SUPERFLUID VORTICES IN A WALL-BOUNDED FLOW

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Ancona, 17 settembre 2009

Struttura della presentazione

1. Introduction
2. Thermal Counterflow
3. Model
4. Results and Conclusions

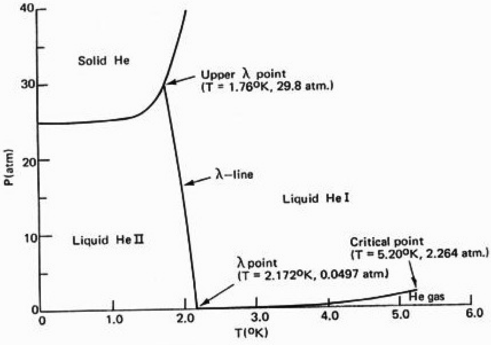
He^4 Characteristics

He peculiarities

- permanent liquid
- distinct liquid phases
- λ-phase-transition

He I: Ordinary fluid

- $\rho = 0.137\ \text{g/cm}^3$
- $\nu = 2.56 \cdot 10^{-8} \text{m}^2/\text{s}$



The phase diagram of He^4.

He II: Quantum fluid

- very low temperatures
- indistinguishable particles
- Bose-Einstein quantum statistics

Two-fluid MODEL

- Tisza (1940) , Landau (1941-1947)

NORMAL

- ρ_n
- $\boldsymbol{v}_n$
- s_n
- ν_n

SUPERFLUID

- ρ_s
- $\boldsymbol{v}_s$
- $s_s = 0$
- $\nu_s = 0$

- $\rho = \rho_n + \rho_s$
- $\boldsymbol{j} = \rho_n \boldsymbol{v}_n + \rho_s \boldsymbol{v}_s$

- $\frac{\partial \boldsymbol{v}_n}{\partial t} = N.S.(\boldsymbol{v}_n)$

- $\nabla \times \boldsymbol{v}_s = 0$

INCOMPLETE

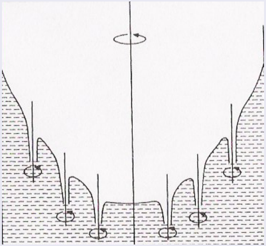
Superfluid Quantized Vortex-lines

Regions of concentrated superfluid vorticity

- very small regions
- circulation of $\boldsymbol{v}_s$ quantized $\Gamma = n\frac{h}{m}\ ,\quad n \in \mathbb{N}$
  - London (1954), Landau & Lifshitz (1955): *vortex-sheets*
  - Onsager (1949), Feynman (1955): *vortex-lines*

Hall & Vinen, 1956

- studies on uniformly rotating He II



- thermodynamical discussion
- experimental study

$\Downarrow$

QUANTIZED VORTEX-LINES

Mutual Friction Force
Hall & Vinen, 1956

vortex lines = *scattering centres*

$\Downarrow$

momentum transfer

collision *rotons, phonons* – vortex lines

$\Downarrow$

MUTUAL FRICTION FORCE

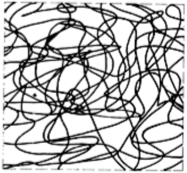
Theory of Mutual Friction in uniformly rotating He II

$$\boldsymbol{f}_D = -\alpha \rho_s \Gamma \widehat{\boldsymbol{\omega}}_s \times \left[\widehat{\boldsymbol{\omega}}_s \times \left(\overline{\boldsymbol{v}_n} - \overline{\boldsymbol{v}_s}\right)\right] - \alpha' \rho_s \Gamma \widehat{\boldsymbol{\omega}}_s \times \left(\overline{\boldsymbol{v}_n} - \overline{\boldsymbol{v}_s}\right)$$

- $\alpha = \alpha(T) \quad , \quad \alpha' = \alpha'(T)$
- $\widehat{\boldsymbol{\omega}_s}$ // axis of rotation
- $\overline{\boldsymbol{v}_n}$ and $\overline{\boldsymbol{v}_s}$: average over $\ell \gg \ell_0$

Mutual Friction
Schwarz, 1978

- *generic* He II system
  - *vortex – tangle*
  - $\overline{\boldsymbol{\omega}}_s = 0$



Theoretical Formulation

- $\boldsymbol{v}_n$ and $\boldsymbol{v}_s$ *local*
- vortex curvature
  - self induction
  - $\boldsymbol{v}_i(\boldsymbol{s}) = \frac{\Gamma}{4\pi}\int_L \frac{(\boldsymbol{z}-\boldsymbol{s})\times d\boldsymbol{z}}{|\boldsymbol{z}-\boldsymbol{s}|^3}$

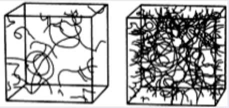
$$\boldsymbol{f}_D = -\alpha\rho_s\Gamma\boldsymbol{s}'\times\left[\boldsymbol{s}'\times(\boldsymbol{v}_n-\boldsymbol{v}_s-\boldsymbol{v}_i)\right]-\alpha'\rho_s\Gamma\boldsymbol{s}'\times(\boldsymbol{v}_n-\boldsymbol{v}_s-\boldsymbol{v}_i)$$

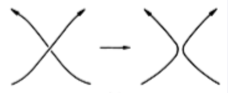
$$\boldsymbol{v}_L = \boldsymbol{v}_s+\boldsymbol{v}_i+\alpha\boldsymbol{s}'\times(\boldsymbol{v}_n-\boldsymbol{v}_s-\boldsymbol{v}_i)-\alpha'\boldsymbol{s}'\times\left[\boldsymbol{s}'\times(\boldsymbol{v}_n-\boldsymbol{v}_s-\boldsymbol{v}_i)\right]$$

NUMERICAL SIMULATIONS

SCHWARZ, 1988

- *prescribed* $\boldsymbol{v}_n$ and $\boldsymbol{v}_s$
- $\boldsymbol{v}_i$ computed with *LIA*
- analysis of vortex *topology*
- preliminary study of *reconnections*

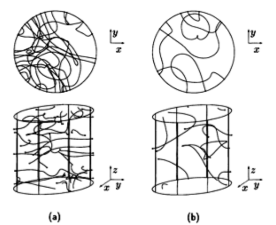




AARTS, DE WAELE, SAMUELS, BARENGHI (1994–1997)

- *kinematic* simulations

  $\boldsymbol{v}_n$ prescribed
  - *uniform*
  - *Poiseuille*
  - *ABC* model flows
- $\boldsymbol{v}_n$ and $\boldsymbol{v}_s$ *coupling*



Numerical Simulations

Barenghi, Samuels (2001) , Idowu *et al.* (2001)

- *self-consistent* algorithm
- *Lagrangian* description $\boldsymbol{v}_L$
    - vortex *filament* method
    - *Local Induced Approximation* for $\boldsymbol{v}_i$
- Navier–Stokes *Eulerian* equations $\boldsymbol{v}_n$
    - $\frac{\partial \boldsymbol{v}_n}{\partial t} + (\boldsymbol{v}_n \cdot \nabla)\, \boldsymbol{v}_n = -\frac{1}{\rho} \nabla p - \frac{\rho_s}{\rho_n} s \nabla T + \nu_n \nabla^2 \boldsymbol{v}_n - \frac{1}{\rho_n} \boldsymbol{F}_{ns}$
    - *local* $\boldsymbol{F}_{ns}$

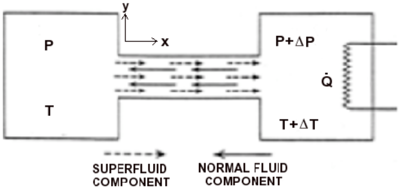
BUT

- *all* past simulations performed in *unbounded* domains

Importance of solid boundaries

- Vortex–lines *nucleation*
- Vortex–lines *dynamics*

THERMAL COUNTERFLOW



IMPORTANCE

- unique phenomenon
- two-fluid model
- fundamentals of vortex–line dynamics

- $\dot{Q} \quad \Rightarrow \quad \boldsymbol{q} = \rho s T \overline{\boldsymbol{v}}_n$
- *two-fluid* model equations

$$\rho_n \frac{\partial \boldsymbol{v}_n}{\partial t} + \rho_n (\boldsymbol{v}_n \cdot \nabla) \boldsymbol{v}_n = \eta_n \nabla^2 \boldsymbol{v}_n - \frac{\rho_n}{\rho} \nabla p - \rho_s s \nabla T$$

$$\rho_s \frac{\partial \boldsymbol{v}_s}{\partial t} + \rho_s (\boldsymbol{v}_s \cdot \nabla) \boldsymbol{v}_s = \frac{\rho_s}{\rho} \nabla p + \rho_s s \nabla T$$

Thermal Counterflow

- *steady-state*
- *small velocities*

$$\Rightarrow \begin{cases} \nabla p = \rho s \nabla T \\ \nabla p = \eta_n \nabla^2 \boldsymbol{v}_n \end{cases}$$

- $\boldsymbol{v}_n(\boldsymbol{x}) = \frac{|\nabla p|}{2\eta_n} y(y - 2\delta)\hat{\boldsymbol{x}}$

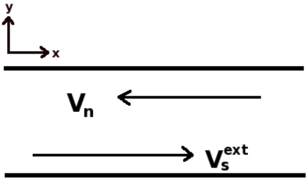
- $\dot{Q} > \dot{Q}_*$ $\Rightarrow$ vortex–lines *nucleation*
  - mutual–friction
  - $\nabla T_{eff} > \nabla T \quad , \quad \nabla p_{eff} > \nabla p$
  - no steady–state

Mass flux condition

- $\overline{\boldsymbol{v}}_s = -\frac{\rho_n}{\rho_s}\overline{\boldsymbol{v}}_n$

Computational Method

- thermal–*channel*–counterflow



- bidimensional domain
- solid boundaries

N vortex–points dynamics

$$\boldsymbol{v}_L(x_k, y_k, t) = \left[(1-\alpha') \left(v_s^{ext} + v_{s,i}^x \right) - \alpha' v_n(y) \pm \alpha v_{s,i}^y \right] \hat{\boldsymbol{x}} +$$

$$+ \left[(1-\alpha') v_{s,i}^y \mp \alpha \left(v_n(y) + v_s^{ext} + v_{s,i}^x \right) \right] \hat{\boldsymbol{y}}$$

- $\boldsymbol{v}_{s,i}(\boldsymbol{x}, t)$: superfluid induced velocity
- $\boldsymbol{v}_n(\boldsymbol{x}) = -v_n(y)\hat{\boldsymbol{x}}$, $v_n(y) \geq 0$, parabolic profile
- $\boldsymbol{v}_s^{ext} = v_s^{ext}\hat{\boldsymbol{x}} = -\dfrac{\rho_n}{\rho_s}\overline{\boldsymbol{v}}_n$: uniform superfluid velocity

Computational Method

- irrotational
- *incompressible*

$\Big\} \Rightarrow$

- *complex-potentials*

Vortex in a plane channel

- *conformal* mapping
- *images* method

$$v_j(z) = v_j^x - i v_j^y = \mp i \frac{\hbar}{m} \frac{\pi}{4\delta} \left\{ \coth\left[\frac{\pi}{4\delta}\left(z - z_j\right)\right] - \coth\left[\frac{\pi}{4\delta}\left(z - \overline{z_j}\right)\right] \right\}$$

- $$v_{s,i}(z_k) = v_{s,i}^x - i v_{s,i}^y = \sum_{j \neq k} v_j(z_k) \pm i \frac{\hbar}{m} \frac{\pi}{4\delta} \coth\left[\frac{\pi}{4\delta}\left(z_k - \overline{z_k}\right)\right]$$

Vortex–points nucleation

- N constant
- $$y^* = \frac{2}{\pi} \arctan\left(\frac{\pi}{4} \frac{1}{v_s^{ext}}\right)$$

NUMERICAL RESULTS

$T(^\circ K)$	1.5
ρ_n/ρ_s	0.143
α	0.078
α'	6.25×10^{-3}
$\Delta p_{tot}(Pa)$	10
$L_x(m)$	0.1
$\eta_n(Pa\cdot s)$	3×10^{-7}
$\delta(m)$	3×10^{-5}

- $\overline{v}_n = 0.10\text{m/s}$
- $v_s^{ext} = 0.014\text{m/s}$

- vortex–points *sign–separation*

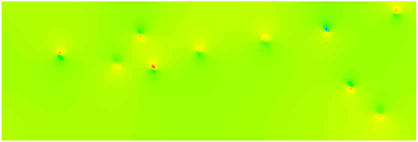
$$v_L^y = (1-\alpha')\, v_{s,i}^y \mp \mp\alpha\left(v_n(y) + v_s^{ext} + v_{s,i}^x\right)$$

  - $+$ vortices $\downarrow$
  - $-$ vortices $\uparrow$

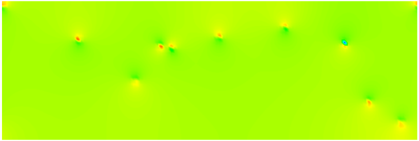
- $y^* \sim \delta$
  - vortex *nucleation* on *centerline*

Numerical results

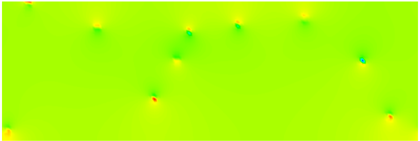
Surface plots of $v^x_{s,i}$



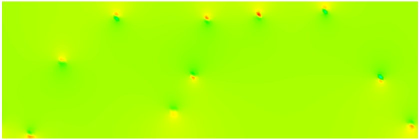
t_1



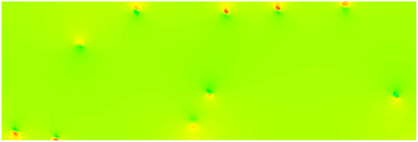
t_2



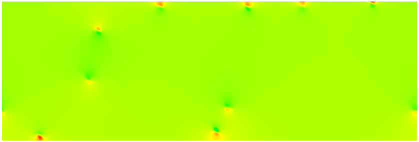
t_3



t_4

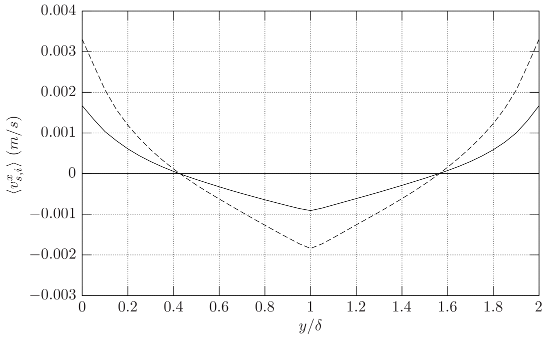


t_5



t_6

Numerical results



- $N = 10$ —- line
- $N = 20$ - - - line

$v_{s,i}$ VS v_n

- *similarities*
  - *qualitative locking*
    – Samuels , 1992
    – Barenghi *et al.* , 1997
- *differences*
  - *centerline peak*
  - $\langle v^x_{s,i} \rangle (y) \Big|_{y=0,2\delta} \neq 0$

Summary and Future work

Summary

- bidimensional numerical simulation of *He* II thermal–*channel–counterflow*
- first simulation in *wall bounded geometry*
- focus on:
  - vortex–points *dynamics*
  - vortex–points *nucleation*
  - superfluid *induced* velocity *profile*
  - $\langle v^x_{s,i} \rangle(y)$ vs $v_n(y)$

Future works

- compute $\boldsymbol{v}_{s,i}$ *mass–flux*
- implement vortex–points *feedback* on normal fluid
- verify numerically *flattening* of *normal fluid profile*