LINEAR STABILITY
OF PLANE POISEUILLE FLOW
OVER A GENERALIZED STOKES LAYER

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1. INTRODUCTION

2. FORMULATION

3. RESULTS
1 INTRODUCTION

2 FORMULATION

3 RESULTS
$w = A \cos (\kappa x - \omega t)$
THE SPANWISE OSCILLATING BOUNDARY LAYER
Quadrio & Ricco, JFM 2011

$w(y, t)$

TSL

$w(y, x)

SSL

$w(y, x - ct)$

GSL
Stability of Poiseuille flow + GSL

M. Quadrio

Introduction

Formulation

Results

TURBULENT DRAG REDUCTION
Quadrio et al., JFM 2009
QUESTION: DO WAVES AFFECT TRANSITION?
A PRELIMINARY SURVEY BY DNS

- Temporal problem (plane channel flow) by DNS, $Re = 2000$
- Transition scenario: oblique waves (Reddy et al., JFM 1998)
- Optimal i.c. for $\alpha = 1, \beta = \pm 1$, 1% random noise
- Initial energy $= 2 \times$ transition threshold
- No generality
OBLIQUE WAVES
AFTER-TRANSIENT DRAG REDUCTION, A = 0.25
OBLIQUE WAVES

\[ \frac{G_{\text{max}}}{G_{\text{max,ref}}}, A = 0.25 \]
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Similar to Orr-Sommerfeld-Squire problem, but...

- Additional transversal stationary base flow

\[ \overline{W}(x, y) = \frac{1}{\text{Ai}(0)} \Re \left\{ e^{j \kappa x} \text{Ai} \left( -\frac{j y}{\delta_x} e^{-j4/3\pi} \right) \right\} \]

with

\[ \delta_x = \left( \nu / \kappa u_{y,0} \right)^{1/3} \]

- Streamwise-varying coefficients!
Fourier transform in $x$

$\overline{W}(x)$ is sinusoidal:

$$\int e^{jp\frac{\kappa}{n}x}e^{j\kappa x}e^{-j\alpha x}dx \neq 0 \quad \text{for} \quad p\frac{\kappa}{n} + \kappa - \alpha = 0$$

$$q(x, y, t) = \sum_{i=-M}^{+M} \hat{q}_i(y, t)e^{j(i+m)\kappa x}$$

- The problem becomes global in $x$
- Block-tridiagonal matrix
- Each block is like a standard Orr-Sommerfeld-Squire problem
- Size of full problem is $(2M + 1)^2 \times$ a single OSSq
A huge parametric study

Large number of parameters:
- Spanwise wavenumber $\beta$ of the perturbation
- Base flow wave number $\kappa$
- Base flow amplitude $A$
- Reynolds number $Re$
- Wall-normal resolution $N$
- Modal truncation (streamwise resolution) $M$
INTRODUCTION

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Changes in long-term stability

$\kappa = 1, \beta = 1.5$
Changes in maximum transient growth

$\kappa = 1, \beta = 1.5$
CONCLUSIONS, PERSPECTIVES

- Large effects on at least one (temporal) transition scenario
- Formulation of the linear stability problem
- Least-stable eigenvalue reduced
- Transient growth weakened
- Energy transfer among wavenumbers?
**OPTIMAL INPUT**

\( \kappa = 1, \beta = 1.5, A = 0 \text{ (TOP)} \text{ vs } A = 1 \text{ (BOTTOM)} \)

\[
\begin{align*}
\textbf{u} & \quad 0.11 - 0.74 \\
\textbf{v} & \quad 2.68 - 2.63 \\
\textbf{w} & \quad 3.28 - 3.79
\end{align*}
\]
OPTIMAL OUTPUT

$\kappa = 1, \beta = 1.5, A = 0$ (TOP) vs $A = 1$ (BOTTOM)

$u$

$481x - 44x$

$v$

$0.47x - 0.51x$

$w$

$0.40x - 0.46x$
Modal truncation (adaptive resolution)