Turbulent drag reduction by feedback: a Wiener-filtering approach

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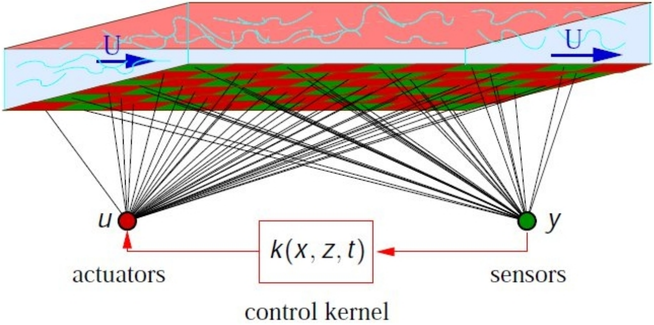
ETC XII – Marburg

Outline

Outline

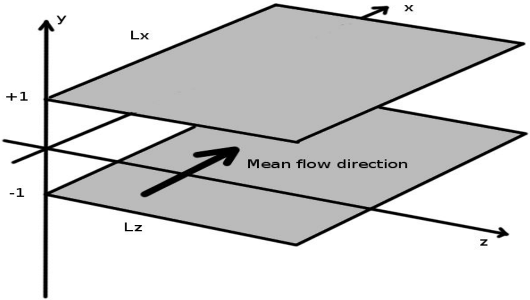
Feedback control of wall turbulence



$$u(x,z,t) = \int y(x',z',t')\, K(x-x',z-z',t-t')\; dx'\; dz'\; dt'$$

- Goal: reduction of friction drag
- Actuators: zero-net-mass-flux wall blowing/suction
- Sensors: pressure and skin friction components

The plant: turbulent plane channel flow



- Flow is spatially invariant in x and z
- Efficient DNS at moderate Re (and $\approx 10^8$ d.o.f.s)
- State variables: v-η

State of the art

A young field

- Hope for linear control (Kim & Lim, 2000)
- Modern Optimal Control Theory, state-space formulation
- Kalman-filter-based estimators: very poor performance
- Additional challenge: billions of d.o.f.

A recent step ahead?

Luchini & Quadrio, PoF 2006

Problem

- Poor system model: NS equations linearized about the mean velocity profile
- Turbulence dynamics is missing

Solution

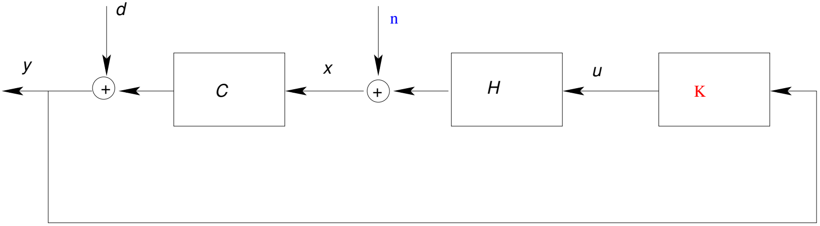
- Enrich the model: the average turbulent linear response function $\mathcal{H}$
- More physics: turbulent diffusion is accounted for (on average)

$\mathcal{H}$ is measured by cross-correlating small space-time white noise wall forcing with the perturbed flow

Goal of the present work

- Devise a strategy for using an impulse response to design the control kernel
- Lay down a computationally-efficient procedure
- Test the procedure with the average impulse response in the full nonlinear problem
- Hope it works...

The feedback control problem



- $\mathcal{H}$ is the average relation between boundary input and (inner) state variables
- n: turbulent fluctuations in the uncontrolled flow
- Aim: design K to minimize

$$J = E\{x^H Q x + u^H R u\}$$

Outline

Switch to frequency domain!

F.Martinelli, PhD thesis, PoliMi 2009

- A state-space realization of $\mathcal{H}$ is unaffordable
- Rewrite the objective functional in frequency:

$$J(f) = \int_{-\infty}^{+\infty} Tr[Q\phi_{xx}(f)] + Tr[R\phi_{uu}(f)]\ df.$$

with $\phi_{xx}(f)$ psd of state.

- Substituting, J is not quadratic in K.
- Minimization w.r.t. K does not lead to a linear problem

Obtaining a quadratic form

J may be written as a quadratic form of the Youla parameter $\overline{K} = (I - KCH)^{-1}K$ as:

$$J = \int_{-\infty}^{+\infty} Tr\Big\{Q\phi_{nn} + QH\overline{K}C\phi_{nn} + Q\phi_{nn}C^H\overline{K}^H H^H + \dots$$
$$\dots + QH\overline{K}C\phi_{nn}C^H\overline{K}^H H^H + QH\overline{K}\phi_{dd}\overline{K}^H H^H\Big\} + \dots$$
$$\dots + Tr\Big\{R\overline{K}C\phi_{nn}C^H\overline{K}^H + R\overline{K}\phi_{dd}\overline{K}^H\Big\}\, df.$$

Minimization yields the best compensator (that is non-causal)

Enforcing causality

Introduce a Lagrange multiplier Λ:

$$J = \int_{-\infty}^{+\infty} Tr\Big\{ Q\phi_{nn} + QH\overline{K}_{+}C\phi_{nn} + Q\phi_{nn}C^{H}\overline{K}_{+}^{H}H^{H} \ldots$$
$$\ldots + QH\overline{K}_{+}C\phi_{nn}C^{H}\overline{K}_{+}^{H}H^{H} + QH\overline{K}_{+}\phi_{dd}\overline{K}_{+}^{H}H^{H}\Big\} + \ldots$$
$$\ldots + Tr\Big\{ R\overline{K}_{+}C\phi_{nn}C^{H}\overline{K}_{+}^{H} + R\overline{K}_{+}\phi_{dd}\overline{K}_{+}^{H}\Big\} + Tr[\Lambda_{-}\overline{K}_{+}^{H}]\, df.$$

A Wiener-Hopf problem

Minimization leads to the (linear) Wiener-Hopf problem:

$$(H^H Q H + R)\overline{K}_+(C\phi_{nn}C^H + \phi_{dd}) + \Lambda_- = -H^H Q \phi_{nn} C^H$$

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3. Scalar equation for the SISO case: superfast FFT-based solution

Outline

The procedure

Measure ⇒ design ⇒ test

- Response function and noise spectral densities are measured via DNS
- Compensator is designed wavenumber-wise by solving a scalar Wiener-Hopf problem
- Compensators are tested in a full nonlinear DNS

Parametric study, more than 300 DNS ($\approx$ 40 years of CPU time)

Results

Measured friction drag reduction

	J=energy			J=dissipation		
Re_τ	τ_x	τ_z	p	τ_x	τ_z	p
100	0%	0%	0%	2%	0%	0%
180	0%	0%	0%	8%	6%	0%

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- Dissipation norm is effective
- Pressure measurement alone is not effective

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- Performance improves with *Re*

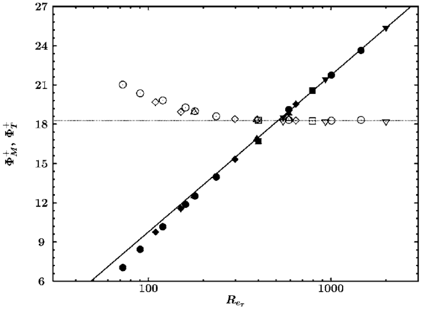
"Inverse" *Re*-effect

$$\left.\frac{d\langle U\rangle}{dy}\right|_w = -\frac{1}{U_B}\left\langle \underbrace{\frac{1}{2}\int_{-1}^{1}\left(\frac{\partial \hat{U}}{\partial y}\right)_{(0,0)}\left(\frac{\partial \hat{U}}{\partial y}\right)^*_{(0,0)} dy}_{D_{mean}} + \underbrace{\sum_{(\alpha,\beta)\neq(0,0)} D(\alpha,\beta)}_{D_{turb}} \right\rangle$$

- D_{mean} is affected indirectly via nonlinear interactions between fluctuations and the mean flow
- D_{turb} is affected directly by zero net mass flux wall blowing/suction

“Inverse” *Re*-effect
Laadhari, PoF 2007

The relative contribution of D_{turb} to the total dissipation increases with *Re*!



Re_τ	D_{turb}	D_{mean}
100	26.8%	73.2%
180	39.5%	60.5%

Critical discussion

Good news

Present compensators are the best possible for LTI systems

Bad news

Their performance is rather poor

Critical discussion

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- Should we blame the cost function?
- Should we blame the linear, time-invariant framework?

Conclusions

- Novel formulation for designing the compensator in frequency domain
- Extremely efficient
- Can exploit a measured linear model of the turbulent channel flow
- The time-space structure of the state noise (turbulence) is accounted for