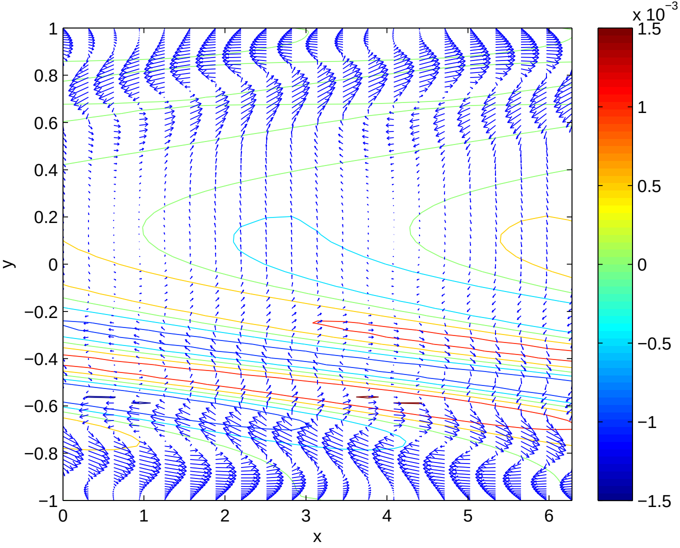


TRANSIENT GROWTH AT $\beta = 0$



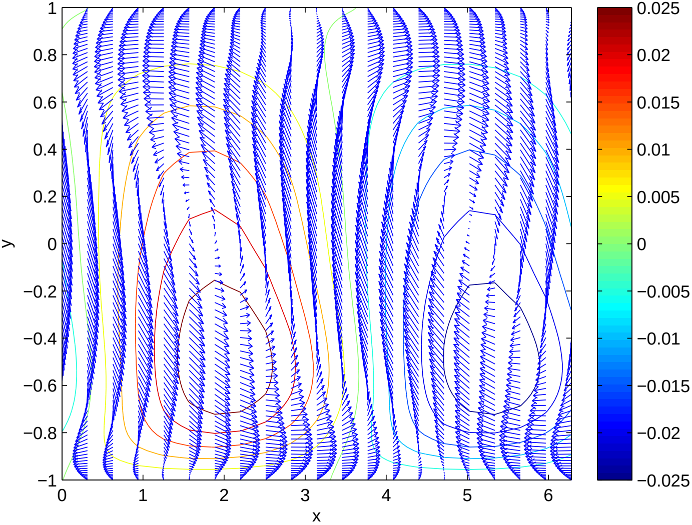
OPTIMAL INPUT FOR $\beta = 0$

ORR MECHANISM. $\alpha = 1$, $\beta = 0$, $Re = 1000$



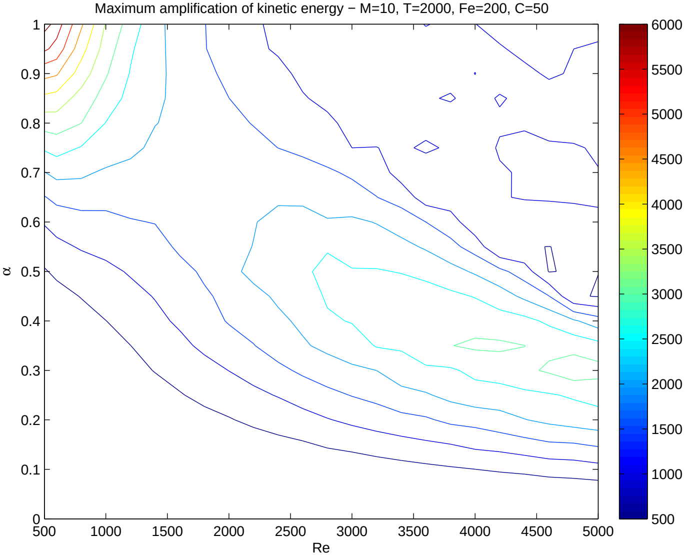
OPTIMAL OUTPUT FOR $\beta = 0$

ORR MECHANISM



DOES EHD ENHANCE TRANSIENT GROWTH?

LOOKING AT KINETIC ENERGY ALONE, $\beta = 0$



OUTLINE

CONCLUSIONS

- Electroconvection (stability) revisited
- Role of diffusion
- Non-modal effects (esp. with cross-flow)
- Non-linear effects?
- EHD as a extremely-low-power flow control device?

DIMENSIONLESS NUMBERS

Reference length, potential, velocity, time and pressure are: h, Φ_0, $K\Phi_0/h$, $h^2/K\Phi_0$ and $\rho K^2\Phi_0^2/h^2$

$$M = \frac{1}{K}\sqrt{\frac{\varepsilon}{\rho}}$$

$$T = \frac{\varepsilon\Phi_0}{\mu K}$$

$$Fe = \frac{K\Phi_0}{D}$$

$$C = \frac{q_0 h^2}{\varepsilon\Phi_0}$$

K is ionic mobility, ρ and μ fluid density and dynamic viscosity, D is charge diffusivity, ε fluid (uniform) fluid permittivity.

DIMENSIONLESS EQUATIONS

$\overline{U}(y)$ and $\overline{\Phi}(y)$ are the base velocity and potential profiles

$$\frac{\partial \hat{\Delta}\hat{v}}{\partial t} = -j\alpha\overline{U}\hat{\Delta}\hat{v} + j\alpha\overline{U}''\hat{v} + M^2\left[\overline{\Phi}'''\kappa^2\hat{\psi} - \overline{\Phi}'\kappa^2\hat{\Delta}\hat{\psi}\right] + \frac{M^2}{T}\hat{\Delta}\hat{\Delta}\hat{v}$$

$$\frac{\partial \hat{\eta}}{\partial t} = -j\beta\overline{U}'\hat{v} - j\alpha\overline{U}\hat{\eta} + \frac{M^2}{T}\hat{\Delta}\hat{\eta}$$

$$\frac{\partial \hat{\Delta}\hat{\psi}}{\partial t} = \overline{\Phi}'\frac{\partial \hat{\Delta}\hat{\psi}}{\partial y} + \overline{\Phi}'''\frac{\partial \hat{\psi}}{\partial y} + 2\overline{\Phi}''\hat{\Delta}\hat{\psi} - j\alpha\overline{U}\hat{\Delta}\hat{\psi} - \overline{\Phi}'''\hat{v} + \frac{1}{Fe}\hat{\Delta}\hat{\Delta}\hat{\psi},$$

EXAMPLE OF FLUID PROPERTIES

DATA FOR PYRALENE 1460

$$K = 3.2E - 9 \qquad D = 8.2E - 11$$

$$\varepsilon = 5.224E - 11 \qquad \mu = 0.01$$

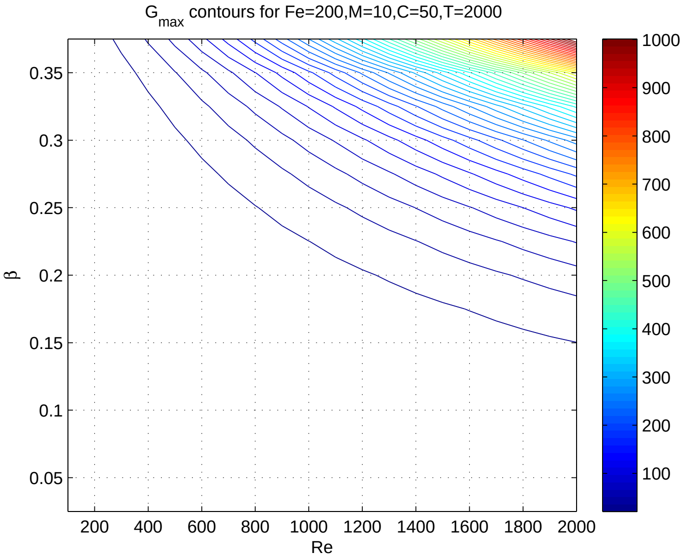
$$\rho = 1.41E3 \qquad M = 60$$

$$T = 1.6325\Phi_0 \qquad Fe = 0.6\Phi_0$$

APPLICATIONS

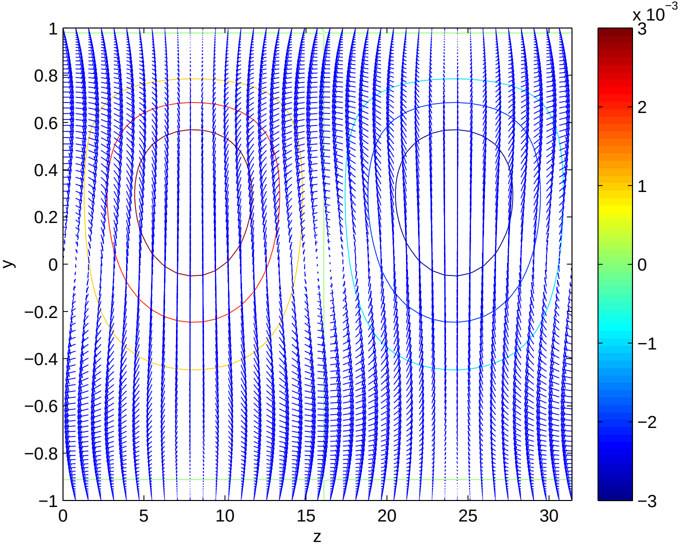
- Ion-drag pumping
- EHD turbulent mixing
- EHD heat transfer augmentation

TRANSIENT GROWTH AT $\alpha = 0$



OPTIMAL INPUT FOR $\alpha = 0$

LIFTUP MECHANISM. $\alpha = 0$, $\beta = 0.2$, $Re = 1000$



OPTIMAL OUTPUT FOR $\alpha = 0$

LIFTUP MECHANISM