Turbulent drag reduction by feedback: a Wiener-filtering approach

F. Martinelli[1], M. Quadrio[1], and P. Luchini[2]

[1]Dipartimento di Ingegneria Aerospaziale, Politecnico di Milano, Milan, Italy
fulvio.martinelli@polimi.it
[2]Dipartimento di Meccanica, Università di Salerno, Fisciano (SA), Italy

Modern control theory has recently been employed in the design of linear controllers[1], state estimators[2], and compensators[3; 4], in an attempt to devise control laws for reducing drag in turbulent wall flows. These approaches led to encouraging results, revealing the potential of linear control in targeting significant dynamics in wall turbulence[5].

All the aforementioned works, however, rely on an approximate state-space representation of the system dynamics, obtained by linearization of the governing equations. The state-space formulation reduces the compensator design problem to the solution of two matrix Riccati equations, a procedure that becomes computationally cumbersome for high dimensional systems. Effects of nonlinearities and modeling errors are accounted for by introducing state and measurement noises with known (approximately modeled) statistics.

In contrast to previously proposed approaches, in this work we employ a linearized model of the wall-forced turbulent channel flow system in the form of an average impulse response function; such model can be directly measured with DNS using the procedure proposed in Luchini et al.[6]. These authors introduced small velocity perturbations at the channel walls in the form of a space-time white noise, and computed runtime the cross-correlation between the flow state and the wall forcing. Leveraging a well known result in linear system theory, they used the computed correlation function to define a linear impulse response function, representing the average linear dynamics of a turbulent channel flow when impulsive wall forcing is applied.

A model given in the form of impulse response function would require first a state-space realization in order for standard Riccati-based control techniques to be applied. Instead of performing such realization – which would be impractical in the present very high-dimensional setting – we employ a frequency domain formulation of the optimal compensator design problem, that allows us to directly use the Fourier transform of the impulse response function (i.e. the frequency response function) in the compensator design procedure. The block diagram of the feedback problem at hand is shown in fig. 1, where the feedback compensator having frequency response function $K(\omega)$ feeds the system - whose frequency response is $H(\omega)$ - with a signal u determined on the basis of real-time measurements y, obtained with the sensor $C(\omega)$. Note that both the measurement and the state x are corrupted by disturbances d and noise n, respectively, that are supposed to be uncorrelated. The spectral density functions of the disturbances and noise will be denoted by $\phi_{dd}(\omega)$ and $\phi_{nn}(\omega)$, respectively, and may be functions of the frequency.

The goal is the design of an optimal feedback compensator such that the usual LQG expectation functional

$$J = E\{x^H Qx + u^H Ru\} \tag{1}$$

is minimized in the feedback controlled system; here, Q is a positive semidefinite hermitian matrix, and R is a positive definite weight of the control effort. Note that substitution of

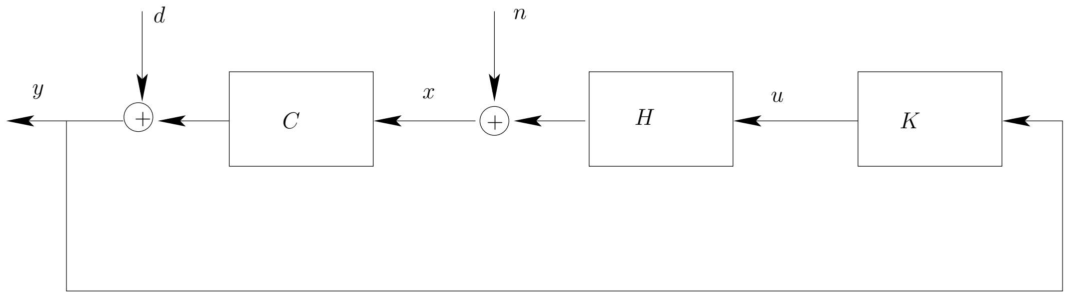

Fig. 1. Standard feedback control loop.

the closed loop relations in this functional leads to a form which is not quadratic in K. However, exploiting an Internal Model Control (IMC) approach, we can define a unique parametrization of all stabilizing compensators as

$$\overline{K} = (I - KCH)^{-1} K, \tag{2}$$

thus actually rewriting the feedback system in an equivalent open-loop form. This is a key step in the formulation, as the functional in (1) can now be written as a quadratic form of $\overline{K}$. Minimization of such functional with respect to $\overline{K}$ then leads to a linear problem; causality of the compensator must be explicitly enforced so that the following Wiener-Hopf problem for $\overline{K}$ is obtained:

$$(H^H Q H + R)\overline{K}_+ (C \phi_{nn} C^H + \phi_{dd}) + \Lambda_- = -H^H Q \phi_{nn} C^H; \tag{3}$$

here, plus and minus subscripts denote frequency response functions of causal and anticausal systems, respectively, and Λ is the Lagrange multiplier associated to the causality constraint for $\overline{K}$. It is noteworthy that this procedure allows to design the optimal compensator in one single step, without the need to resort to the separation theorem. Note also that, in the single-input/single-output case, the coefficients of (3) are scalars, so that the optimal feedback compensator can be obtained from the solution of a scalar problem instead of two matrix Riccati equations, no matter how large is the number of states.

Note that the spectral density functions of noise and disturbances appear in their functional form in the coefficients of (3); therefore, arbitrarly "colored" perturbations can be easily handled. We exploit this property by using, in the compensator design procedure, the true statistics of the flow as measured by DNS of an uncontrolled turbulent channel, thus accounting for the full spatio-temporal structure of the state noise; this is a fundamental difference with respect to previously proposed approaches, where approximate models for the noise statistics have been employed.

In the linearized setting, after Fourier transformation in streamwise and spanwise directions, the full control problem is reduced to a set of single-input/single-output control problems for each wavenumber pair (α, β). Control laws are tested performing a DNS of turbulent channel flows, whose boundary condition on wall-normal velocity is computed runtime from the convolution integral

$$\hat{v}_{wall}(\alpha, \beta, t) = \int_0^t \hat{K}(\alpha, \beta, \tau) \hat{m}(\alpha, \beta, t - \tau)\, d\tau, \qquad \forall \alpha, \beta;$$

here, hats denote Fourier coefficients, $\hat{K}(\alpha, \beta, t)$ is the impulse response function of the optimal compensator, and $\hat{m}(\alpha, \beta, t)$ denotes the history of either one of the wall measurements. Upon inverse Fourier tranformation to physical space, the spatial structure of the compensator can be recovered in the form of a convolution kernel; fig. 2 shows such kernel at zero time lag, i.e. the function $K(x, z, 0)$, for a kernel based on streamwise skin friction measurements.

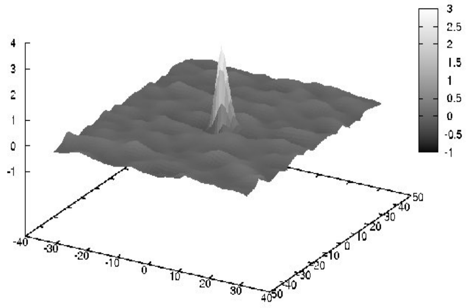

Fig. 2. Spatial representation of the impulse response function of the control kernel at zero time lag, $K(x, z, 0)$, when measurement of streamwise skin friction is employed.

We designed and tested feedback compensators for turbulent channel flows at $Re_\tau = 100$ and $Re_\tau = 180$, using two state weighting matrices (namely, derived from the energy and dissipation norms), actuating with the wall-normal velocity at the walls, and measuring either one of the skin friction components of pressure fluctuations; this required about 300 DNS of controlled turbulent channel flows, that were performed on a cluster dedicated to the simulation of wall turbulence and located at the University of Salerno.

A summary of the performance assessment is reported in Table 1. Results indicate that,

	Dissipation norm			Energy norm		
Re_τ	τ_x	τ_z	p	τ_x	τ_z	p
100	2%	0%	0%	0%	0%	0%
180	8%	6%	0%	0%	0%	0%

Table 1. Best performance drag reduction results, as a function of Re, state weighting and measurement. Accuracy of the drag reduction is estimated to be $\approx \pm 1\%$. The values 0% indicate that no measurable difference in the average skin friction was obtained with respect to the uncontrolled case.

employing any of the available measurements and for the two values of Re_τ considered, weighting matrix derived directly from the energy norm is ineffective in providing drag reducing compensators; this confirms previous findings by Lim [4], who managed to obtain drag reduction results only employing weighting matrices derived from other relations, for instance from the output equation. On the other hand, our results demonstrate that state weighting matrix derived directly from the dissipation rate of turbulent kinetic energy leads to effective compensators. Among these, the best performing require measurements of wall skin friction components, and the overall best performance result – obtained using streamwise skin friction measurements – lead to a maximum drag reduction of $\approx 8\%$ at $Re_\tau = 180$. The net power saved when control is applied may be quantified by the following ratio:

$$P.R. = 100\frac{P_r - P_c}{P_r},$$

where P_r is the power required to drive the uncontrolled flow against viscous stresses, while P_c is that required to drive the controlled flow plus the control action, estimated as suggested in [9]. The power required for the control action was found to be $\approx 0.2\%$ of P_r, and the corresponding power reduction index is $P.R. \approx 7.7\%$.

It is interesting to note that compensators designed using the dissipation norm have better performance at higher values of Re_τ. This "inverse" effect can be attributed to the fact that the ratio between the turbulent component (associated to turbulent fluctuations) and the mean component (associated to the mean flow) of the total dissipation rate increases with Re_τ. As the present optimal compensators are designed to directly target turbulent fluctuations only (and, therefore, the turbulent component of the dissipation), their drag reducing capability increases with Re_τ, probably up to a certain saturation limit to be determined in future work.

Since the present formulation allows the design of compensators accounting for the measured structure of the noise statistics, as well as the measured average dynamics of wall-forced turbulent channel flow, it actually represents a tool to devise the best possible linear time-invariant feedback control strategy for the problem at hand. In light of the results obtained in the present work, as well as of those reported in the recent literature, we emphasize the importance of selecting an appropriate objective function in the control design, that appears to be the only important degree of freedom for the present problem. Therefore, in future work we aim at exploiting the computational effectiveness of the present compensator design strategy to test a wide variety of objective functions, in order to quantitatively assess limiting performance of linear optimal compensators for drag reduction in wall turbulence.

References

[1] M. Högberg, T.R. Bewley, and D. Henningson. Relaminarization of $Re_\tau = 100$ turbulence using gain scheduling and linear state-feedback control. *Phys. Fluids*, 15(11):3572–3575, 2003.

[2] M. Chevalier, J. Höpffner, T. R. Bewley, and D.S. Henningson. State estimation in wall-bounded flow systems. Part 2. Turbulent flows. *J. Fluid Mech.*, 552:167–187, 2006.

[3] K. H. Lee, L. Cortelezzi, J. Kim, and J. Speyer. Application of reduced-order controller to turbulent flows for drag reduction. *Phys. Fluids*, 13(5), 2001.

[4] J. Lim. Control of wall-bounded turbulent shear flows using modern control theory Ph.D. Thesis, Univ. of California, Los Angeles, 2003

[5] J. Kim and J. Lim. A linear process in wall-bounded turbulent flows. *Phys. Fluids*, 12(8):1885–1888, 2000.

[6] P. Luchini, M. Quadrio, and S. Zuccher. Phase-locked linear response of a turbulent channel flow. *Phys. Fluids*, 18(121702):1–4, 2006.

[7] P. Luchini, T.R. Bewley, M. Quadrio. An optimal feedback controller for the reduction of turbulent energy in 3D plane-duct flow. APS Meeting, Chicago, 2005

[8] P. Luchini. Phase-locked linear response and the optimal feedback control of near-wall turbulence. Mathematical Physics Models and Engineering Sciences, Liguori Editore, Naples, 2008.

[9] T.R. Bewley, P. Moin, R. Temam DNS-based predictive control of turbulence: an optimal benchmark for feedback algorithms *J. Fluid Mech.*, 447:179–225, 2001.