## Spanwise generalized Stokes layer and turbulent drag reduction

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This paper considers plane channel flows modified by waves of spanwise wall velocity which travel along the streamwise direction. The laminar and turbulent streamwise flows are both studied. The interest of this wall forcing resides in the recent discovery<sup>1</sup> of the turbulent drag-reduction potential of such waves, together with the extremely limited energetic cost of producing them.

When laminar, the streamwise flow is unaffected by the alternating spanwise flow induced by the waves. This flow is a thin, unsteady and streamwise-modulated boundary layer that can be given an analytical expression in terms of the Airy function of the first kind. We name it the generalized Stokes layer because it reduces to the classical oscillating Stokes layer in the limit of infinite wave speed.

When the streamwise flow is turbulent, the laminar generalized Stokes layer solution describes well the space-averaged turbulent spanwise flow, provided that the phase speed of the waves is sufficiently different from the turbulent convection velocity, and that the time scale of the forcing is smaller than the life time of the near-wall turbulent structures. Under these conditions, the drag reduction is found to scale linearly with the Stokes layer thickness, as illustrated in fig. 1, where turbulent drag reduction corresponding to different waves is plotted againts the generalized Stokes layer thickness.

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<sup>&</sup>lt;sup>1</sup>Quadrio et al, J. Fluid Mech. **627** 161–178, 2009



Figure 1: Drag reduction rate R versus thickness  $\delta^+_{lam}$  of the generalized Stokes layer obtained from the laminar solution. Points are from different datasets, computed either from constant-flow-rate (circles) or constant-pressure-gradient (squares) DNS of turbulent plane channel flow at  $Re_{\tau} = 200$ . Only black points are expected to satisfy a linear scaling.

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