Active control and drag reduction in turbulent wall flows

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Abstract

Control of turbulent flows is possibly one of the major challenges of today's engineering. Many technological processes may be substantially improved, with significant gains in terms of efficiency, with properly applied small control efforts; in particular, active feedback techniques have been recently considered as a possible mean of obtaining substantial skin friction drag reduction in turbulent wall flows. The present work is focused on the design of active feedback controllers with the capability of significantly reducing skin friction drag in turbulent channel flows. The design procedure is briefly outlined, and direct numerical simulations are performed with the aim of assessing the Reynolds number dependence of active controllers' performance.

1 Introduction

Everyday experience gives many opportunities to observe turbulent flows: the smoke from a chimney, water vortices in a river or the buffeting of the wind near a flag are just few examples of the ubiquity of turbulent flows in Nature. Many technological applications rely on, or are affected by, turbulent behavior of a working fluid as well; then, improvement in understanding the fundamental dynamics of turbulent flows is of utmost importance.

From the technological point of view, efficient control of turbulent flows is possibly one of the major challenges of today's engineering. Up to the last decade, turbulent flows were controlled with passive means (e.g. proper shaping of bodies and surfaces) or active feedforward techniques (e.g. continuous or variable blowing-suction in boundary layers). It is only very recently that active *feedback* control techniques have been considered as a possible viable mean of achieving a desired global effect on a complex flow with limited control effort. At present, only the first steps in the application of modern feedback control theory to turbulent flows have been moved. In particular, the only relevant results can be ascribed to the group of T. Bewley alone [1], who used modern linear control theory to design optimal controllers capable of reducing skin friction drag in a turbulent channel flow up to 25%, via distributed wall blowing-suction. However, this encouraging result has been obtained for a turbulent flow at Reynolds number as low as $Re = 1450$; then, the assessment of this result at higher values of Re is mandatory.

The present paper attempts to clarify whether this kind of closed-loop control of turbulent wall flows performs well at higher Reynolds numbers. To this aim, suitable optimal control kernels are first designed and computed according to the technique introduced in Ref. [2]. Well resolved direct numerical simulations of turbulent channel flow controlled through distributed, time-dependent blowing and suction applied at the channel walls are then performed at $Re = 1450$ and $Re = 3333$. The main results in terms of skin friction drag reduction and turbulent kinetic energy suppression are reported.

2 Computational procedure

2.1 Design of the active feedback controller

We shall consider the incompressible flow in a plane channel having dimensions L_x, 2δ and L_z in the streamwise ($\hat{x}$), wall-normal ($\hat{y}$) and spanwise ($\hat{z}$) directions, respectively; corresponding velocity components are denoted by $\hat{u}$, $\hat{v}$ and $\hat{w}$.

In this study, the controller is designed along the lines developed by Bewley and coworkers, in particular as described in Ref. [2]. The governing incompressible Navier-Stokes equations are linearized around the laminar Poiseuille solution and rewritten in the well-known v-η formulation, where η denotes the wall-normal vorticity component. This formulation offers the well-known advantages that pressure disappears from the evolutive equations and that the computation becomes optimally fast when a Fourier expansion is adopted for the homogeneous directions. In the present context, it is worth noting that the implicit satisfaction of the incompressibility constraint eases setting up the control problem, by bringing to light the two degrees of freedom of the mathematical problem.

Fourier transforming the v and η equations in the homogeneous directions $\hat{x}$ and $\hat{z}$ yields:

$$\begin{aligned} \Delta\dot{v} &= [-i\alpha U\Delta + i\alpha U'' + \Delta\Delta/Re]v = \mathbf{L}v \\ \dot{\eta} &= [-i\beta U']v + [-i\alpha U + \Delta/Re]\eta = \mathbf{C}v + \mathbf{S}\eta \end{aligned} \tag{1}$$

which corresponds to a transformation into the well-known Orr-Sommerfeld-Squire form; here α and β denote the wavenumber in x and z directions, respectively; variables with the hats dropped denote Fourier coefficients, the dot denotes time differentiation, and $U(y)$ is the laminar Poiseuille solution.

The governing equations (1), after proper discretization in wall-normal direction and taking into account inhomogeneous boundary conditions due to the control action at the walls, can be recast in standard state-space form:

$$\dot{x} = Ax + B\dot{\phi}$$

Note that $\dot{\phi}$ is the time derivative of the wall blowing/suction velocity v, i.e. the time derivative of the control that will have to be computed and applied runtime. Controllers are designed by applying optimal control theory to this system. In particular, the problem is stated as the search for a proportional controller K such that the control law $\dot{\phi} = Kx$ stabilizes the system while minimizing some cost functional. A convenient quadratic cost function can be derived from the energy of the flow perturbations; within the present framework, energy can be conveniently written as

$$\mathbf{E} = \sum_{\alpha,\beta} \frac{1}{8k^2} \int_{-1}^{1} w(y) \left(k^2|v|^2 + |\frac{\partial v}{\partial y}|^2 + |\eta|^2 \right) dy = \sum_{\alpha,\beta} E(\alpha,\beta)$$

where $E(\alpha,\beta)$ denotes the contribution of single wavenumber pairs to the total energy of flow perturbations.

The function $w(y)$ is an arbitrary weighting function that can be used to assign different weights to states located in different positions along the wall-normal direction; when $w(y) = 1$, the usual definition of turbulent kinetic energy is obtained. It is straightforward to rewrite $E(\alpha,\beta)$ as a quadratic function of the state, as follows:

$$E(\alpha,\beta) = x^H Q x \tag{2}$$

where H denotes conjugate traspose and Q is an hermitian nonnegative definite matrix.

Let us introduce the following quadratic functional:

$$J = \int_0^{+\infty} (x^H Q x + u^H R u)\, dt \tag{3}$$

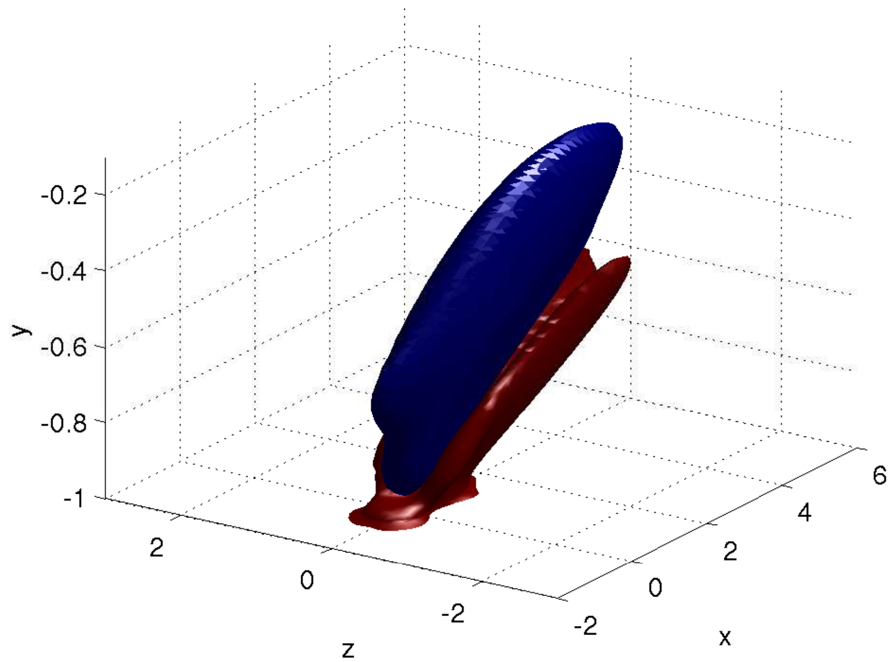

Figure 1: Three-dimensional view of the velocity convolution kernel $K_{\hat{v}}(\hat{x}, \hat{y}, \hat{z})$ for the forcing at the lower wall. Isosurfaces refer to the ±5% of the peak value. The spatial convolution of this kernel in $\hat{x}$ and $\hat{z}$ directions with the wall normal velocity component v yields the first part of the control signal $\dot{\phi}(x, z, t)$ at the wall at each time instant, as indicated by (4).

where R is a positive definite hermitian matrix, corresponding to the weight of the control effort. The optimal control problem is now reduced to the minimization of the functional J, constrained by the state-space equation. Matrices Q and R are design parameters; it can be shown [3] that the optimal feedback gain matrix K can be found by

$$K = -R^{-1} B^H P$$

where P is the so-called stabilizing solution to the following algebraic Riccati equation:

$$PA + A^H P - PBR^{-1} B^H P + Q = 0$$

The solution to the optimal control problem described above is computed for each wavenumber pair (α, β) by solving a number of one-dimensional problems; controllers are then reconstructed for the full velocity and vorticity fields v and η. Fourier transforming back to physical space yields the so-called control convolution kernels; these kernels relate the control signal $\dot{\phi}(\hat{x}, \hat{z}, t)$ at a given time to the velocity and vorticity fields in the whole domain via the following convolution integrals:

$$\dot{\phi}(\hat{x}, \hat{z}, t) = \int K_{\hat{v}}(\hat{x}-\overline{x}, \overline{y}, \hat{z}-\overline{z}, t)\hat{v}(\overline{x}, \overline{y}, \overline{z})\ d\overline{x}d\overline{y}d\overline{z} + \int K_{\hat{\eta}}(\hat{x}-\overline{x}, \overline{y}, \hat{z}-\overline{z}, t)\hat{\eta}(\overline{x}, \overline{y}, \overline{z})\ d\overline{x}d\overline{y}d\overline{z} \quad (4)$$

A three-dimensional view of a control kernel $K_{\hat{v}}(\hat{x}, \hat{y}, \hat{z})$ is shown in fig.1, whereas fig. 2 shows a three-dimensional view of a control kernel $K_{\hat{\eta}}(\hat{x}, \hat{y}, \hat{z})$.

2.2 Direct Numerical Simulation of turbulent channel flow

The governing Navier-Stokes equations are solved numerically by using the computer code and computing system developed by Luchini & Quadrio and described in [4]. The code employs a mixed spatial discretization, based on Fourier expansion in the streamwise and spanwise directions and on fourth-order accurate compact finite differences in wall-normal direction. The

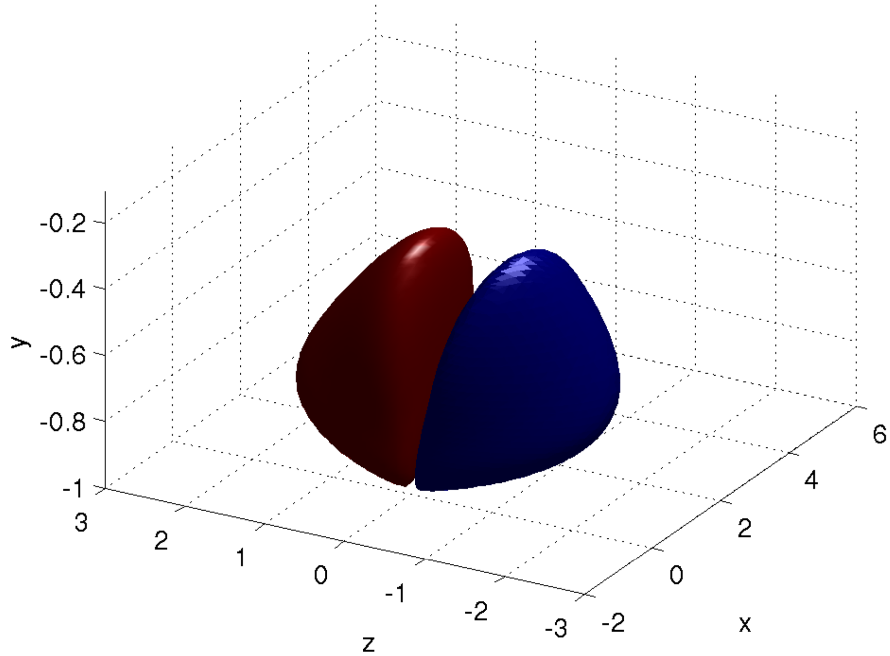

Figure 2: Three-dimensional view of the vorticity convolution kernel $K_{\hat{\eta}}(\hat{x}, \hat{y}, \hat{z})$ for the forcing at the lower wall. Isosurfaces refer to the $\pm 25\%$ of the peak value. The spatial convolution of this kernel in $\hat{x}$ and $\hat{z}$ directions with the wall normal vorticity component η yields the second part of the control signal $\dot{\phi}(x, z, t)$ at the wall at each time instant, as indicated by (4).

localized nature of compact finite difference stencil allows to exploit a simple partitioning of the data along the computing system, obtaining a very efficient parallel performance. The amount of communication is then reduced so that a dedicated computing machine can be built by using a particular connection topology but without the need of expensive networking hardware. These machines - called Personal Supercomputers - are available both at the Dipartimento di Ingegneria Aerospaziale del Politecnico di Milano, and at the Dipartimento di Meccanica dell'Università di Salerno; the former machine is made by 10 dual-CPU single core Intel Xeon machines, while the latter is assembled with 64 dual-CPU dual-core AMD Opteron processors. Time discretization is performed with a partially-implicit approach, where viscous terms are treated implicitly and nonlinear terms are advanced explicitly with an efficient, low storage, fourth order accurate Runge-Kutta scheme.

Computing the control kernels requires the efficient solution of the algebraic Riccati equations, one for each wavenumber pair. These equations are solved using the generalized Schur method [5] implemented in the Matlab routine `care`. This method has a complexity of order $\approx O(N^3)$, where N is the number of states; thus the overall complexity of the algorithm for the computation of the whole kernel is $\approx O(n_x \cdot n_z \cdot n_y^3)$, where n_x and n_z denote the number of modes in streamwise and spanwise directions, respectively, whereas n_y is the number of points in the wall-normal direction.

2.3 Discretization and computational parameters

Two direct numerical simulations of the controlled turbulent channel flow are performed at $Re = 1450$ and $Re = 3333$. Re is defined based on the channel half-width δ and the bulk velocity U_b. The dimensions of the channel in homogeneous directions are $L_x/\delta = 4\pi$ and $L_z/\delta = 2\pi$. In the $Re = 1450$ case, the domain is discretized using 64 grid points in y direction and 128×128 Fourier modes in $\hat{x}$ and $\hat{z}$ directions; in the $Re = 3333$ case, 128 grid points in the $\hat{y}$ direction and 256×256 Fourier modes are used.

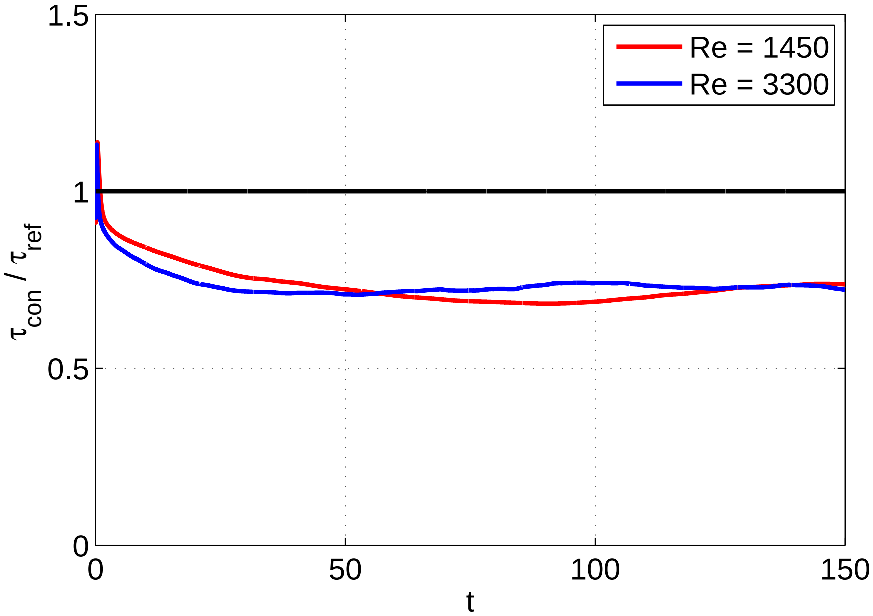

Figure 3: Reduction of friction drag achieved by the active controller at $Re = 1450$ and $Re = 3300$. τ_{con} is the space-time mean value of the friction in the controlled case, and τ_{ref} is the corresponding value of the uncontrolled case.

Control kernels presented here have been computed for each wavenumber pair using an energy weighting function $w(y) = 1 + U'(y)^2$, as suggested by Högberg et al. in Ref. [1]. Moreover, the turbulent mean flow profile $U(y)$ has been used as base flow, instead of the laminar profile, in order to obtain a state space representation which is closer to the real flow system. A control effort weighting matrix $R = \rho I$, where I denotes the identity matrix, has been used for each wavenumber pair, with $\rho = 0.01$.

The same temporal integration scheme used to advance the governing equations is used also to integrate the control derivative $\dot{\phi}(\hat{x}, \hat{z}, t)$ to obtain the control history. At each time step, the control signal $\dot{\phi}(\hat{x}, \hat{z}, t)$ is first computed directly in Fourier space, for each wavenumber pair. The wall blowing-suction distribution is then obtained by time integration, and the boundary conditions are eventually updated to advance the solution to the next time step.

Simulations are carried out for a time interval $T = 1400h/U_b$ for the $Re = 1450$ case and $T = 500h/U_b$ for the $Re = 3333$ case. The time step size is set at $0.030h/U_b$ and $0.012h/U_b$, respectively. The severe stability limitations (and the consequent required minuscule time step size) that would have occurred had a Chebyshev discretization of the wall-normal direction been employed are avoided by our finite-difference discretization.

3 Results

Controlling the turbulent channel flow with the optimal controllers designed to minimize turbulent fluctuations and (indirectly) friction drag leads to the results reported in figs. 3 and 4.

Looking first at the low-Re case, in fig. 3 it is shown that, after an initial transient in which drag increases, an equilibrium state is reached in which the controller is capable to reduce drag by approximately 30%; fig. 4 shows that turbulent kinetic energy, which is the main target of the control strategy, is reduced up to 45%, after the initial transient rise. These results agree with what can be inferred from Ref. [2] at the same Re without the use of the gain-scheduling technique.

The same qualitative behavior is obtained for the case at higher Re; results are summarized

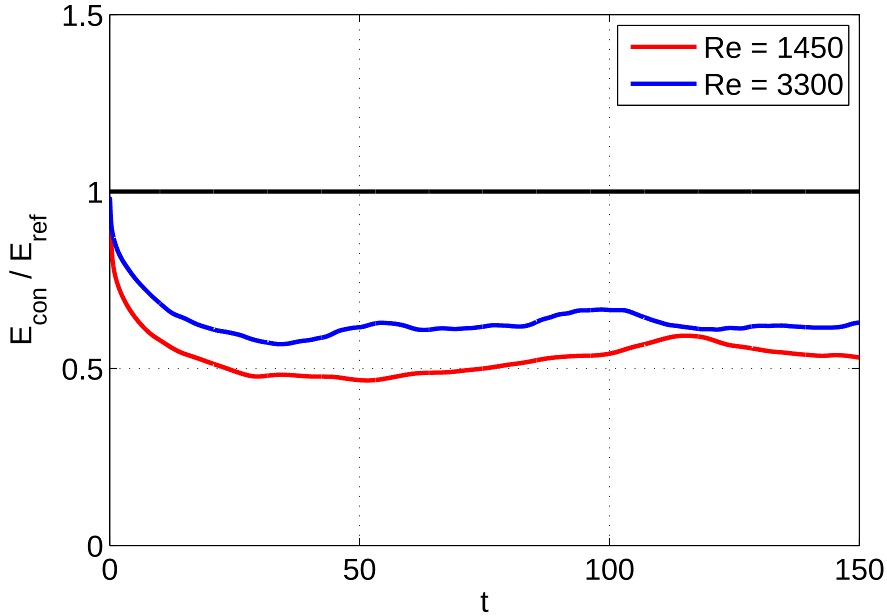

Figure 4: Reduction of turbulent kinetic energy achieved by the active controller at $Re = 1450$ and $Re = 3300$. E_{con} is the mean value of the fluctuations energy in the controlled case, and E_{ref} is the corresponding value of the uncontrolled case.

Re	% skin friction reduction	% turbulent kinetic energy reduction
$Re = 1450$	28.2 %	45.7 %
$Re = 3333$	27.2 %	36.6 %

Table 1: Summary of controller performance at different values of Re.

in table 1. Since no earlier attempt of essaying the control performance at this high value of Re is available in the literature, these results are of interest. They indicate that no significant drop in the amount of drag reduction takes place when Re is almost doubled, and this is very good news. It must be said, however, that a more substantial increase in Re would be needed to draw definite conclusions on this matter, since $Re = 3333$ is still low-Re. The key issue here is the capability of a wall-based control to interact with the overlying boundary layer in its full extent, and it is well-known that increasing Re increases the relative importance of the outer layer when compared to the wall-layer. From this viewpoint, observing a Re-dependency on the decrease of turbulent kinetic energy is reasonable, since energy is an integral quantity which is affected both by outer and inner layer dynamics.

4 Conclusions

In this work, the Reynolds-number dependence of the performance of optimal controllers aimed at reducing skin friction drag in turbulent channel flow is addressed.

Concerning the fundamental question of whether the performance of the controllers is deemed to decrease with Re, our preliminary results indicate that the performance of optimal controllers in reducing skin friction is insensitive to the value of Re, at least in the limited range of Re tested. However, the controller's ability of reducing turbulent kinetic energy, i.e. its primary target, is reduced by doubling Re.

Further studies at even higher values of Re are required, and are currently being performed by our research group.

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