LINEAR STABILITY OF PLANE POISEUILLE FLOW OVER A GENERALIZED STOKES LAYER

M.Quadrio[1,2], F.Martinelli[2] & P.Schmid[2]

[1]Dip. Ing. Aerospaziale, Politecnico di Milano (I)
[2]LadHyx, École Polytechnique (F)

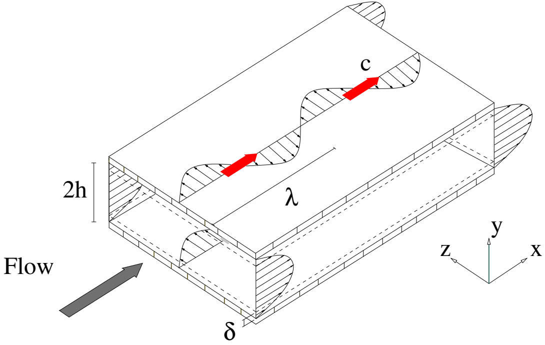
ETC 13, Warsaw, September 2011

OUTLINE

Outline

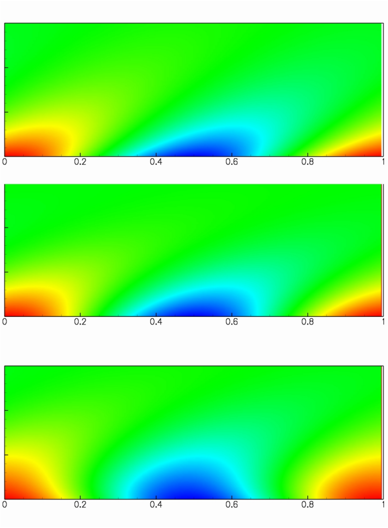
POISEUILLE + GSL
QUADRIO, PHIL.TRANS.R.SOC.A 2011

$$w = A\cos(\kappa x - \omega t)$$



The spanwise oscillating boundary layer

Quadrio & Ricco, JFM 2011



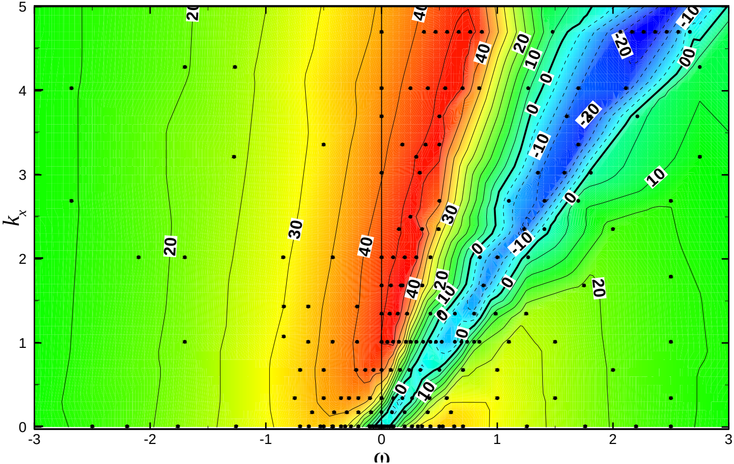
$w(y, t)$
TSL

$w(y, x)$
SSL

$w(y, x - ct)$
GSL

TURBULENT DRAG REDUCTION

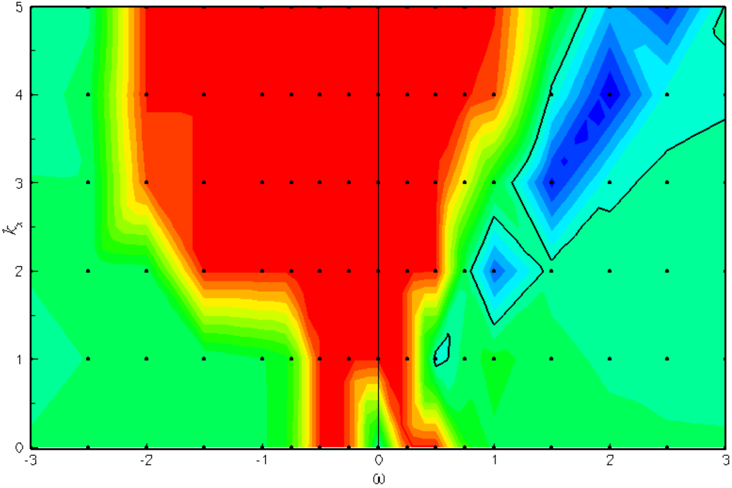
QUADRIO ET AL., JFM 2009



QUESTION: DO WAVES AFFECT TRANSITION?
A PRELIMINARY SURVEY BY DNS

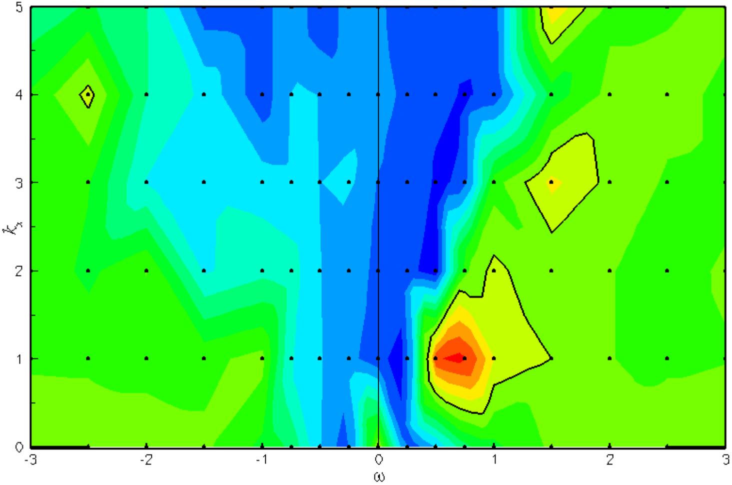
- Temporal problem (plane channel flow) by DNS, $Re = 2000$
- Transition scenario: oblique waves (Reddy et al., JFM 1998)
- Optimal i.c. for $\alpha = 1, \beta = \pm 1$, 1% random noise
- Initial energy = 2 $\times$ transition threshold
- No generality

OBLIQUE WAVES
AFTER-TRANSIENT DRAG REDUCTION, $A = 0.25$



OBLIQUE WAVES

$G_{max}/G_{max,ref}$, $A = 0.25$



Outline

Linear stability: formulation

Modal and non-modal characteristics

Similar to Orr-Sommerfeld-Squire problem, but...

- Additional transversal stationary base flow $\overline{W}(x,y) = \frac{1}{\mathrm{Ai}(0)} \Re \left\{ \mathrm{e}^{j\kappa x} \mathrm{Ai} \left(-\frac{jy}{\delta_x} \mathrm{e}^{-j4/3\pi} \right) \right\}$ with $\delta_x = (\nu/\kappa u_{y,0})^{1/3}$
- Streamwise-varying coefficients!

Fourier transform in x

$\overline{W}(x)$ is sinusoidal:

$$\int \mathrm{e}^{jp\frac{\kappa}{n}x}\mathrm{e}^{j\kappa x}\mathrm{e}^{-j\alpha x}dx \neq 0 \quad \text{for} \quad p\frac{\kappa}{n}+\kappa-\alpha=0$$

$$\boxed{q(x,y,t)=\sum_{i=-M}^{+M}\hat{q}_i(y,t)\mathrm{e}^{j(i+m)\kappa x}}$$

- The problem becomes global in x
- Block-tridiagonal matrix
- Each block is like a standard Orr-Sommerfeld-Squire problem
- Size of full problem is $(2M+1)^2\times$ a single OSSq

A HUGE PARAMETRIC STUDY

Large number of parameters:

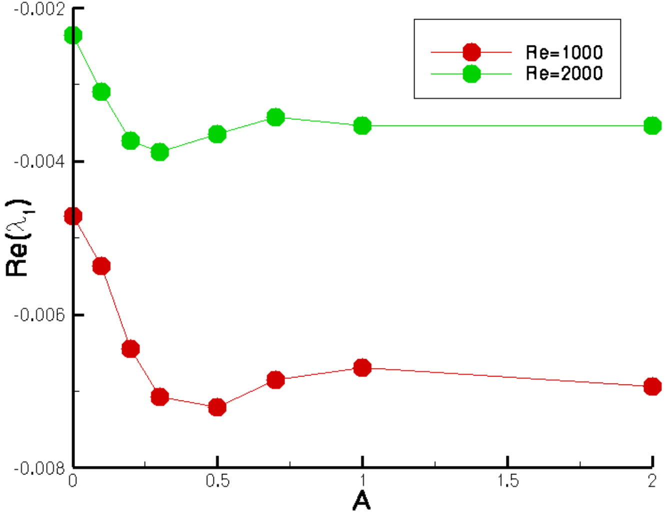
- Spanwise wavenumber β of the perturbation
- Base flow wave number κ
- Base flow amplitude A
- Reynolds number Re

- Wall-normal resolution N
- Modal truncation (streamwise resolution) M

Outline

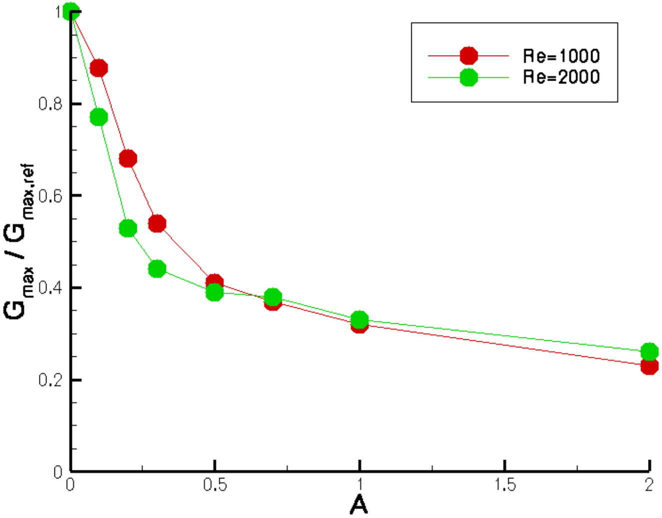
Changes in long-term stability

$\kappa = 1$, $\beta = 1.5$



Changes in maximum transient growth

$\kappa = 1$, $\beta = 1.5$



Conclusions, perspectives

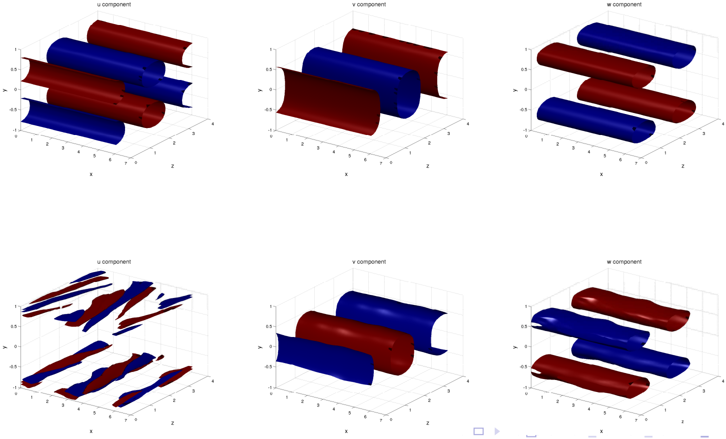
- Large effects on at least one (temporal) transition scenario
- Formulation of the linear stability problem
- Least-stable eigenvalue reduced
- Transient growth weakened

- Energy transfer among wavenumbers?

OPTIMAL INPUT

$\kappa = 1$, $\beta = 1.5$, $A = 0$ (TOP) VS $A = 1$ (BOTTOM)

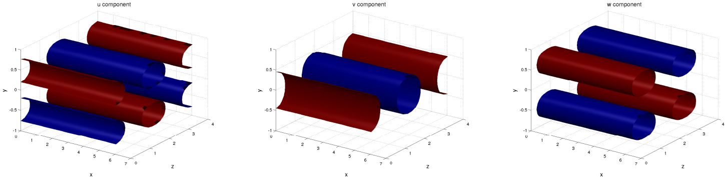
u	v	w
0.11 - 0.74	2.68 - 2.63	3.28 - 3.79

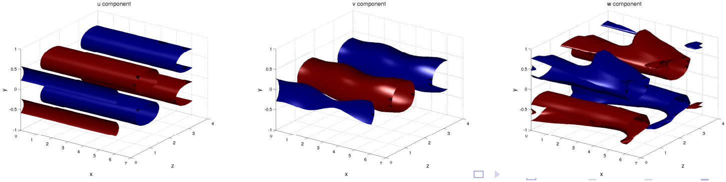


OPTIMAL OUTPUT

$\kappa = 1, \beta = 1.5, A = 0$ (TOP) VS $A = 1$ (BOTTOM)

u	v	w
481x - 44x	0.47x - 0.51x	0.40x - 0.46x





MODAL TRUNCATION (ADAPTIVE RESOLUTION)