Feedback control of transient energy growth in subcritical plane Poiseuille flow

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Motivation

In subcritical plane Poiseuille flow:

1. Is complete feedback suppression of transient growth possible, when employing wall-based actuation?
2. How to design a feedback controller that directly targets the transient growth mechanism?

Model of the system

- Orr-Sommerfeld-Squire stability equations, *y* discretization by Chebyshev expansion
- For each wavenumber pair (α, β), wall actuation accounted for by a lifting procedure.

Standard state-space form:

$$\dot{x} = Ax + Bu, \qquad x(0) = x_0, \qquad \forall \alpha, \beta$$

- most general case: $u = (\dot{u}_u, \dot{u}_l, \dot{v}_u, \dot{v}_l, \dot{w}_u, \dot{w}_l)^T$ ("vectorized" transpiration at both walls).
- input: rate of change of transpiration velocity (due to lifting)
- rescaling of state variables: energy is $||x||^2 = x^H x$

Closed loop monotonic stability

Theorem (Whidborne & McKernan, 2007)

A static, state-feedback control law $u = Kx$ exists such that the closed-loop system is monotonically stable if and only if:

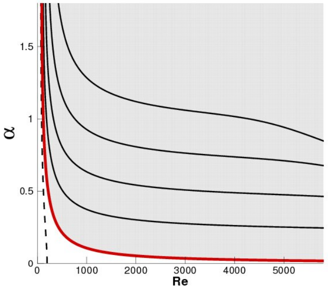
$$B^{\perp}\left(A + A^{H}\right)B^{\perp H} < 0 \text{ or } BB^{H} > 0,$$

where $B^{\perp}$ is the left null space of B.

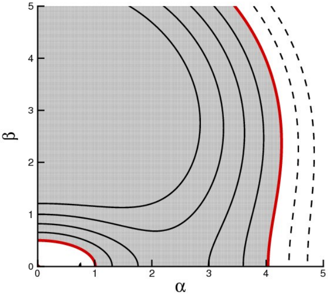
- Second criterion never satisfied using wall forcing
- First criterion verified numerically

Numerical verification of the algebraic criterion

2D, *v* actuation, $\beta = 0$



3D, full actuation, $Re = 120$



Contours of $T(\alpha, \beta, Re) = \lambda_{max}(B^{\perp}\,(A + A^{H})\,B^{\perp H})$

It is not possible to completely suppress the transient growth mechanism by feedback wall forcing.

Upper bound on maximum growth

Linear, time-invariant stable system (open-loop):

$$\dot{x} = Ax, \qquad x(0) = x_0$$

Upper bound to the maximum transient energy growth G:

$$G_u = \lambda_{max}(P)\lambda_{max}(P^{-1}) \geq G.$$

$P = P^H > 0$ satisfies the Lyapunov inequality:

$$PA + A^H P < 0.$$

Minimization of the upper bound

G_u depends on the choice of P; to minimize it:

$$\begin{aligned} &\min \gamma : \\ &PA + A^H P < 0, \quad P = P^H > 0 \\ &I < P < \gamma I \end{aligned}$$

- Linear Matrix Inequality (LMI) generalized eigenvalue problem.
- Last inequality ensures that $\gamma > G_u$.
- Standard solvers exist.

Feedback minimization of the upper bound

Full-state feedback control law $u = Kx$. Closed loop dynamics:

$$\dot{x} = (A + BK)x, \qquad x(0) = x_0.$$

Lyapunov inequality:

$$PA + A^H P + PBK + K^H B^H P < 0$$

which is bilinear in K and P.

Feedback minimization of the upper bound

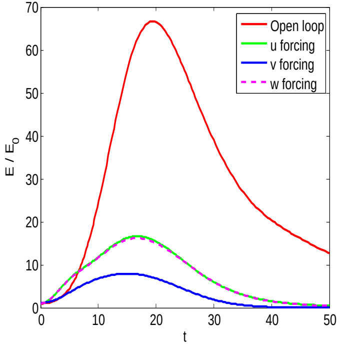
Introducing $Q = P^{-1}$ and $Y = KQ$, and an additional constraint such that $\max_{t \geq 0} ||u||^2 < \mu^2$, the optimization problem reads:

$$
\begin{aligned}
&\min \gamma : \\
&AQ + QA^H + BY + Y^H B^H < 0, \quad Q = Q^H > 0 \\
&I < Q < \gamma I \\
&\begin{pmatrix} Q & Y^H \\ Y & \mu^2 I \end{pmatrix} > 0
\end{aligned}
$$

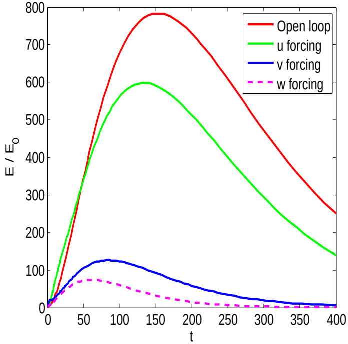
which is a LMI problem. Controller gains recovered from $K = YQ^{-1}$.

Effect of different actuation components - linear case

Wave ($\alpha = 1, \beta = 1$).



Vortex ($\alpha = 0, \beta = 2$).



Tests ($Re = 2000$, $\mu = 100$) show the bound γ is conservative of a factor ≈ 2.

Closed loop transition thresholds

Direct numerical simulations at $Re = 2000$:

- Pair of oblique waves: $\alpha_0 = 1, \beta_0 = \pm 1$
- Streamwise vortices: $\alpha_0 = 0, \beta_0 = 2$
- Random noise (Stokes modes) on the array $(0, \pm 1, \pm 2)\alpha_0$ and $(0, \pm 1, \pm 2)\beta_0$, 1% of total perturbation energy
- Controller designed on the same array, $\mu = 100$
- Performance: improvement factor = $\dfrac{E_{0,control}^{(thres)}}{E_{0,free}^{(thres)}}$

	Improvement factor		
	u	v	w
Oblique waves	≈ 6.0	≈ 20.7	1
Streamwise vortices	1	≈ 2.0	≈ 1.6

Conclusions

- An algebraic criterion to identify the possibility of feedback suppression of transient growth has been presented.
- Closed-loop monotonic stability is not possible when using wall actuation in subcritical plane Poiseuille flow.
- A new, LMI-based control design technique – directly targeting the growth mechanism – has been proposed.
- In terms of transition thresholds modification, wall forcing with u and w components is less effective than forcing with v.