Phase-locked linear response of a turbulent channel flow

P.Luchini[1], M.Quadrio[2], S.Zuccher[2]

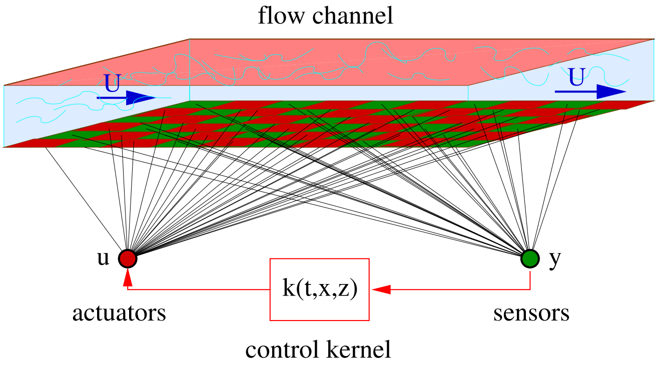
[1] Università di Salerno, Dip. Ing. Meccanica
[2] Politecnico di Milano, Dip. Ing. Aerospaziale

EDRM 2006, Ischia, 10-13 April

Outline

Active-feedback turbulent drag reduction

Our long-term objective



The control kernel

$$u(t,x,z) = \int y\left(t',x',z'\right) k\left(t-t',x-x',z-z'\right) \mathrm{d}t'\mathrm{d}x'\mathrm{d}z'$$

ACTIVE-FEEDBACK DRAG REDUCTION: HISTORY (1)

- Instantaneous feedback ('opposition') control of a turbulent flow: Choi, Moin & Kim (JFM 1994)
- Spatially localized convolution kernel through modern (Kalman-filter based) optimal control applied to the linearized Navier–Stokes equations: Högberg & Bewley (Automatica, 2001)

ACTIVE-FEEDBACK DRAG REDUCTION: HISTORY (2)

- Realization that more physical information could be embodied in the controller if the linearized NS problem is replaced by a mean linear response of the full turbulent flow to external disturbances (Luchini, unpub. 2001)
- First measurement via DNS of such response function (Quadrio & Luchini, IX Eur.Turbulence Conf. 2002)
- Deadlock in 2002: How to design the controller?

Solution: Wiener-based LQ control! (Luchini, Bewley & Quadrio, APS meeting 2005)

The linear response

Average response of the turbulent flow to a Dirac $\delta(t, x, z)$:

$$u_j(t,x,y,z) = \int H_{ij}(t-t', x-x', y, z-z')\delta_i(t',x',z')\mathrm{d}t'\mathrm{d}x'\mathrm{d}z', \quad i = 1,2,3$$

H answers the question: How to act at the wall here and now to achieve the desired effect there and after a given time?

Problem: turbulence!

- A turbulent flow has large noise, while forcing amplitude must be small
- Solution: phase-locked averages to extract deterministic part of the signal from background noise (Employed by Hussain & Reynolds, JFM 1972)

Two possible strategies

Response to a δ function

– Small amplitude for linearity, since forcing power is concentrated

+ All frequencies obtained at once

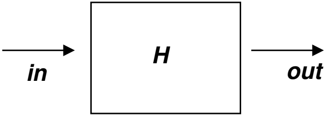
Response to sinusoidal forcing

+ Large amplitude for linearity, since forcing power is distributed

– One single frequency obtained at a time

Both approaches require unaffordable computational loads

THE RIGHT WAY TO GO
MEASURING THE INPUT-OUTPUT CORRELATION



$$R_{in,out}(t) = \int H(t-t')R_{in,in}(t')\mathrm{d}t'$$

If input is white noise then $R_{in,in} = \delta(t')$ and $R_{in,out} = H$

- Turbulent fluctuations will be averaged out just as in phase-locking
- Forcing power is uniformly distributed, all frequencies are obtained at once

Computational parameters

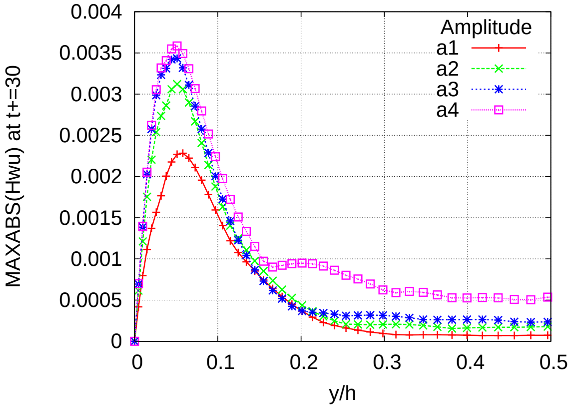
DNS with b.c. for u_i at one wall made by a (space-time) uniform distribution with given amplitude.

- Turbulent channel flow at $Re_\tau = 180$
- Standard domain size: $L_x = 4\pi h$, $L_y = 2h$ and $L_z = 4.2h$
- Standard spatial resolution: $N_x = 192$, $N_y = 128$ and $N_z = 128$
- Averaging time $\sim 10^5$ viscous time units

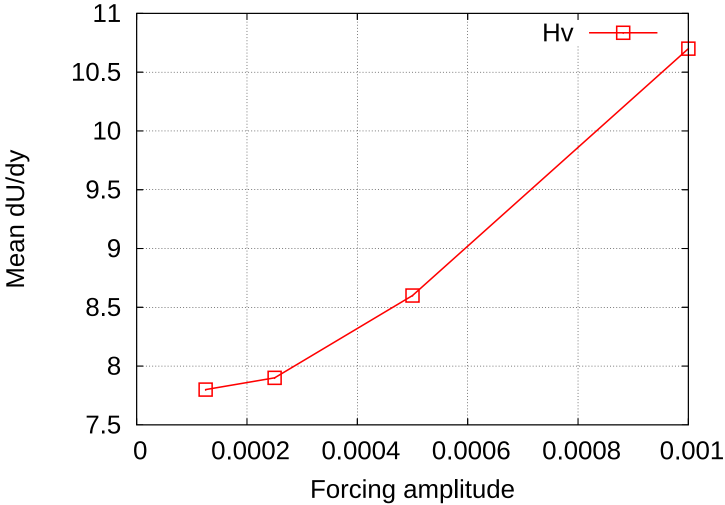
Computational tools

- DNS pseudo-spectral code
- Parallel strategy to exploit commodity hardware (Luchini & Quadrio JCP 2006)
- The 'Personal Supercomputer': Powerful dedicated system with 128 Opteron CPUs, 100GB RAM, 4TB disk

LINEARITY CHECK (1)

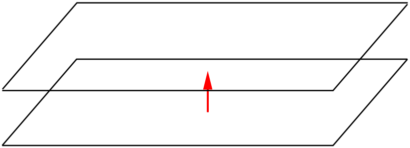


Linearity check (2)



The H_v response at $t = 0$

An analytical solution

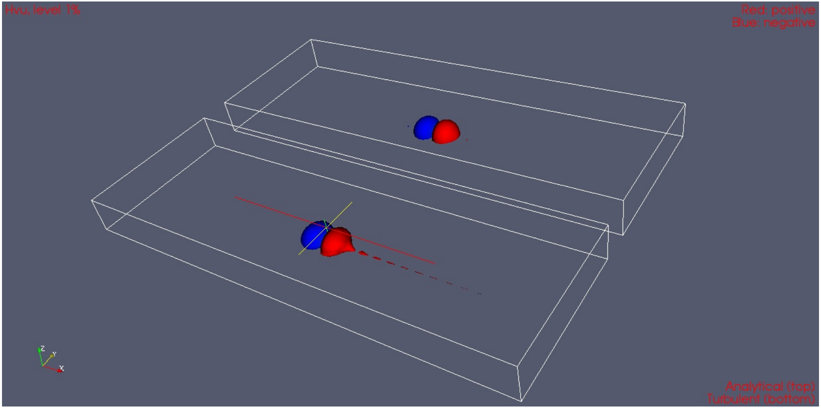


- Laplace eq. in 3d for potential $\varphi(x, y, z)$
- Decouples for each wavenumber pair after Fourier transf.
- Simple b.c. in Fourier space: $\widehat{\varphi}_y(0) = 1, \quad \widehat{\varphi}_y(2) = 0$
- Solution:

$$\boxed{\widehat{\varphi}(y) = -\frac{\mathrm{Cosh}[\kappa(2-y)]}{\kappa \mathrm{Sinh}(2\kappa)}} \qquad \kappa = \sqrt{\alpha^2 + \beta^2}$$

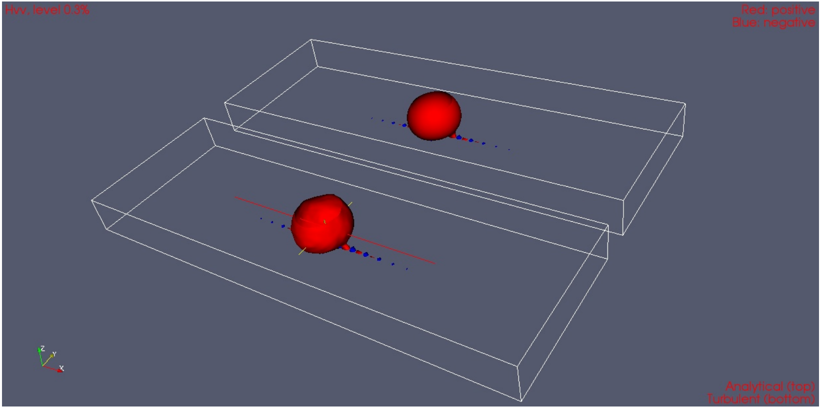
THE POTENTIAL RESPONSE

H_{vu}, ANALYTICAL (TOP) VS COMPUTED TURBULENT (BOTTOM)



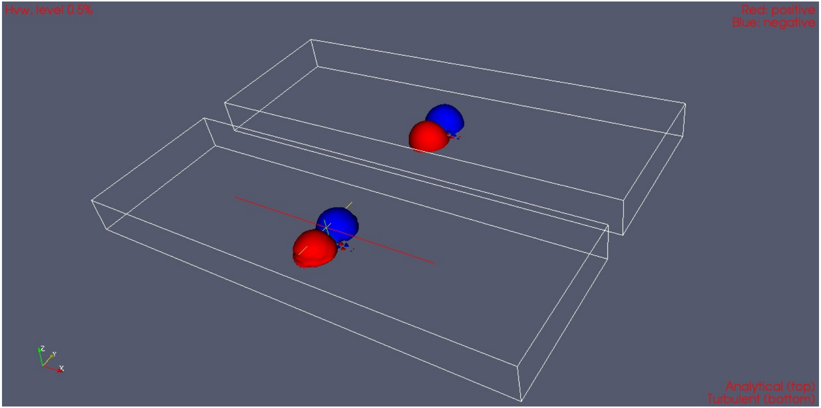
THE POTENTIAL RESPONSE

H_{vv}, ANALYTICAL (TOP) VS COMPUTED TURBULENT (BOTTOM)



THE POTENTIAL RESPONSE

H_{vw}, ANALYTICAL (TOP) VS COMPUTED TURBULENT (BOTTOM)



3D, TIME-VARYING PLOTS

- Comparison between computed turbulent and computed 'laminar' response
- Visualization of the full tensor $H_{ij}(t, x, y, z)$

CONCLUSIONS

- Complete turbulent response tensor shown here for the first time
- Analytical potential component at $t = 0$ compares very well with measurements
- Turbulent H differs significantly from laminar H!!
- Possible significant implication for feedback control

CONTROLLER KERNELS AVAILABLE

We have the spatio-temporal kernels for drag reduction, if someone has the (MEMS) technology to test them

FUTURE WORK

- Full understanding of spatio-temporal structure of H
- Further (more refined?) measurements
- Laboratory experiments welcome for higher-Re flows