On the Evaluation of Control Performance in Drag Reducing Flows

*Money versus Time*

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Skin Friction Drag Reduction Technology

- Key Aspects of Practical Fluid Transport Systems
  - **Convenience**
    - flow rate in pipeline
    - travel speed of vehicle
  - **Energy Saving**
    - energy consumption to achieve certain “Convenience”

- Evaluation of Control Performance in Fundamental Studies
  - **Constant Flow Rate (CFR)**: wall friction is changed by control
    - Successful Control
      - Reduction of wall friction (reduction of pumping power)
  - **Constant Pressure Gradient (CPG)**: wall friction is kept constant by design
    - Successful Control
      - Increase of flow rate (increase of pumping power)
Internal Flow

Flow rate \( U_b \)

Pumping Energy \( E_p \)

Duct properties:
- Cross sectional area : \( A \)
- Wetted perimeter: \( C \)
- Hydraulic diameter: \( D = 4A/C \)

- Fluid travel time per unit length: \( 1/U_b \)
- Pumping energy per unit wetted area:

\[
E_p = \frac{\tau_w V}{A} = \frac{M U_b^2 C_f}{2A}
\]

Friction coefficient

\[
C_f = \frac{\tau_w}{\frac{1}{2} \rho U_b^2}
\]
Energy Saving vs Convenience

\[ E_p = \frac{\tau_w V}{A} = \frac{MU_b^2 C_f}{2A} \]

\[ E_p \propto \left(U_b\right)^{7/4} \]

**CPI line**
(Constant Power Input)

**CFR line**

\[ E_p \propto U_b \]

\[ U_b^{-1} \]

\[ C_f \propto U_b^{-1} : \text{laminar} \]

\[ C_f \propto U_b^{-1/4} : \text{turbulent} \]
Active Control of Internal Flow

Flow rate $U_b$

Pumping Energy $E_p$

Fluid travel time per unit length: $1/U_b$

Total energy consumption per unit wetted area:

$$E_t = E_p + E_c$$

Energy Consumption for Control: $E_c$

Volume: $V$
Mass: $M = \rho V$
No flow states below the laminar curve
Bewley 2009, Fukagata et al., 2009

\[ E_t = E_p + E_c \]

Turbulent (uncontrolled)
\[ E_p \propto (U_b)^{7/4} \]

CPI line
(Constant Total Power Input)

UB

\[ E_t = E_p + E_c \]

CFR line

\[ E_p \propto U_b \]

CPG line

laminar (uncontrolled)

No flow states below the laminar curve
Bewley 2009, Fukagata et al., 2009
Example

Cost function: \[ J = E_t^2 + \left( \frac{1}{U_b} \right)^2 \]

Isoline of \( J \)

Optimal in uncontrolled flow

CPI line
(Constant Total Power Input)

Turbulent (uncontrolled)

Optimal

Laminar (uncontrolled)

\[ E_p \propto \left( U_b \right)^{7/4} \]

\[ E_p \propto U_b \]

(Inconvenience: time) \( U_b^{-1} \)
Non-dimensionalization

- **Convenience (Fluid travel time per unit length)**

\[ T_c = \frac{1}{U_b} \quad \rightarrow \quad \left( \frac{1}{U_b} \right) \left( \frac{\nu}{D} \right) = \frac{\nu}{U_b D} = \text{Re}_b^{-1} \]

- **Energy Expenditure**

  ✓ **Pumping Energy**

\[ E_p = \frac{MU_b^2 C_f}{2A} \quad \rightarrow \quad C_f = E_p \left( \frac{2A}{MU_b^2} \right) \quad \rightarrow \quad C_f \text{Re}_b^2 = E_p \left( \frac{2AD^2}{M\nu^2} \right) \]

  ✓ **Total Energy (Pumping + Control)**

*Effective wall friction*

\[ \tau_w^e = \frac{P_p + P_c}{U_b} = \tau_w + \frac{P_c}{U_b} \quad \rightarrow \quad C_f^e \text{Re}_b^2 = E_t \left( \frac{2AD^2}{M\nu^2} \right) \]
The value of $C_f$ does not represent energy consumption, e.g., $C_f$ decreases with increasing Re.

Comparison of $C_f$ at different Re does not make sense.
New Plots

\[ C_f \, \text{Re}^2 - \text{Re}^{-1} \text{ plot} \]

\[ C_f \propto \text{Re}_m^{-1/4} \]

\[ C_f \propto \text{Re}_m^{-1} \]

turbulence

laminar

\[ C_f \, \text{Re}^2 - \text{Re}^{-1} \text{ plot} \]
Application to External Flow

- **Convenience (traveling time per unit distance)**

\[
\frac{1}{U_\infty} \quad \Rightarrow \quad \frac{\nu}{(U_\infty l)} = \frac{1}{Re_l}
\]

- **Propulsion energy per unit fluid-contacting area and unit distance**

\[
E_p = \frac{1}{2} \rho U_\infty^2 \overline{C_f} \quad \Rightarrow \quad \overline{C_f} \frac{Re_l^2}{Re} = E_p \left( \frac{\rho \nu^2}{2l^2} \right)
\]

\(C_f Re^2 - Re^{-1}\) plot can also be used for external flows.
Conclusions

- In real applications, a compromise between *Convenience (Time)* and *Energy expenditure (Money)* has to be reached so as to accomplish a goal which in general depends on a specific application.

- Based on this idea, we propose a new evaluation plane (money-time plane), which can be viewed as an improved version of the conventional Cf-Re plot.

- The new plane consists of two dimensionless parameters $Re^{-1}$ and $CfRe^2$ which represent the flow rate (convenience) and the energy expenditure required to achieve that flow rate, respectively.

- The new evaluation plane is useful to seek the optimal control strategy for minimizing the application-dependent cost function.

- The above considerations can be easily extended to external flows.