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WIENER-HOPF DESIGN OF FEEDBACK COMPENSATORS FOR DRAG REDUCTION IN TURBULENT CHANNELS

Fulvio Martinelli[⋆], **Maurizio Quadrio**[⋆] **and Paolo Luchini**[†]

[⋆]Dipartimento di Ingegneria Aerospaziale
Politecnico di Milano, Via La Masa 34, 20156 Milano, Italy
e-mail: fulvio.martinelli@polimi.it
[†]Dipartimento di Meccanica
Università di Salerno, Via Ponte don Melillo, 84084 Fisciano (SA), Italy

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Abstract. *In the last decade, fundamental research has shown the potential of feedback control as applied to turbulent flows for drag reduction purposes. This potential has not been fully exploited yet, since present-day results – though encouraging – have been obtained numerically for very low Reynolds number turbulence; furthermore, designing optimal control laws is a computationally demanding procedure, that requires the solution of high-dimensional matrix Riccati equations. In these approaches, the system dynamics is defined through a linear model, which has been always obtained via linearization of the governing equations about a reference velocity profile.*

Application of optimal feedback control laws at substantially higher Reynolds number would benefit from a computationally tractable formulation of the control design procedure, as well as from the capability of directly employing linear models as measured, for instance, from laboratory experiments. In this case, test of the control laws at high Reynolds numbers might be performed experimentally, thus bypassing the obvious limitations of direct numerical simulations of wall turbulence.

Aim of the present work is to provide a computationally effective formulation of the optimal compensator design problem, that offers the additional feature of being well suited for the use of linear models. This formulation is exploited here in a proof-of-principle numerical verification, that highlights the phases of a control design and test procedure exactly as would be performed experimentally. Furthermore, we report on the effectiveness of different objective functions and measurements in the design of compensators, as well as on the effects of varying Reynolds number on the overall performance.

1 INTRODUCTION

In an attempt to devise feedback laws aimed at reducing drag in wall turbulence, modern optimal control theory has recently been employed for the design of linear controllers [1], estimators [2], and compensators [3, 4]. These attempts led to significant positive results, revealing the potential of linear control in targeting at least some fundamental dynamics of turbulent wall flows [5].

All such approaches, however, are based on an approximate state-space representation of the system dynamics, that iinvariably obtained by linearization of the governing equations about a reference velocity profile. Effects of nonlinearities are accounted for *a posteriori* by appropriate definition of state and measurement noises with given statistics. The state-space formulation reduces the compensator design problem to the solution of two algebraic matrix Riccati equations, a procedure that is simply computationally unaffordable for high-dimensional systems, such as those arising from discretization of moderate-to-high Reynolds number fluid flow problems.

In the present work, we employ an alternative formulation of the optimal compensator design problem, which is based on a frequency-domain approach. In contrast to its dual state-space formulation, the present approach allows us to directly work with a system model expressed in terms of response functions, as would be obtained from an experimental measurement. Therefore, we employ a linearized model of the wall-forced turbulent channel flow system in the form of an average impulse response function, that is directly *measured* with DNS using the procedure proposed by Luchini et al. [6]. Thanks to the frequency-domain approach, no state-space realization for such model is needed. An additional feature of this formulation is the capability of accounting for arbitrarly "colored" noise on the state equations (i.e. nontrivial time structure of the state noise) without increasing the complexity of the control design procedure.

As a model problem, we consider the control of the incompressible turbulent flow in an indefinite, plane channel. Streamwise, spanwise and wall-normal directions will be denoted by x, z and y, and we will consider the governing equations written in the well-known v-η formulation, that provides the minimum number of degrees of freedom for the present problem. As usual, we exploit spatial invariance in x and z direction by expanding the solution in Fourier series, with the advantage of providing a framework for both optimally efficient DNS and easing the control problem in the linearized setting, thanks to wavenumber decoupling.

2 A MEASURED LINEAR MODEL OF THE WALL-FORCED TURBULENT FLOW

We employ a linearized model of the wall-forced turbulent channel flow system in the form of an average impulse response function; such model is directly *measured* with DNS using a procedure proposed by Luchini et al. in [6]. In the linear setting, introducing small wall-normal velocity perturbations $v_W(x, z, t)$ at the channel walls in the form of a space-time white Gaussian noise results in a perturbed velocity-vorticity field in the form

$$v_{tot}(x, y, z, t) = \overline{v}(x, y, z, t) + v(x, y, z, t)$$
$$\eta_{tot}(x, y, z, t) = \overline{\eta}(x, y, z, t) + \eta(x, y, z, t),$$

where $(\overline{v}, \overline{\eta})$ indicate the unperturbed field. Cross-correlating the perturbed field and the wall-forcing:

$$E\{v_{tot}(x'+x, y, z'+z, t'+t) v_W^*(x', z', t')\} = \dots$$
$$\dots \underbrace{E\{\overline{v}(x'+x, y, z'+z, t'+t) v_W^*(x', z', t')\}}_{=0} + \dots$$
$$\dots + E\{v(x'+x, y, z'+z, t'+t) v_W^*(x', z', t')\},$$

$$E\{\eta_{tot}(x'+x, y, z'+z, t'+t) u_{i,w}^*(x', z', t')\} = \dots$$
$$\dots \underbrace{E\{\overline{\eta}(x'+x, y, z'+z, t'+t) v_W^*(x', z', t')\}}_{=0} + \dots$$
$$\dots + E\{\eta(x'+x, y, z'+z, t'+t) v_W^*(x', z', t')\}.$$

The first terms on the right hand sides vanish, as the undisturbed turbulent flow is uncorrelated to the wall forcing.

Since both the perturbation fields and the wall forcing are statistically stationary, we can define the second terms on the right hand side as

$$\begin{aligned} H_{v,i}(x, y, z, t) &= E\{v(x'+x, y, z'+z, t'+t) v_W^*(x', z', t')\} \\ H_{\eta,i}(x, y, z, t) &= E\{\eta(x'+x, y, z'+z, t'+t) v_W^*(x', z', t')\}. \end{aligned} \tag{1}$$

It is well known from system theory that the input-output cross-correlation of a linear time-invariant (LTI) system driven by a unit intensity white Gaussian noise equals the impulse response of the system. Hence, eq. (1) can be thought of as the *definition* of a LTI system representing the average response of the perturbation field v, η when a small impulsive input $v_W(x, z, t) = \delta(x, z, t)$ is applied at the channel walls.

3 FORMULATION OF THE CONTROL PROBLEM

The feedback control loop considered in this work is shown in fig. 1, where a feedback compensator $K(\omega)$ feeds a system, having frequency response $H(\omega)$, with an input signal u determined from a real-time measurement y; the sensor's transfer function is denoted by $C(\omega)$. Both the measurement and the state x are corrupted by disturbances d and noise n, respectively, that are supposed to be uncorrelated. The spectral density functions of the disturbances and noise will be denoted by $\phi_{dd}(\omega)$ and $\phi_{nn}(\omega)$, respectively, and are not restricted to be constant (i.e. associated to white noises), but may be functions of the frequency, provided they are positive definite for all ω.

The aim of the control design procedure is finding an optimal feedback compensator K such that the usual LQG expectation functional

$$J = E\{x^H Q x \, + \, u^H R u\} \tag{2}$$

is minimized in the closed-loop system. This functional is defined by the positive semidefinite hermitian matrix Q, weighting the departure of the state from the origin of the state space, and the positive definite weight R, introduced as a regularization parameter to limit the control effort. Note that substitution of the closed loop relations in this functional leads to a form which is not quadratic in K. However, assuming that the system H is asymptotically stable, we can define a parametrization of in the form

$$\overline{K} = (I - KCH)^{-1} K, \tag{3}$$

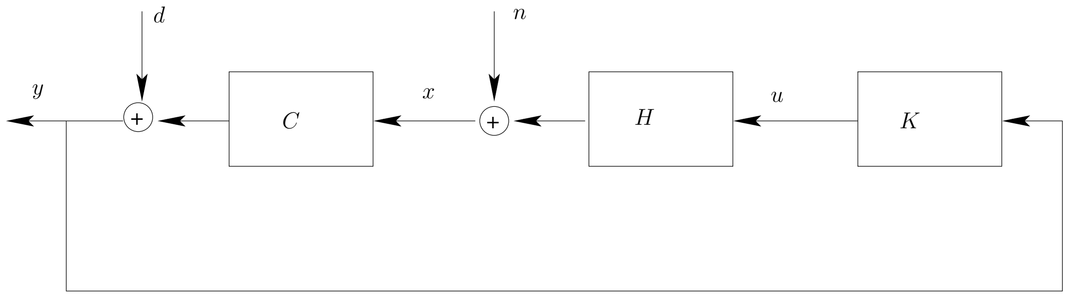

Figure 1: Standard feedback control loop.

where $\overline{K}$ is the frequency response of a stable compensator. This change of variables – known as Youla's parametrization – is equivalent to rewriting the feedback system in an open-loop form; therefore, all the sensitivity functions for this feedback loop can be written as affine functions of $\overline{K}$, and furthermore the functional in (2) results in a quadratic form of $\overline{K}$. Minimization of such functional with respect to $\overline{K}$ then leads to a linear problem, where causality of the compensator must be explicitly enforced so that the following Wiener-Hopf problem for $\overline{K}$ is obtained:

$$(H^H Q H + R)\overline{K}_+(C\phi_{nn}C^H + \phi_{dd}) + \Lambda_- = -H^H Q \phi_{nn} C^H. \tag{4}$$

In the above equation, plus and minus subscripts denote frequency response functions of causal and anticausal systems, respectively, and Λ is the Lagrange multiplier associated to the causality constraint for $\overline{K}$.

It is noteworthy that this procedure allows to design the optimal compensator's frequency response in one sole step, without needing to resort to the separation theorem. Note also that, in the single-input/single-output case, the coefficients of (4) are *scalars*, so that the optimal feedback compensator can be obtained from the solution of a scalar problem instead of two matrix Riccati equations, no matter how large the number of states is.

It should be also emphasized that the spectral density functions of noise and disturbances appear in their functional form in the coefficients of (4). This allows to easily handle arbitrarly "colored" perturbations to the state and the measurement, and we exploit this property by using, in the compensator design procedure, the true statistics of the flow as measured by DNS of an uncontrolled turbulent channel, thus accounting for the full spatio-temporal structure of the state noise. This is a fundamental difference with respect to previously proposed approaches, where approximate models for the noise statistics have been employed (see, for instance, [2]).

4 CONTROL DESIGN AND TESTING PROCEDURE

In the linearized setting, after Fourier transformation in streamwise and spanwise directions, the linear dynamics at different wavenumbers decouples, and the full control problem is reduced to a set of single-input/single-output control problems, parametrized on the wavenumber pair (α, β). Therefore, feedback control laws are designed wavenumber-wise and independently; upon inverse Fourier transformation to physical space, control gains take the form of a convolution kernel $K(x, z, t)$, relating the feedback signal, i.e. the instantaneous wall-normal velocity

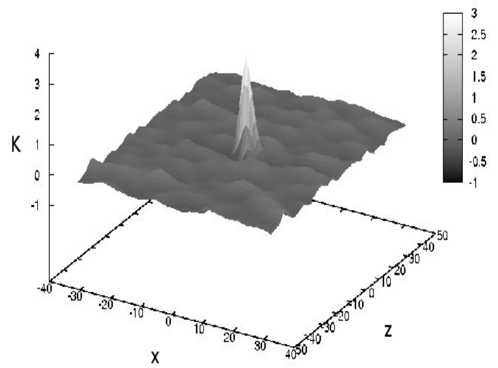

Figure 2: Spatial representation of the impulse response function of the control kernel at zero time lag, $K(x, z, 0)$, when measurement of streamwise skin friction is employed.

	Dissipation			Energy		
Re_τ	τ_x	τ_z	p	τ_x	τ_z	p
100	2%	0%	0%	0%	0%	0%
180	8%	6%	0%	0%	0%	0%

Table 1: Best performance drag reduction results, as a function of Re, state weighting and measurement. Accuracy of the drag reduction is estimated to be $\approx \pm 1\%$. The values 0% indicate that no measurable difference in the average skin friction was obtained with respect to the uncontrolled case.

to be applied at the walls, to the spatial distribution and time history of the measurement y as follows:

$$u(x, z, t) = \iiint K(\overline{x}, \overline{z}, \tau) y(x - \overline{x}, z - \overline{z}, t - \tau)\, d\overline{x} d\overline{z} d\tau$$

Fig. 2 shows the convolution kernel at zero time lag, i.e. the function $K(x, z, 0)$, for a kernel based on streamwise skin friction measurement.

5 RESULTS

We have designed and tested feedback compensators for turbulent channel flows at $Re_\tau = 100$ and $Re_\tau = 180$, using two state weighting matrices (namely, derived from the energy and dissipation norms), actuating with the wall-normal velocity at the walls, and measuring either one of the skin friction components or pressure fluctuations. This required about 300 DNS of controlled turbulent channel flows, that were performed on a computing system, located at the University of Salerno, dedicated to the simulation of wall turbulence.

A summary of the performance assessment is reported in Table 1. Results indicate that, employing any of the available measurements and for the two values of Re_τ considered, a weighting matrix derived directly from the energy norm is ineffective in providing drag reducing compensators. This is consistent with previous findings by Lim [4], who succeded in obtaining drag reduction only by employing weighting matrices derived from other relations, for instance from the output equation. On the other hand, our results show that a state weighting matrix derived directly from the dissipation rate of turbulent kinetic energy leads to effective compensators. Among these, the best performing ones require the measurement of wall skin friction components, and the overall best performance result – obtained using streamwise skin friction measurement – yields a maximum drag reduction of $\approx 8\%$ at $Re_\tau = 180$.

Actuation with the wall-normal velocity component requires an energetic expenditure, and it is therefore interesting to quantify the net power saved when control is applied; we introduce the following ratio:

$$P.R. = 100\frac{P_r - P_c}{P_r},$$

where P_r is the power required to drive the uncontrolled flow against viscous stresses, while P_c is that required to drive the controlled flow plus the control action, estimated as suggested in [9]. The power required for the control action was found to be $\approx 0.2\%$ of P_r, and the corresponding power reduction index is $P.R. \approx 7.7\%$.

An interesting feature of the present results is the fact that dissipation-based compensators yield better performance at higher values of Re; this "inverse" Re-effect may be explained by the following argument. In the uncontrolled flow, it can be shown that the dissipation rate of turbulent kinetic energy equals the average skin friction:

$$\frac{d\langle U\rangle}{dy}\Big|_w = -\frac{1}{U_B}\Big\langle \underbrace{\sum_{(\alpha,\beta)\neq(0,0)} D(\alpha,\beta)}_{D_{turb}} + \underbrace{\frac{1}{2}\int_{-1}^{1}\Big(\frac{\partial \hat{U}}{\partial y}\Big)_{(0,0)}\Big(\frac{\partial \hat{U}}{\partial y}\Big)^*_{(0,0)} dy}_{D_{mean}} \Big\rangle.$$

Here, braces denote ensemble average and the function $D(\alpha, \beta)$ corresponds to the contribution to the total rate of dissipation of turbulent kinetic energy from a given wavenumber pair. The total dissipation can be written as the sum of two terms, corresponding to the contribution due to flow fluctuations (D_{turb}) and to the mean flow (D_{mean}); it has been recently shown in [10] that their ratio D_{turb}/D_{mean} increases with Re in the uncontrolled flow. As the present compensators apply a zero-net-mass-flux forcing, they target directly the turbulent component D_{turb} and affect the mean component only indirectly, via nonlinear triadic interactions. Therefore, we argue that the improvement in the drag reduction performance of our compensators with Re is due to the natural increase of the relative importance of D_{turb} in the turbulent channel flow; we also conjecture that performance might increase even more at higher Re, probably up to a certain saturation limit to be explored in future work.

6 CONCLUSIONS

In this paper, we have presented a computationally effective procedure to design linear feedback laws for drag reduction in wall turbulence. This formulation allows the design of compensators accounting for the measured temporal and spatial structure of the noise statistics, as well as the measured average dynamics of wall-forced turbulent channel flow. As such, it actually represents an effective tool to devise the best possible LTI feedback control strategy for the problem at hand, even on the basis of experimentally measured data. In light of the results obtained in the present work, as well as of those reported in the recent literature, we emphasize the importance of selecting an appropriate objective function in the control design, as it appears to be the most important degree of freedom for the present problem. Therefore, in future work we suggest to exploit the computational effectiveness of the present strategy to test a wide variety of objective functions, in order to quantitatively assess limiting performance of linear optimal compensators for drag reduction in wall turbulence. Our aim is also to quantify to what extent, and up to which value of Re, compensators designed to minimize the dissipation rate of turbulent kinetic energy yield increasing drag reduction performance.

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