Wiener-Hopf design of feedback compensators for drag reduction in turbulent channels

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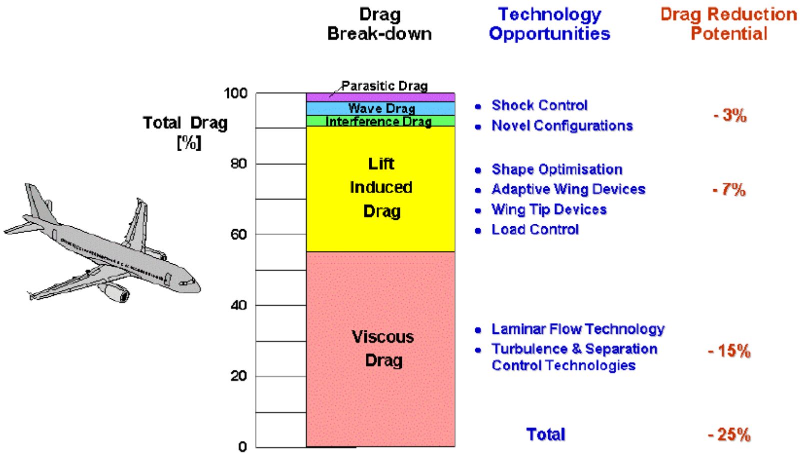
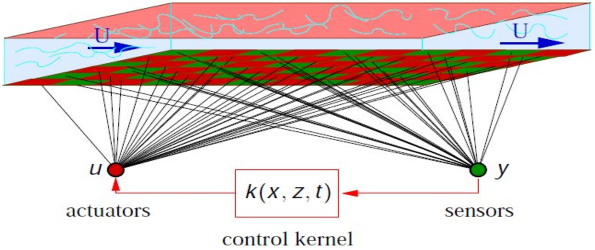
XX AIDAA Conference – Milano
29 Giugno 2009

Outline

Turbulent drag reduction
Source: Airbus



Feedback control of wall turbulence

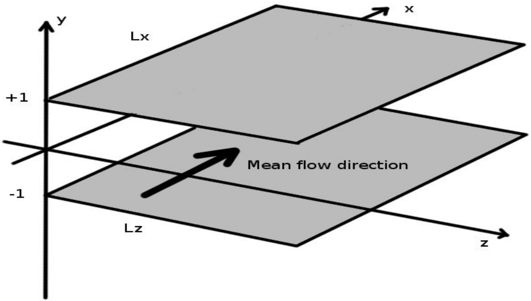


$$u(t,x,z) = \int y(x',z',t')k(x-x',z-z',t-t')\,\mathrm{d}x'\mathrm{d}z'\mathrm{d}t'$$

- Actuators: zero-net-mass-flux wall blowing/suction
- Sensors: pressure and skin friction components

The plant: turbulent channel flow

Incompressible flow between two plane, parallel, infinite walls



- Flow is spatially invariant in x and z
- Efficient DNS at moderate Re (and $\approx 10^8$ d.o.f.s)
- State variables: v-η

State of the art

A very young field

- Hope for linear control (Kim & Lim, 2000)
- Modern Optimal Control Theory
- Unsolved problem of Kalman-filter-based estimator
- State-space formulation
- Additional challenge: billions of d.o.f.

A recent step ahead?

Luchini & Quadrio, PoF 2006

The problem

- Poor system model: NS equations linearized about the mean velocity profile
- Turbulence dynamics is missing

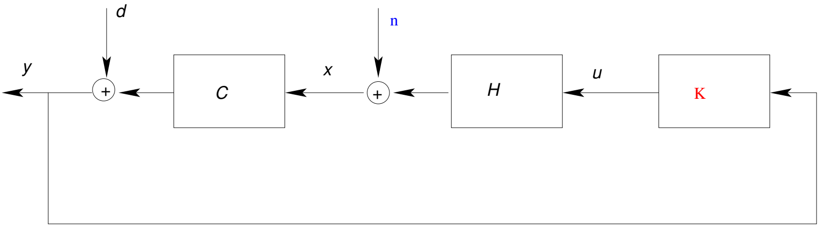
The solution

- Enrich the model: the average linear response function
- Turbulent diffusion is accounted for

Goal of the present work

- Use the average impulse response in the full control problem
- Lay down a computationally-efficient procedure

The feedback control problem



- n: turbulent fluctuations in the uncontrolled flow
- Aim: design K to minimize

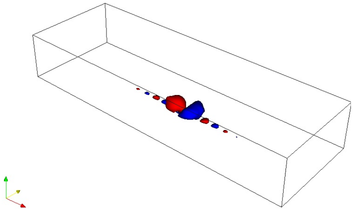
$$J = E\{x^H Q x + u^H R u\}$$

What is H?

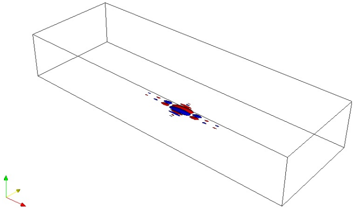
A numerical measurement

- Measurement technique: wall forcing with a small space-time white Gaussian noise
- Cross-correlating the perturbed field with the wall forcing defines H
- H embodies more physics: turbulent diffusion is accounted for (on average)

The average impulse response
Wall-normal forcing

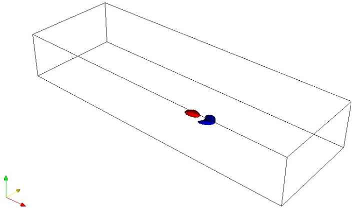


$H_v(x, y, z, t^+ = 5)$

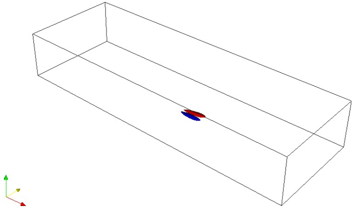


$H_\eta(x, y, z, t^+ = 5)$

The average impulse response

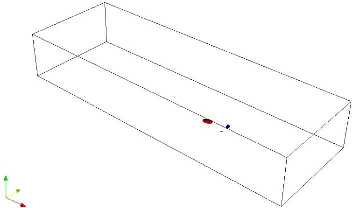


$H_v(x, y, z, t^+ = 15)$

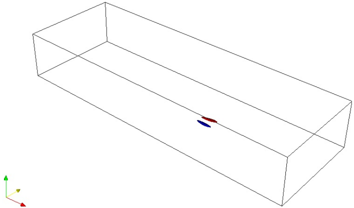


$H_\eta(x, y, z, t^+ = 15)$

The average impulse response



$H_v(x, y, z, t^+ = 25)$



$H_\eta(x, y, z, t^+ = 25)$

Why we've got stuck in 2006

Comment

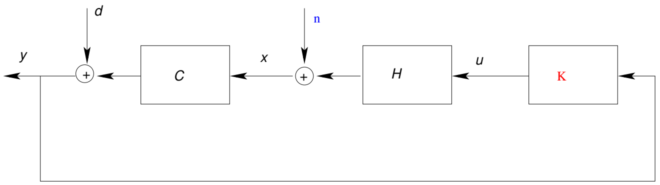
- A state-space realization of H is computationally unaffordable
- Our system is LTI with stationary stochastic forcing
- Our system has $N_{in}, N_{out} \ll N_{states}$

Implication

- Standard control design techniques cannot be employed
- A frequency domain approach is feasible and convenient
- This can (must) be exploited for efficiency

Switch to frequency domain!

F.Martinelli, PhD thesis, PoliMi 2009



Rewriting the objective functional in frequency:

$$J = \int_{-\infty}^{+\infty} Tr[Q\phi_{xx}(f)] + Tr[R\phi_{uu}(f)]\ df.$$

Substituting, J is not quadratic in K.

The optimal compensator in frequency domain

J may be written as a quadratic form of the Youla parameter $\overline{K} = (I - KCH)^{-1}K$ as:

$$J = \int_{-\infty}^{+\infty} Tr\Big\{Q\phi_{nn} + QH\overline{K}C\phi_{nn} + Q\phi_{nn}C^H\overline{K}^H H^H + \dots$$
$$\dots + QH\overline{K}C\phi_{nn}C^H\overline{K}^H H^H + QH\overline{K}\phi_{dd}\overline{K}^H H^H\Big\} + \dots$$
$$\dots + Tr\Big\{R\overline{K}C\phi_{nn}C^H\overline{K}^H + R\overline{K}\phi_{dd}\overline{K}^H\Big\}\, df.$$

Minimization yields the best compensator (that is non-causal)

How to enforce causality?

Introduce a Lagrange multiplier Λ:

$$J = \int_{-\infty}^{+\infty} Tr\Big\{ Q\phi_{nn} + QH\overline{K}_{+}C\phi_{nn} + Q\phi_{nn}C^H\overline{K}_{+}^{H}H^H \dots$$
$$\dots + QH\overline{K}_{+}C\phi_{nn}C^H\overline{K}_{+}^{H}H^H + QH\overline{K}_{+}\phi_{dd}\overline{K}_{+}^{H}H^H \Big\} + \dots$$
$$\dots + Tr\Big\{ R\overline{K}_{+}C\phi_{nn}C^H\overline{K}_{+}^{H} + R\overline{K}_{+}\phi_{dd}\overline{K}_{+}^{H} \Big\} + Tr[\Lambda_{-}\overline{K}_{+}^{H}]\, df.$$

A Wiener-Hopf problem

Minimization leads to the (linear) Wiener-Hopf problem:

$$(H^H QH + R)\overline{K}_+(C\phi_{nn}C^H + \phi_{dd}) + \Lambda_- = -H^H Q\phi_{nn}C^H$$

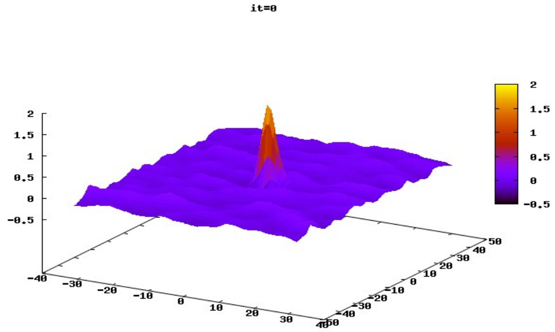
- Solution yields directly the compensator's frequency response (no separation theorem required)
- ϕ_{nn} appears in functional form: full space-time structure of the noise easily accounted for
- Scalar equation for the SISO case: superfast FFT-based solution

The procedure

Measure ⇒ design ⇒ test

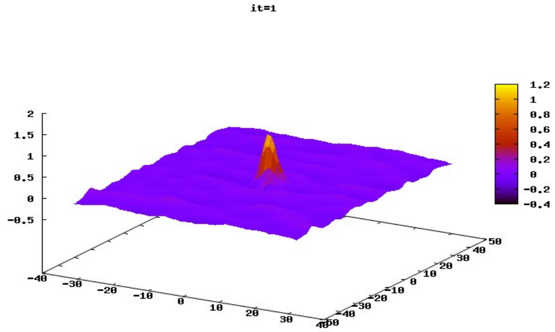
- Response function and noise spectral densities are measured via DNS and Fourier transformed in x and z
- Compensator is designed by solving the Wiener-Hopf problem wavenumber-wise
- Compensators are tested in a full nonlinear DNS

Compensator kernel in physical space



$$u(x, z, t) = \int K(x - x', z - z', t - t') y(x', z', t') \, dx' dz' dt'$$

Compensator kernel in physical space



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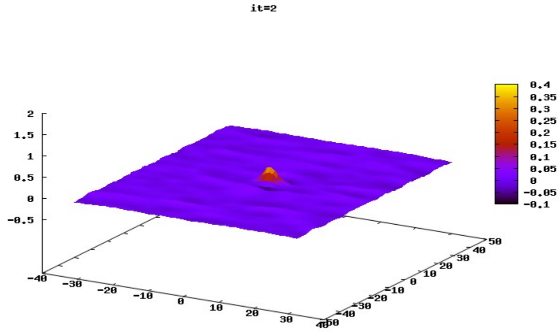

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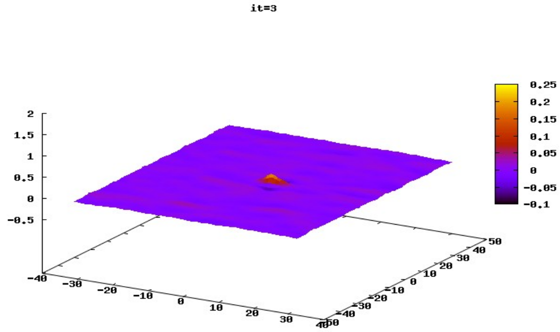

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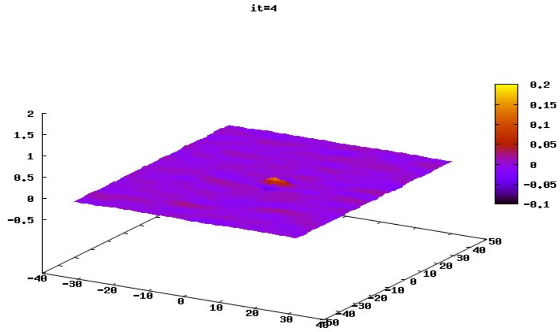

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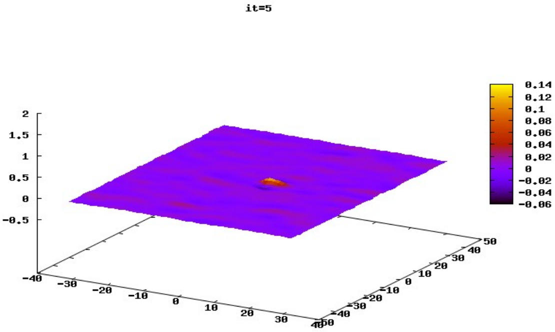


$$u(x,z,t) = \int K(x-x', z-z', t-t') y(x', z', t')\, dx' dz' dt'$$

Performance assessment

Parametric study addressing:

- Choice of the objective functional
- Experimenting with R and ϕ_{dd}
- Effectiveness of the sensors
- *Re* effects

More than 300 DNS ($\approx$ 40 years of CPU time) run at the supercomputing system located at the University of Salerno.

Best performance results

	Dissipation			Energy		
Re_τ	τ_x	τ_z	p	τ_x	τ_z	p
100	2%	0%	0%	0%	0%	0%
180	8%	6%	0%	0%	0%	0%

- Energy norm is not effective
- Dissipation norm is effective
- Pressure measurement is useless
- With dissipation, performance improves with Re

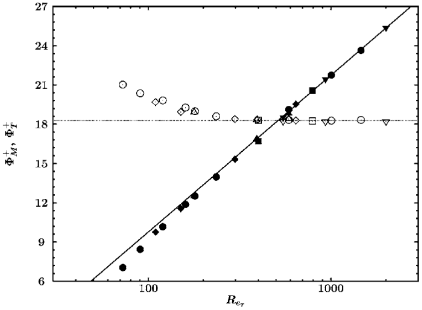
"Inverse" *Re*-effect

$$\frac{d\langle U\rangle}{dy}\Big|_w = -\frac{1}{U_B}\Big\langle \underbrace{\sum_{(\alpha,\beta)\neq(0,0)} D(\alpha,\beta)}_{D_{turb}} + \underbrace{\frac{1}{2}\int_{-1}^{1}\left(\frac{\partial\hat{U}}{\partial y}\right)_{(0,0)}\left(\frac{\partial\hat{U}}{\partial y}\right)^*_{(0,0)} dy}_{D_{mean}}\Big\rangle$$

- D_{turb} is affected directly by zero net mass flux wall blowing/suction
- D_{mean} is affected indirectly via nonlinear interactions between fluctuations and the mean flow

“Inverse” *Re*-effect
Laadhari, PoF 2007

The relative contribution of D_{turb} to the total dissipation increases with *Re*!



Re_τ	D_{turb}	D_{mean}
100	26.8%	73.2%
180	39.5%	60.5%

Critical discussion

Importance of selecting the cost function

- Present compensators are the best possible LTI
- Their performance is poor

The cost function is probably the sole important degree of freedom

Conclusions

- A novel compensator design formulation in frequency domain has been proposed
- It is extremely efficient
- It exploits a measured linear model of the turbulent channel flow
- The time-space structure of the state noise is accounted for
- Feedback compensators can be designed from experimentally measured data