An optimal feedback controller for the reduction of turbulent energy in 3D plane-duct flow

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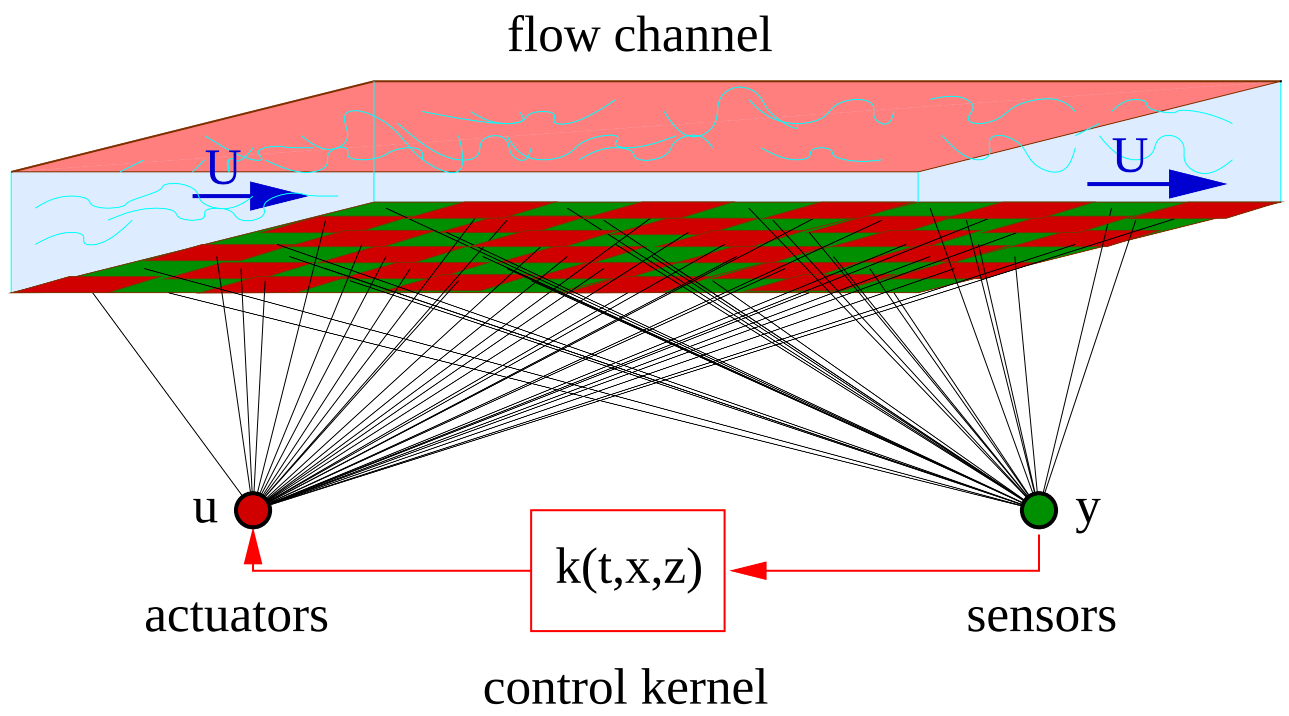
Active-feedback drag reduction

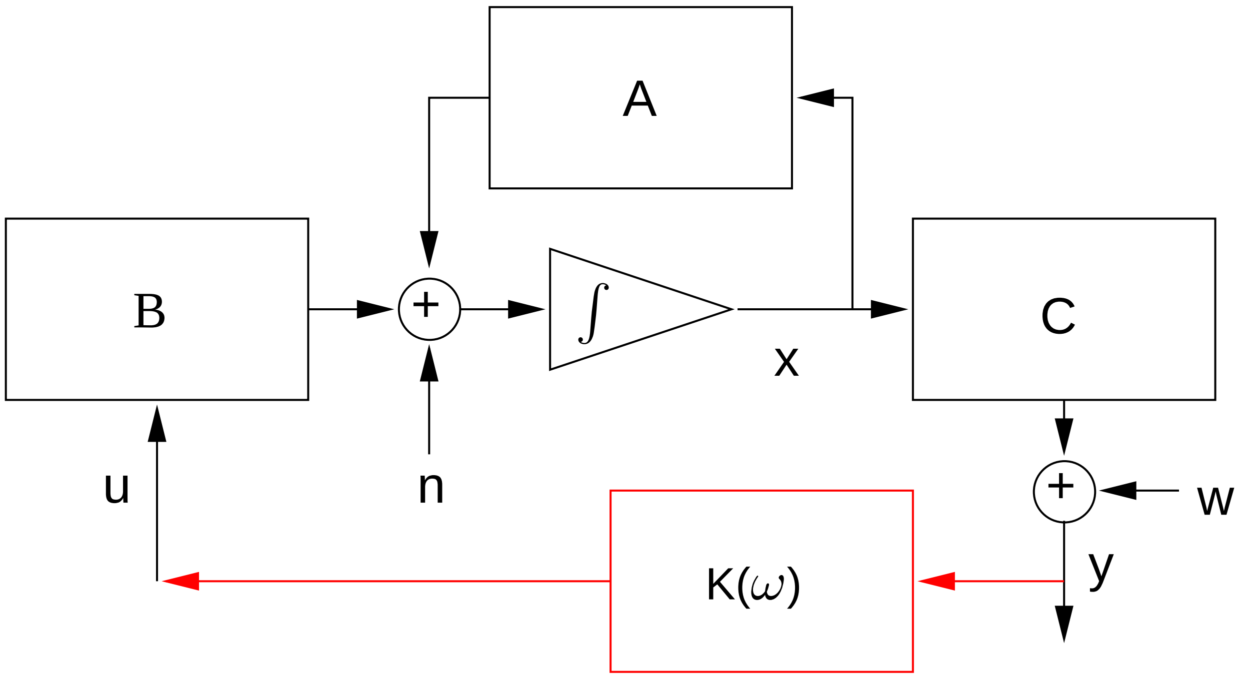


$$u(t,x,z) = \int y(t',x',z')k(t-t',x-x',z-z')\mathrm{d}t'\mathrm{d}x'\mathrm{d}z'$$

A brief history of active-feedback drag reduction

- Instantaneous feedback control ("opposition" control) of a turbulent flow: Choi, Moin & Kim (JFM, 1994)
- Spatially localized convolution kernel through modern (Kalman-filter based) optimal control applied to the linearized Navier-Stokes equations: Högberg & Bewley (Automatica, 2001)
- Realization that a larger amount of physical information could be embodied in the controller if the linearized NS problem was replaced by a mean linear response of the full turbulent flow to external disturbances: Luchini (at a dinner in Stockholm, 2001)
- First computation ("measurement") of such mean linear response from a DNS: Quadrio & Luchini (9th Eur. Turbulence Conference, 2002)
- Deadlock (in 2002): how to design the controller?

State-based representation

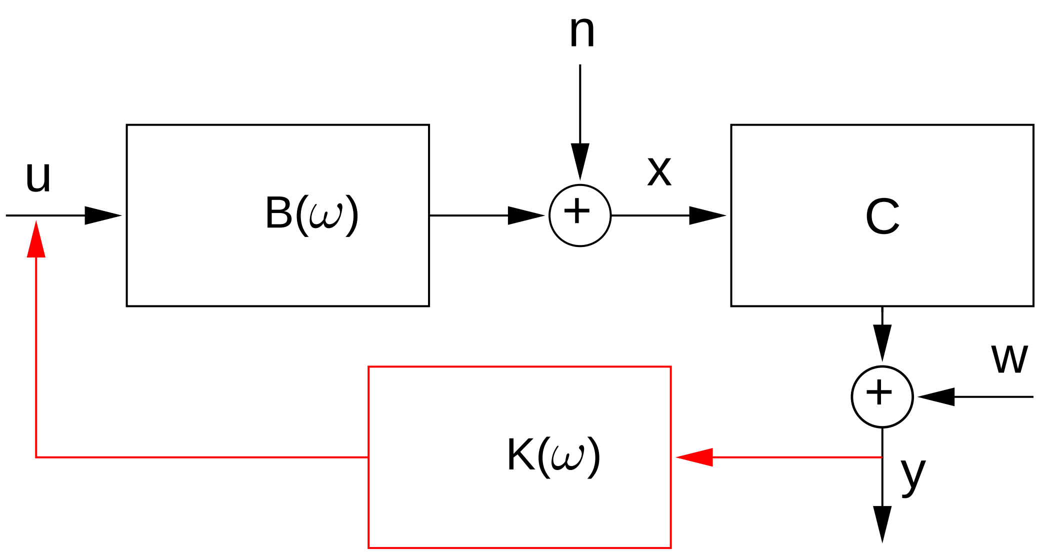


Matrix **A** contains the linearized Navier-Stokes eqs. **n** is a white noise.

y can be any or all wall stress components, **u** any or all wall velocity components.

Quadratic objective function: $x^*Qx + u^*Ru$

Input-output representation



$B(\omega)$ is the measured mean linear response of the turbulent flow.

y can be any or all wall stress component, **u** any or all wall velocity component.

n is the measured turbulent-flow fluctuation.

Quadratic objective function: $x^*Qx + u^*Ru$

Step 1 (2002): The mean linear response

- in frequency domain: $B(\omega)$ is the response to sinusoidal forcing of varying angular frequency ω
- in time domain: $b(t)$ is the response to a Dirac δ function applied at $t = 0$

Linearity requires the perturbations to be smaller than turbulent fluctuations.

$\Rightarrow$ Conceptual solution: phase-locked averaging (either with impulsive or sinusoidal forcing) to extract deterministic part of the signal out of turbulent noise.

- Main difficulty: to obtain a sufficient signal-to-noise (S/N) ratio.

Hussain & Reynolds, (*JFM*, 1970-1972) measured the evolution of small (t-sinusoidal, z-uniform, x-impulsive) disturbances applied at the wall of a turbulent channel flow, at 4 discrete frequencies, by averaging over, typically, 100.000 cycles.

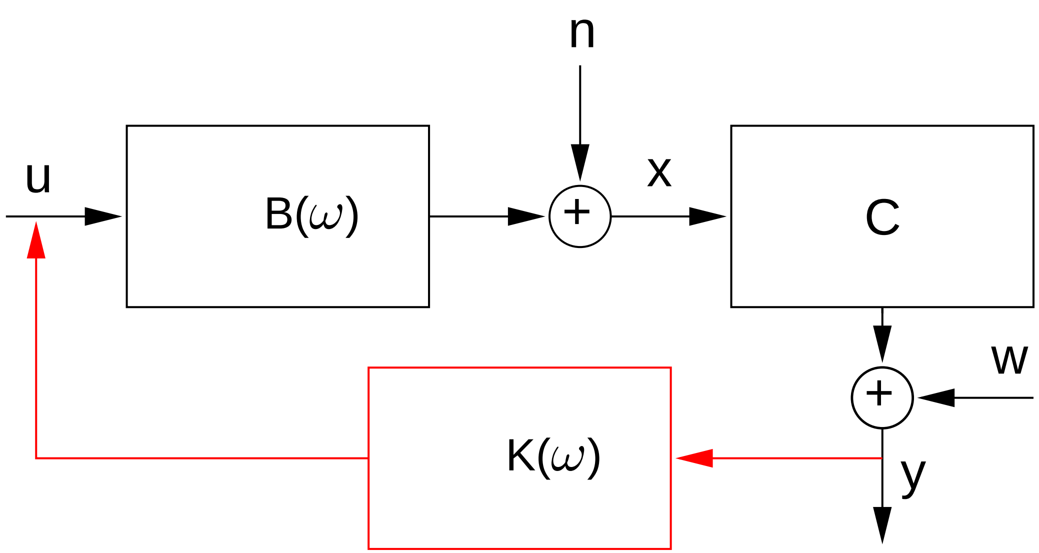
Our solution: forcing with a random signal

$>$From signal theory: when a **white noise** is passed through a linear filter, the **correlation between input and output** is proportional to the impulse response of the system.

$$R_{io}(t,x,z) = \int R_{ii}(t',x',z')b(t-t',x-x',z-z')\mathrm{d}t'\mathrm{d}x'\mathrm{d}z'$$

- If $R_{ii} = \delta(t,x,z)$ then $R_{io} = b$.
- Turbulent fluctuations will be averaged out just as in phase-locking.
- Forcing power is uniformly distributed (in a statistical sense) over time and space; amplitude required for linearity can be as large as for sinusoidal forcing but the whole response is obtained at once.

Step 2 (2005): Design of the optimal controller



Problem: to find $K(\omega)$ such that $x^*Qx + u^*Ru$ is a minimum when $x = Bu + n$; $y = Cx + w$; $u = Ky$, w is a white measurement noise and n has the statistics of the actual, measured, turbulent-flow fluctuations.

After Fourier transformation:

space structure is trivial: wavenumbers decouple;

time dynamics is the true difficulty: **causality** has to be enforced.

The standard solution: state-model based LQG control

- A separation principle applies: the problem can be split into dual *control* and *estimation* Kalman-filter problems.
- Each of the control and estimation problems is solved by a codified technique based on the solution of an algebraic Riccati equation.
- The outcome is an instantaneous input matrix for a controller based on the same state equations as the original system.

However:

- The standard solution does not apply to an input-output formulation.
- The standard solution constrains the fluctuations to have the same statistics as the control.
- The outcome we actually need is a controller with independent dynamics.

The estimation problem: Wiener versus Kalman optimal filtering

- **Kalman** (1960): find an instantaneous K such that $(x-\tilde{x})^*Q(x-\tilde{x})$ is a minimum with $dx/dt = Ax + KC\tilde{x}$; $d\tilde{x}/dt = A\tilde{x} + n$ and n a white gaussian noise.

  Solution: matrix Riccati equation (standard controls-people machinery).

- **Wiener** (1939): find a causal $K(\omega)$ such that $(x-\tilde{x})^*Q(x-\tilde{x})$ is a minimum with $x = KC(\tilde{x}+n)$, where $\tilde{x}$ is a signal and n a noise, both of known but arbitrary (and unrelated) spectrum.

  Solution:

  – Wiener-Hopf method: extremely fast O($N \log N$), S.I.S.O.

  – Levinson algorithm: intermediate speed O(N^2), M.I.M.O.

  – standard LU decomposition: slow O(N^3), straightforward.

The unlocking step: Wiener-based LQ(G) control

1. Separation principle $\longrightarrow$ conversion of the feedback into a feedforward problem:

$$K = (1 + K'CB)^{-1}K'$$

2. K' obeys a **linear-quadratic** optimization problem; setting its gradient to zero yields a **linear system**. K is **causal** if and only if K' is.

3. Enforcing causality of K' is a **Wiener-Hopf** problem: the relevant SISO or MIMO methodology can be applied.

Critical issue: will it work with noisy response function and/or correlation data?

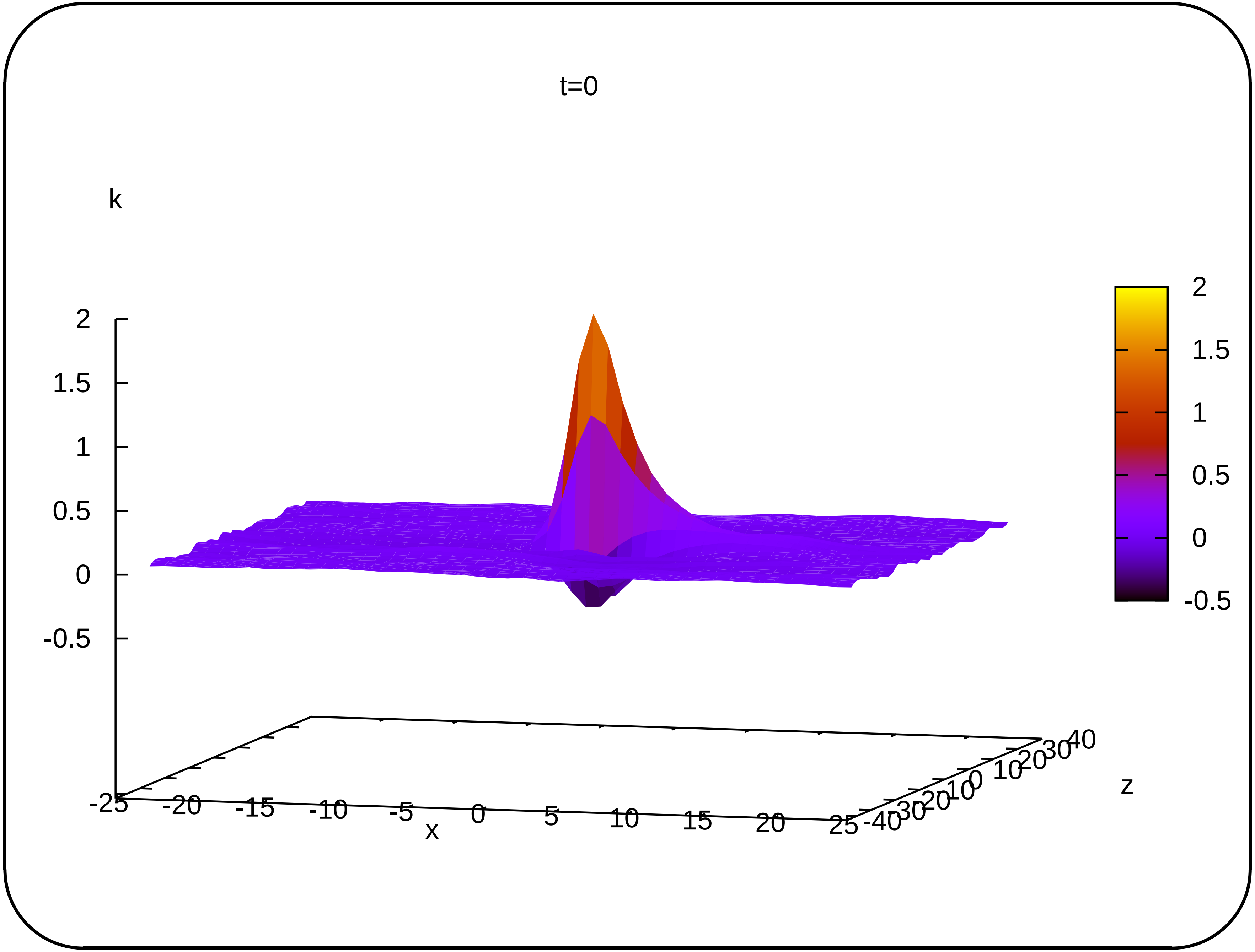
Empirical (and very recent) answer: YES

A control-kernel example

- Sensor y: **all three stress components** (can be any or all wall stress components). The longitudinal skin-friction component is displayed.
- Actuator u: **wall-normal velocity** (can be any or all wall velocity components).
- Objective function: **dissipation** (can be kinetic energy, weighted kinetic energy, dissipation).

Dissipation is a quadratic function that even in the full nonlinear problem exactly equals external work (difference of skin-friction gain and energy spent to operate the actuators).

- Control cost $R = 0.1$; measurement noise $W = 0.1$.
  R and W play a double role as smoothing factors.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1
0.5
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-25 -20 -15 -10 -5 0 5 10 15 20 25
x
-40 -30 -20 -10 0 10 20 30 40
z
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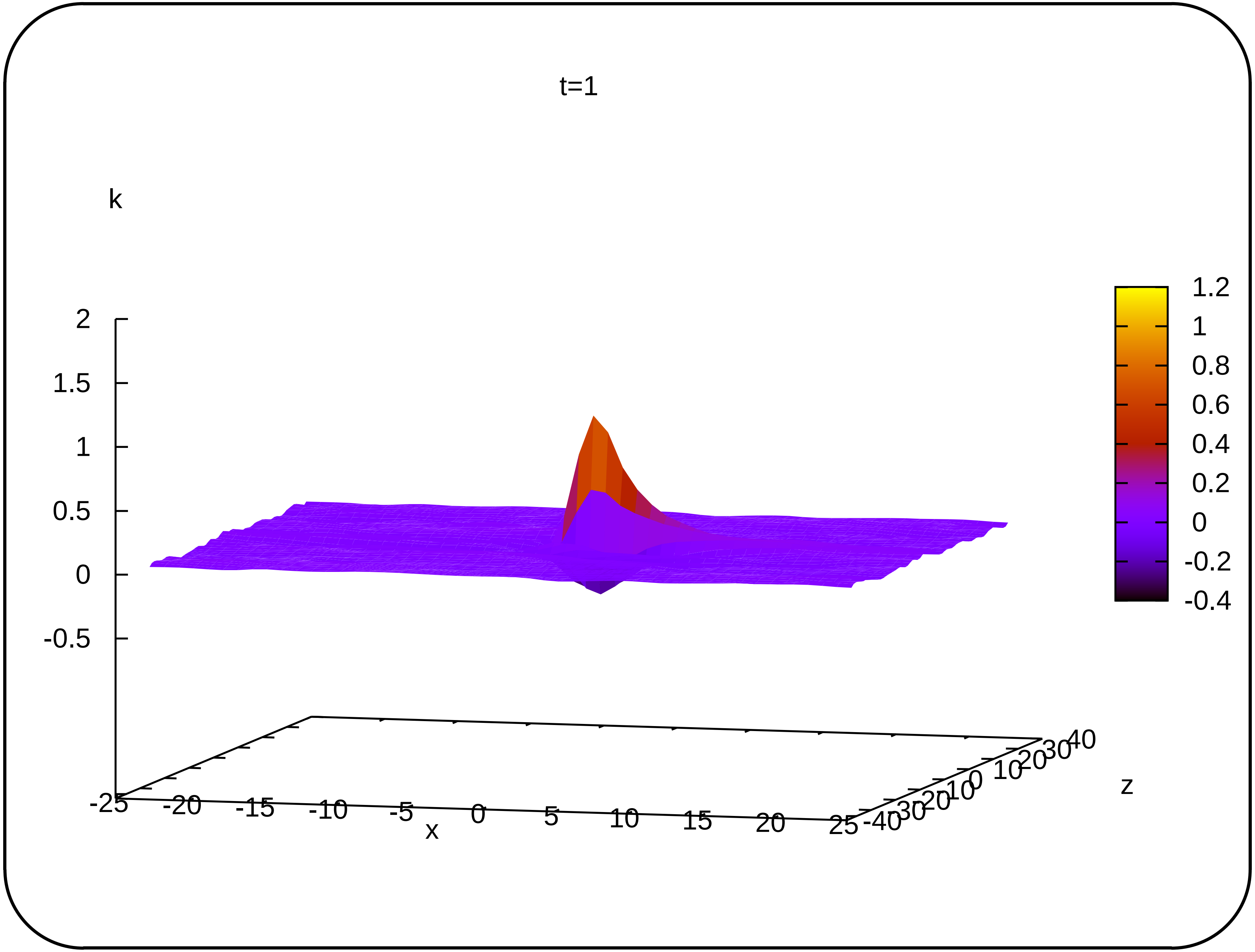
0.5
0
-0.5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x
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0.4
0.2
0
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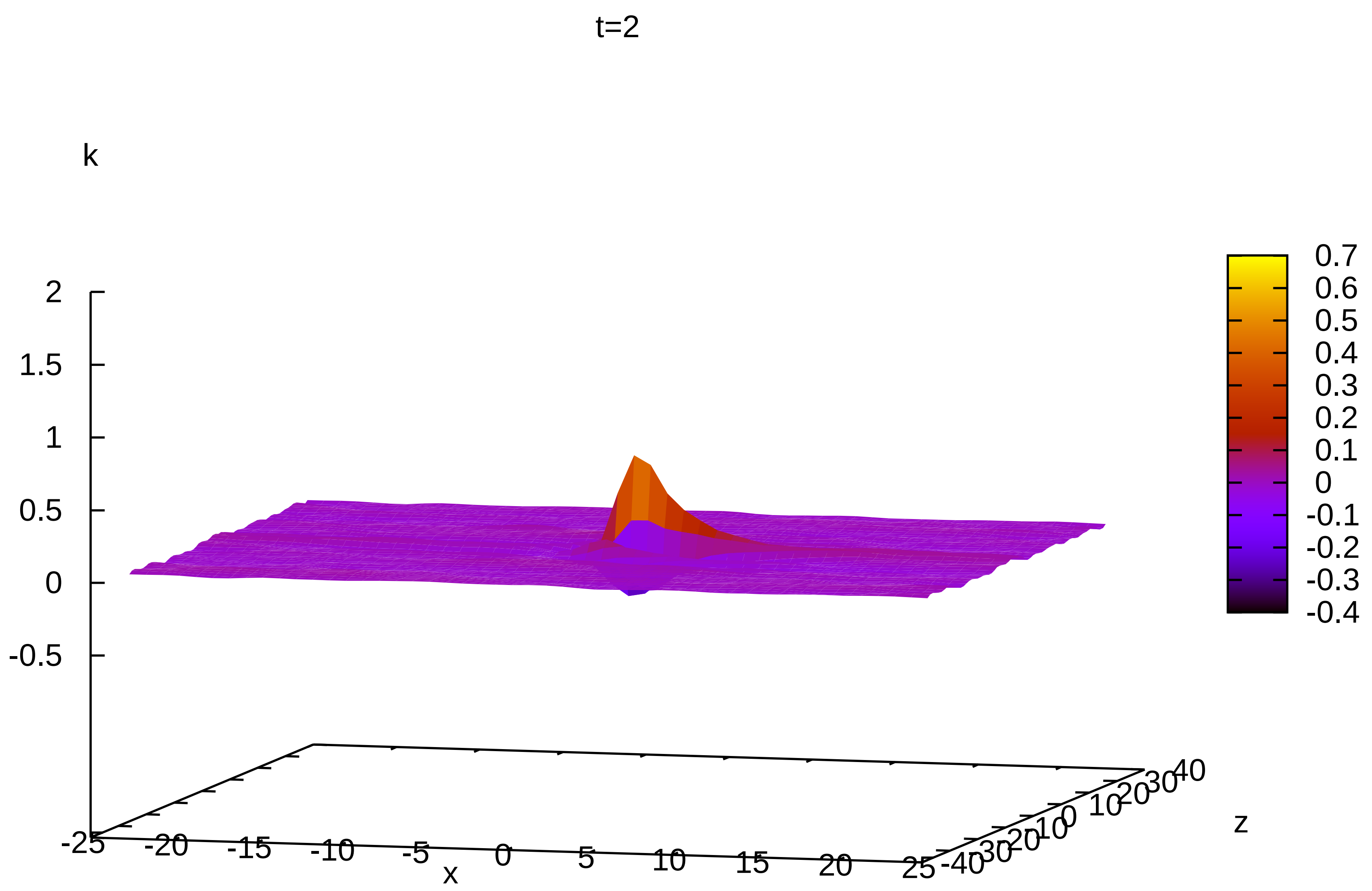
-0.4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z
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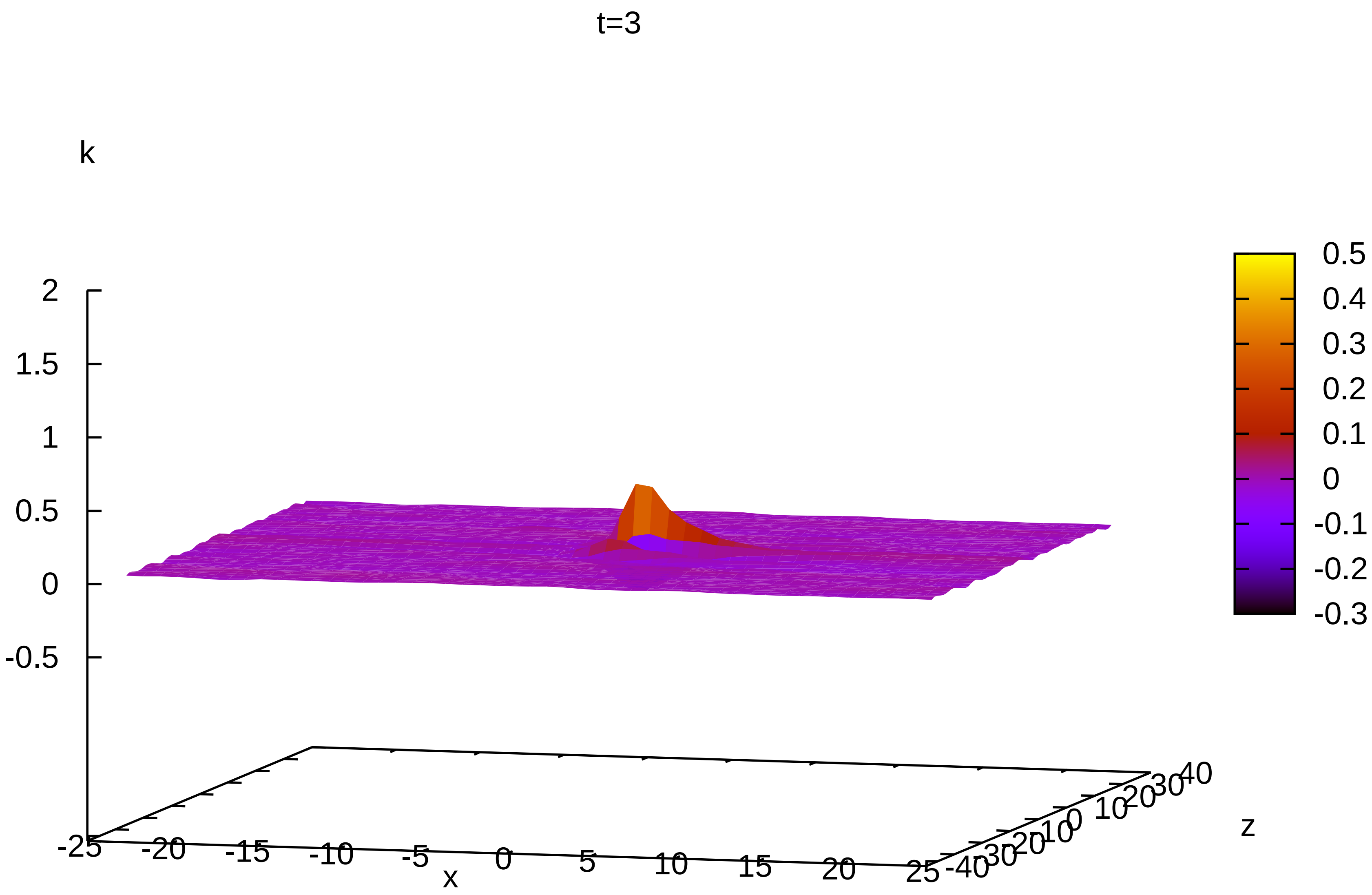
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4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z
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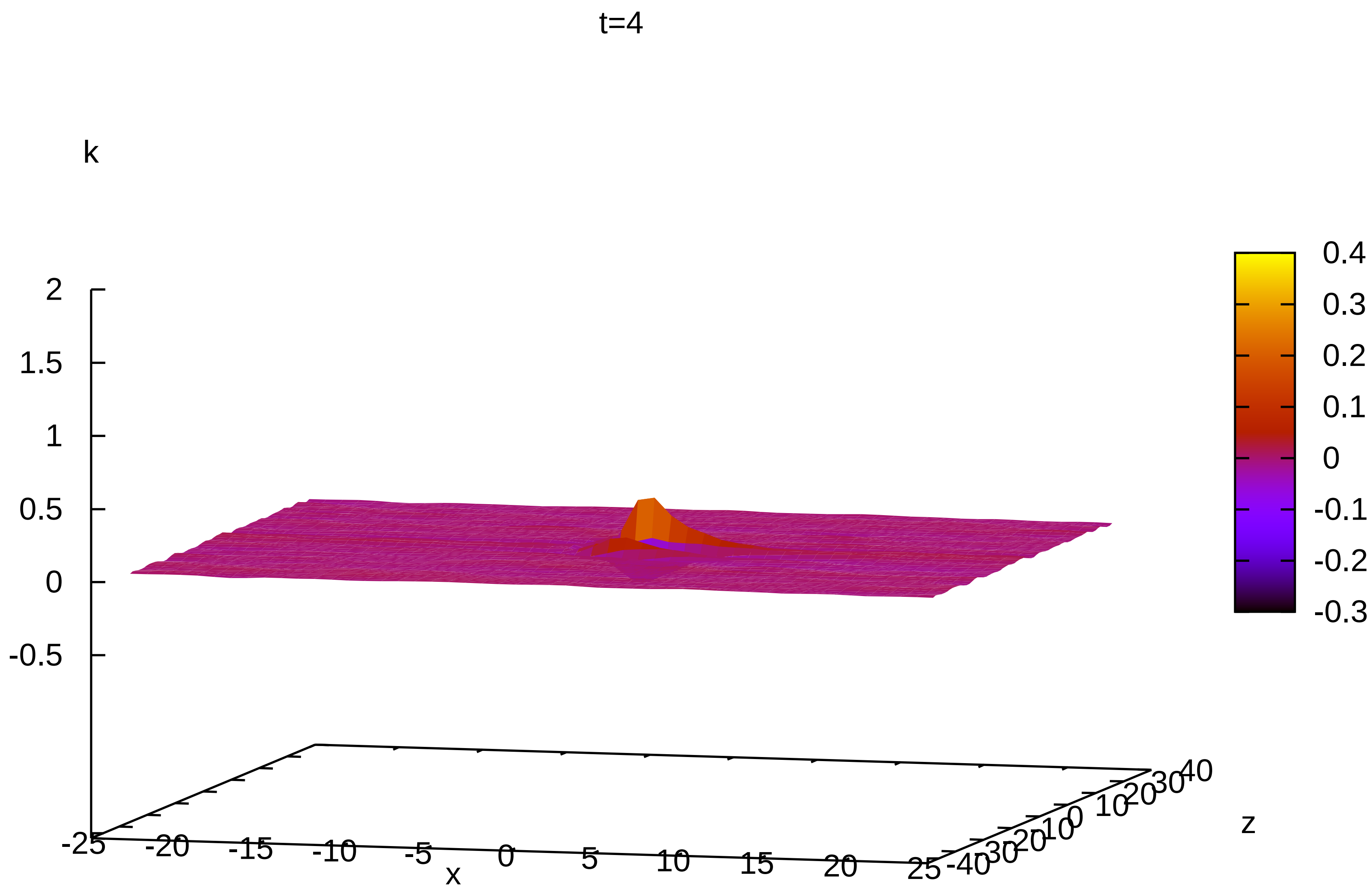
0.3
0.2
0.1
0
-0.1
-0.2
-0.3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x
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z
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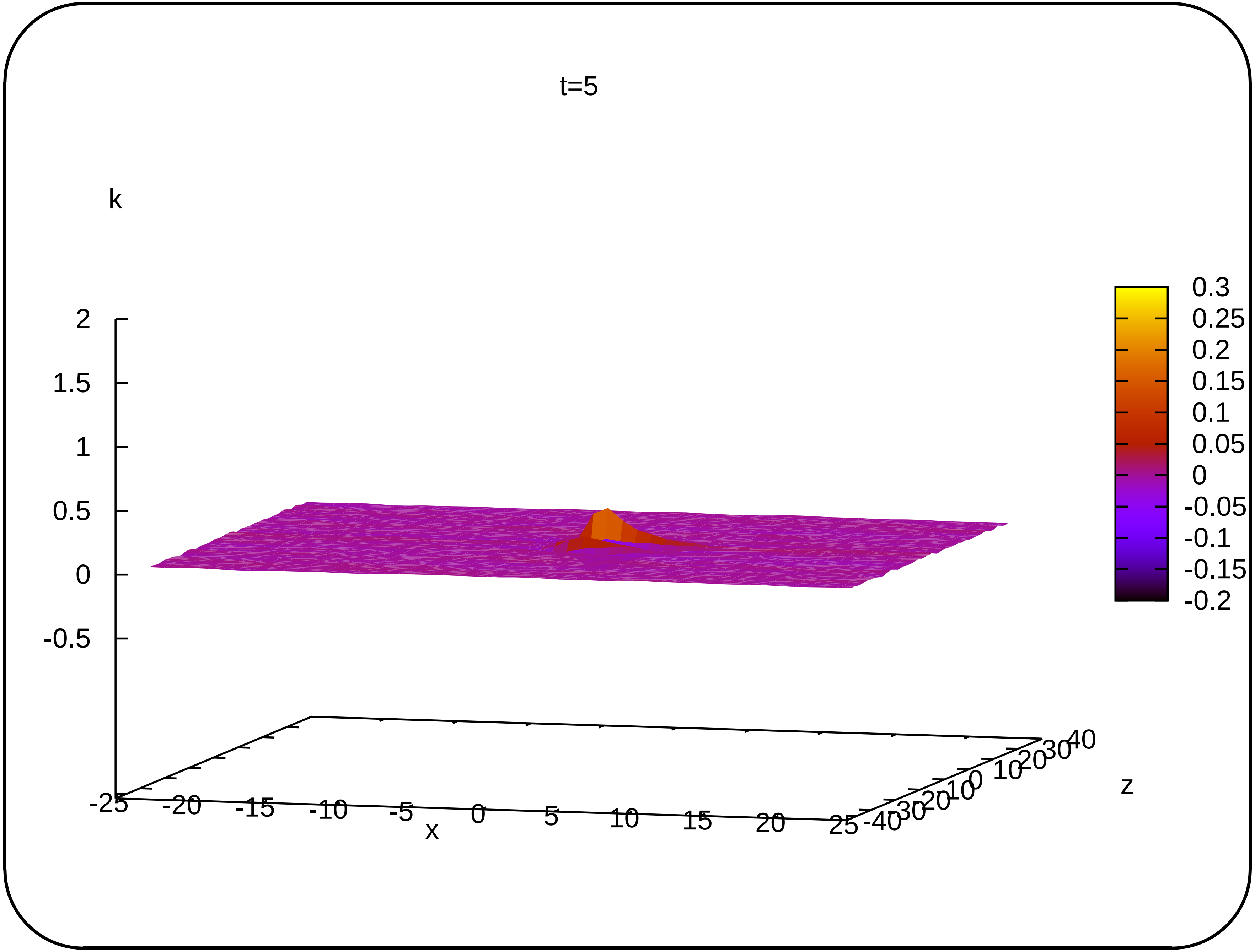
0.3
0.2
0.1
0
-0.1
-0.2
-0.3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-25
-20
-15
-10
-5
0
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15
20
25
x
-40
-30
-20
-10
0
10
20
30
40
z
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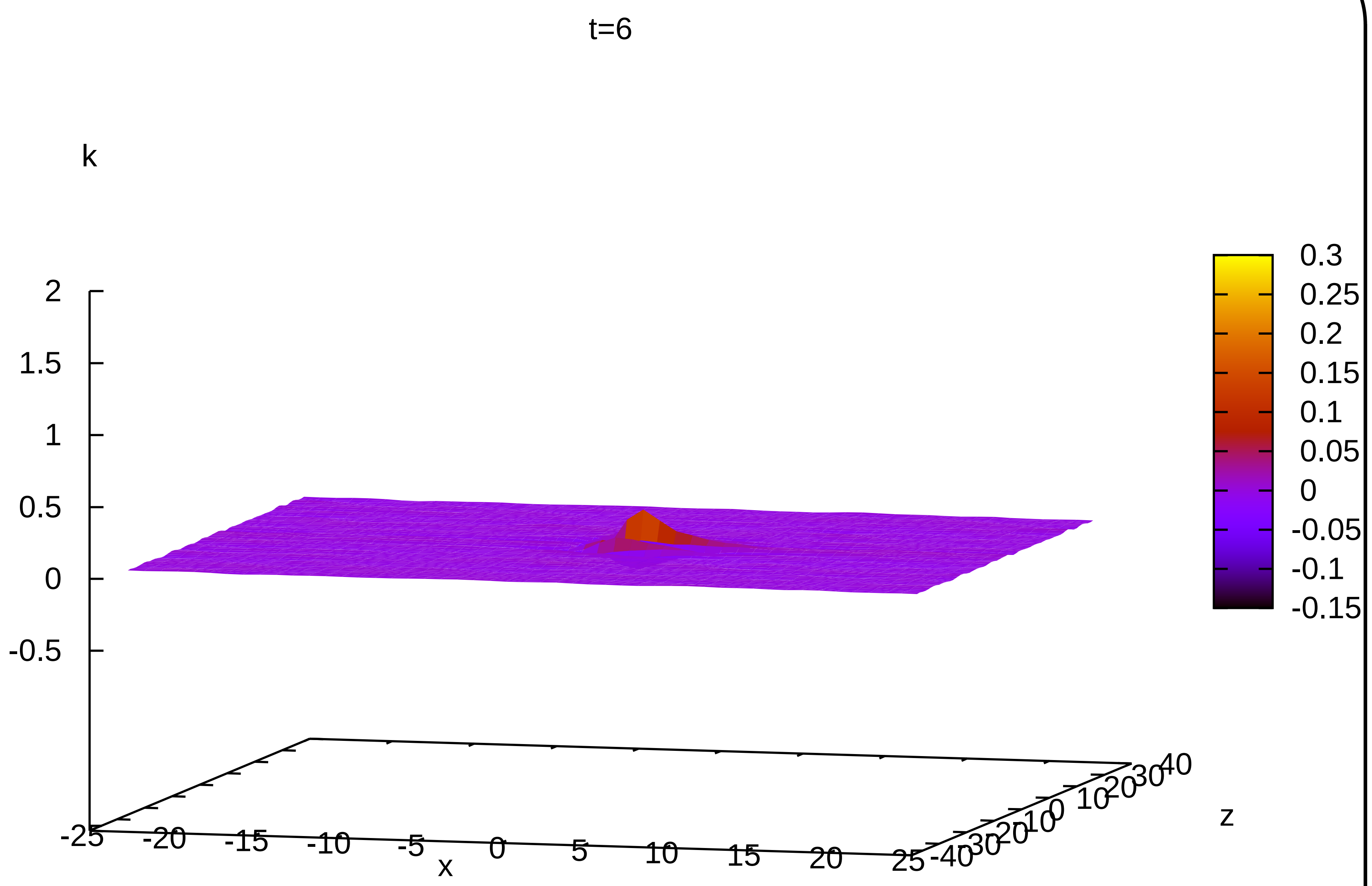
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2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-20
-15
-10
-5
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15
20
25
x
-40
-30
-20
-10
0
10
20
30
40
z
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0.2
0.15
0.1
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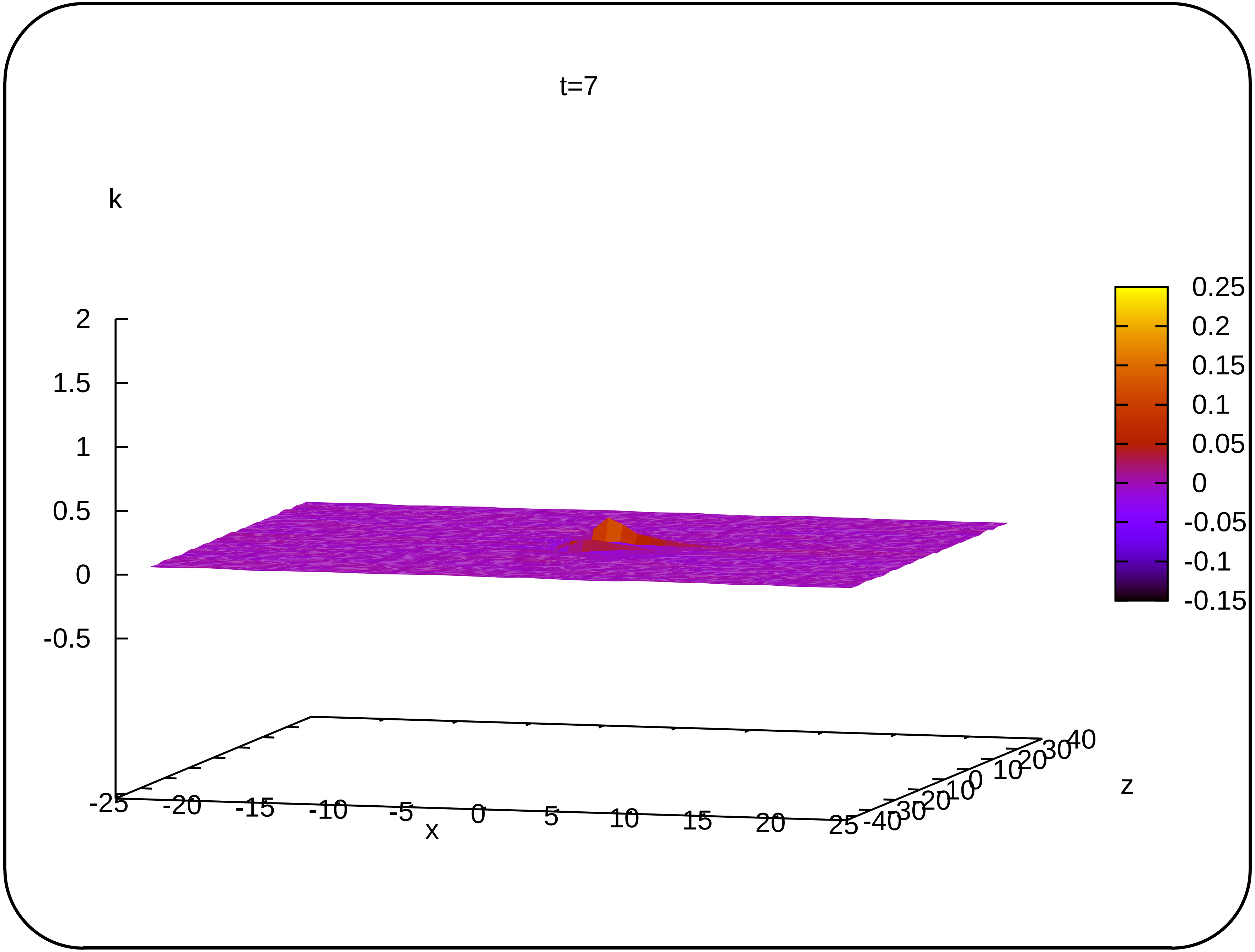
-0.1
-0.15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x
-40 -30 -20 -10 0 10 20 30 40
z
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0.15
0.1
0.05
0
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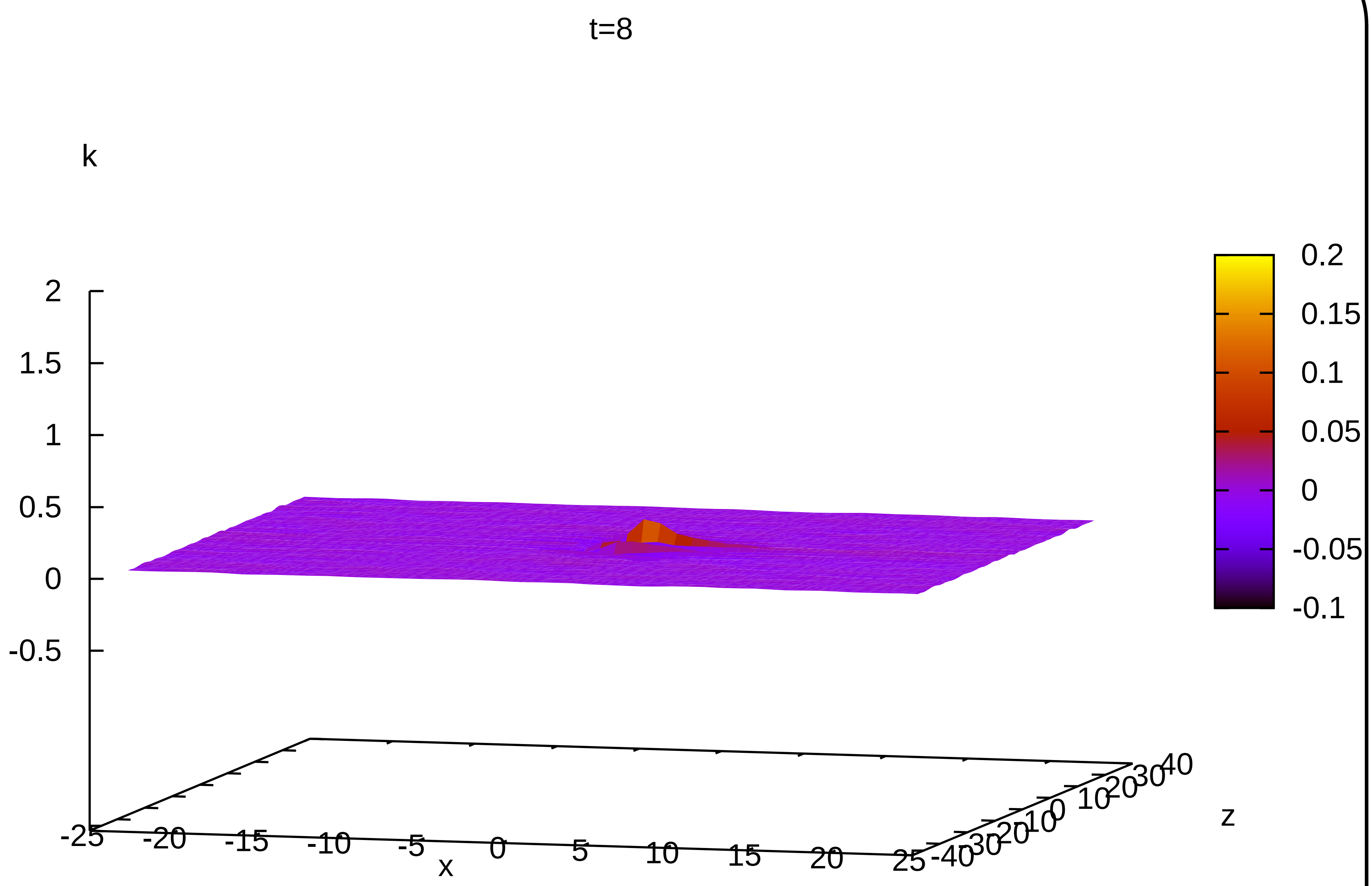
-0.1
-0.15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-10
-5
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5
10
15
20
25
x
-40
-30
-20
-10
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10
20
30
40
z
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0.15
0.1
0.05
0
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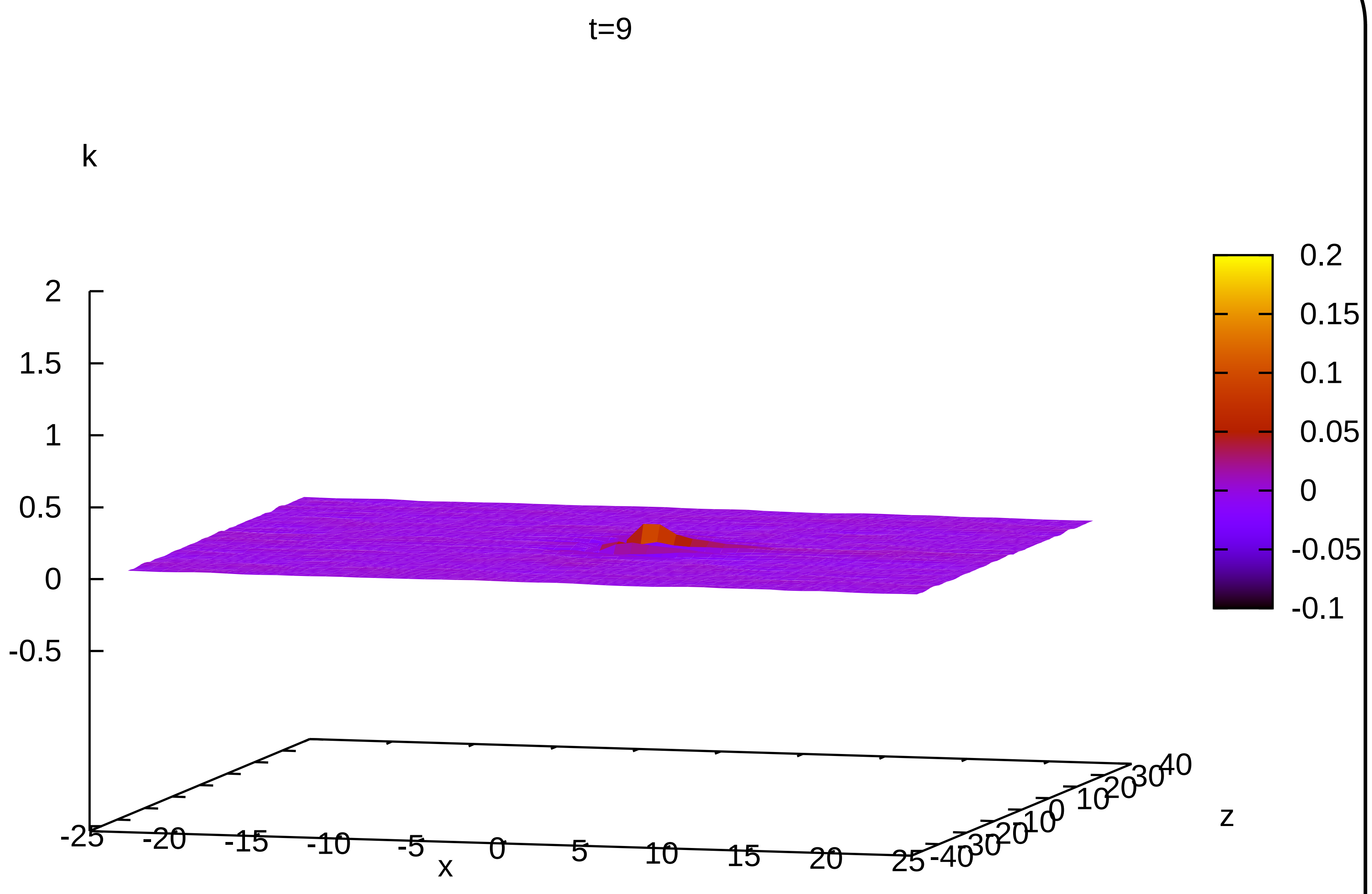
-0.1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0.5
0
-0.5
-25 -20 -15 -10 -5 0 5 10 15 20 25
x
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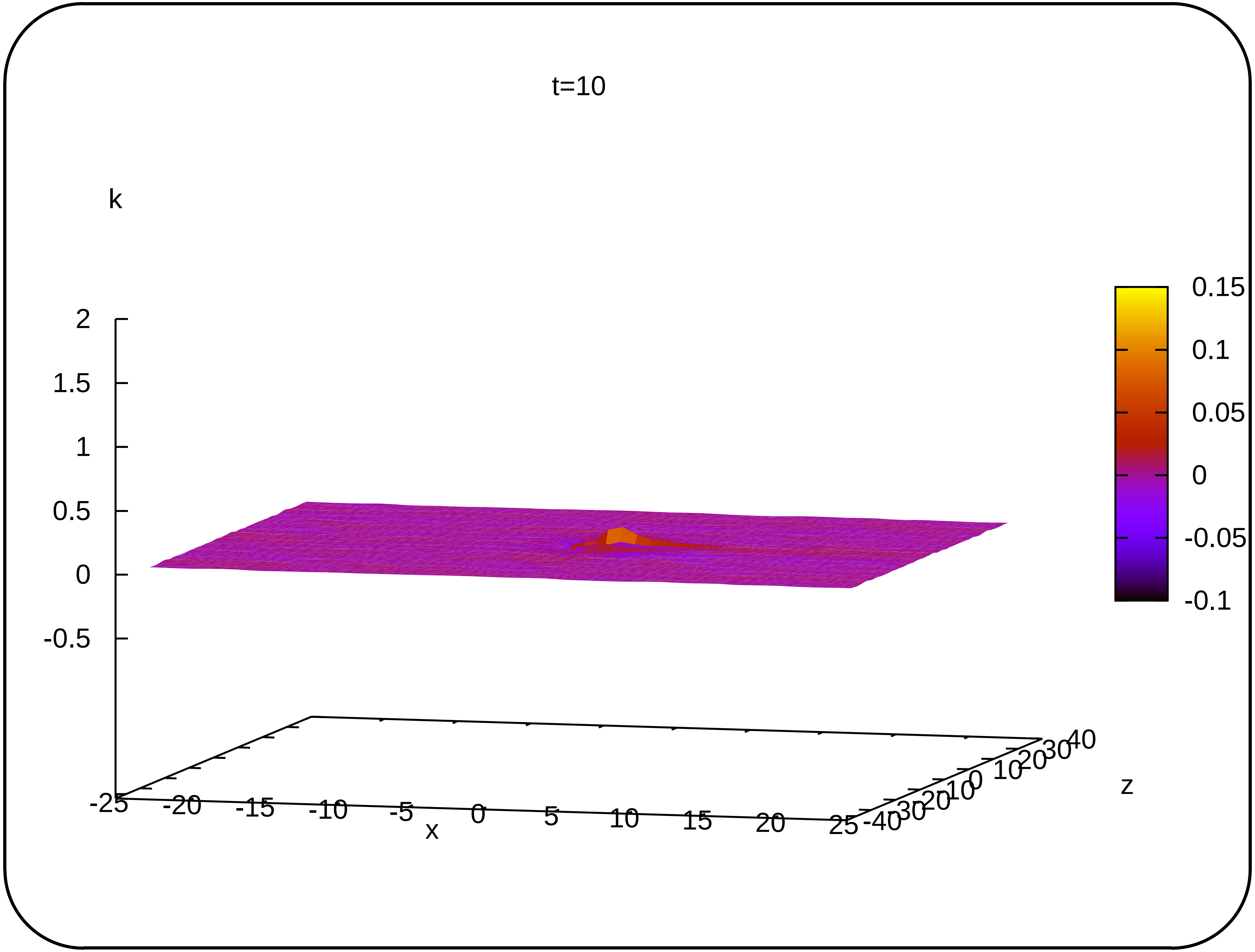
z
0.2
0.15
0.1
0.05
0
-0.05
-0.1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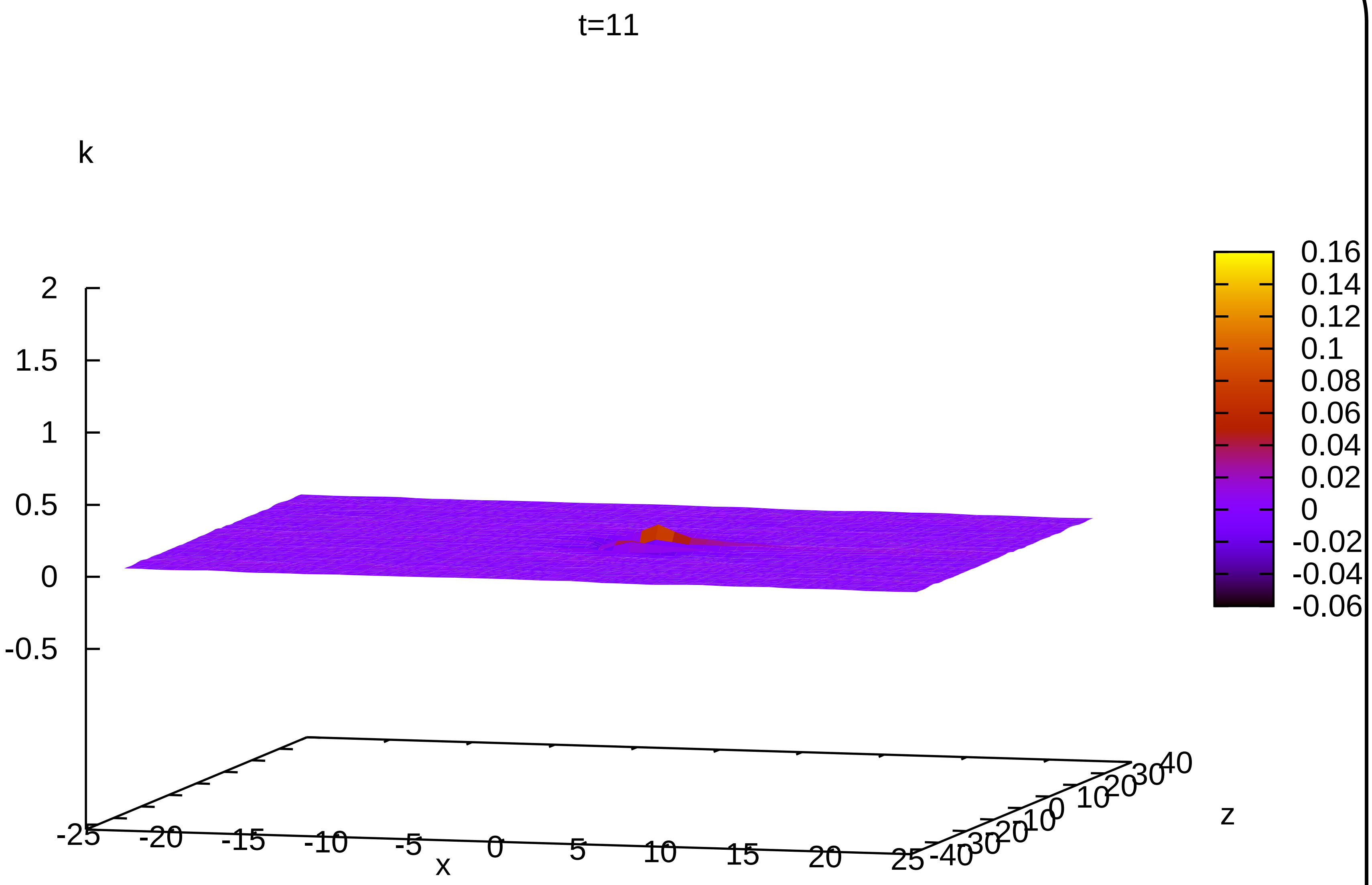
1
0.5
0
-0.5
-25 -20 -15 -10 -5 0 5 10 15 20 25
x
-40 -30 -20 -10 0 10 20 30 40
z
0.15
0.1
0.05
0
-0.05
-0.1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1
0.5
0
-0.5
-25 -20 -15 -10 -5 0 5 10 15 20 25
x
-40 -30 -20 -10 0 10 20 30 40
z
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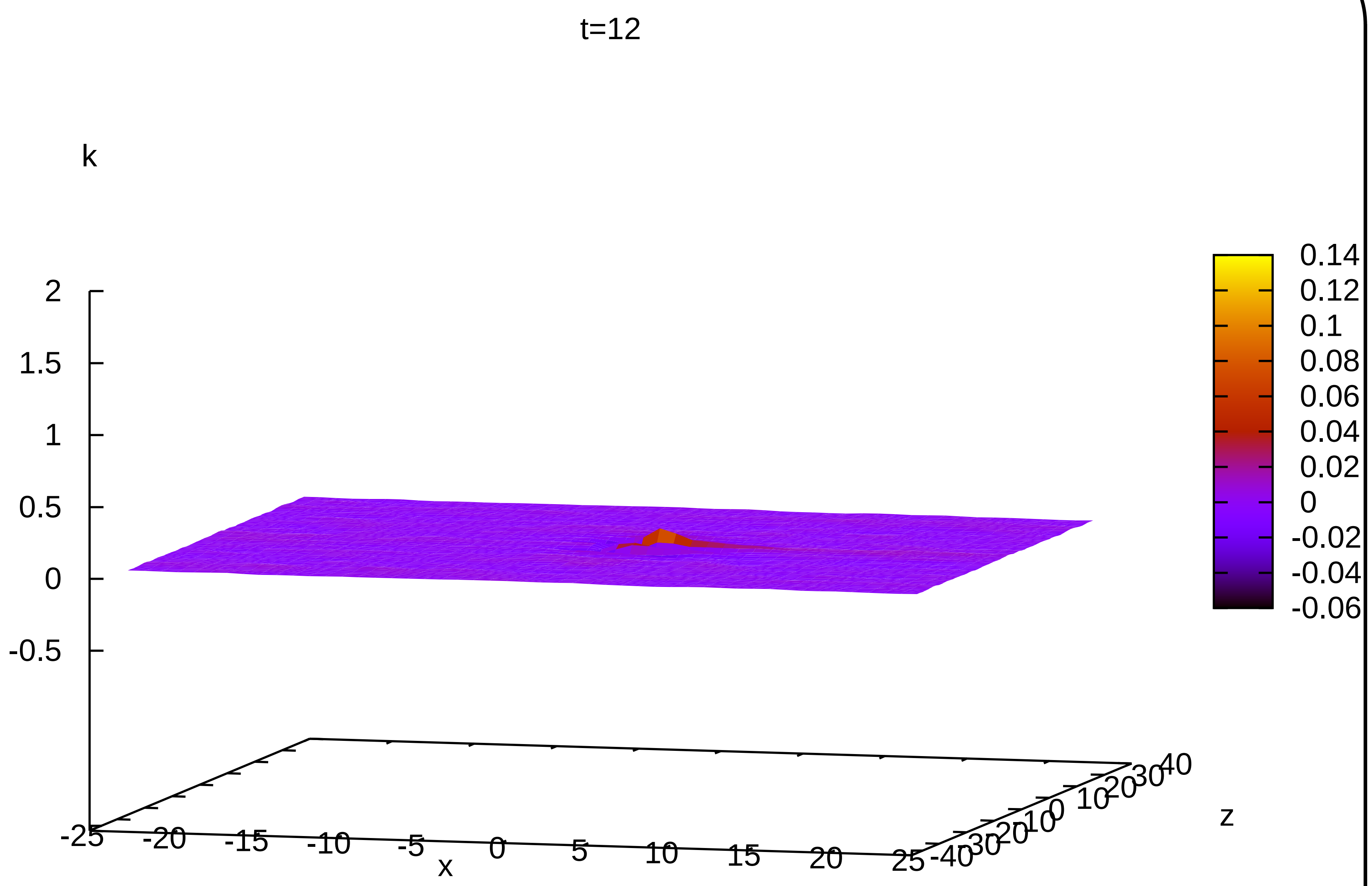
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06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0
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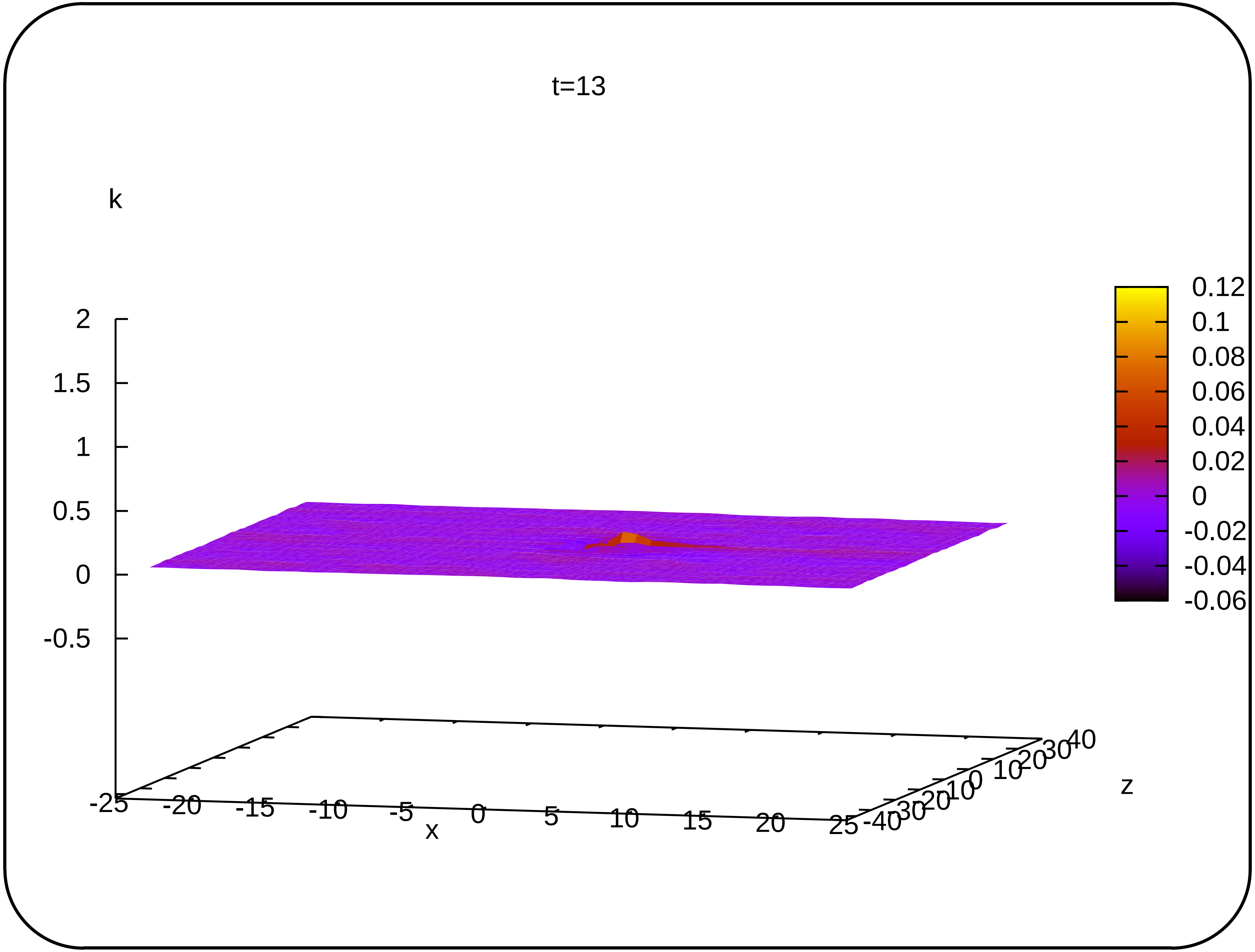
x
-40 -30 -20 -10 0 10 20 30 40
z
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06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0.5
0
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0.1
0.08
0.06
0.04
0.02
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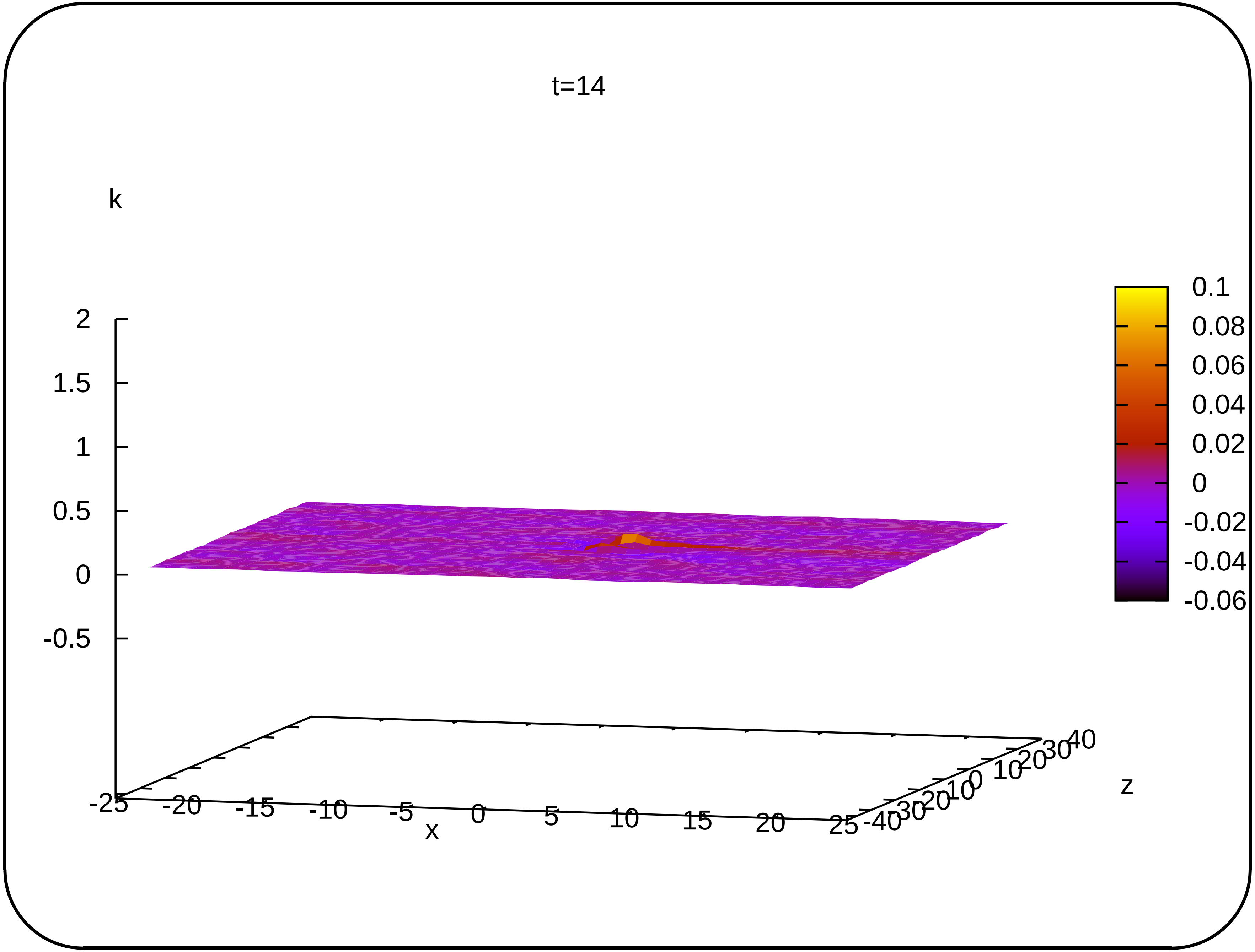
-25 -20 -15 -10 -5 0 5 10 15 20 25
x
-40 -30 -20 -10 0 10 20 30 40
z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x
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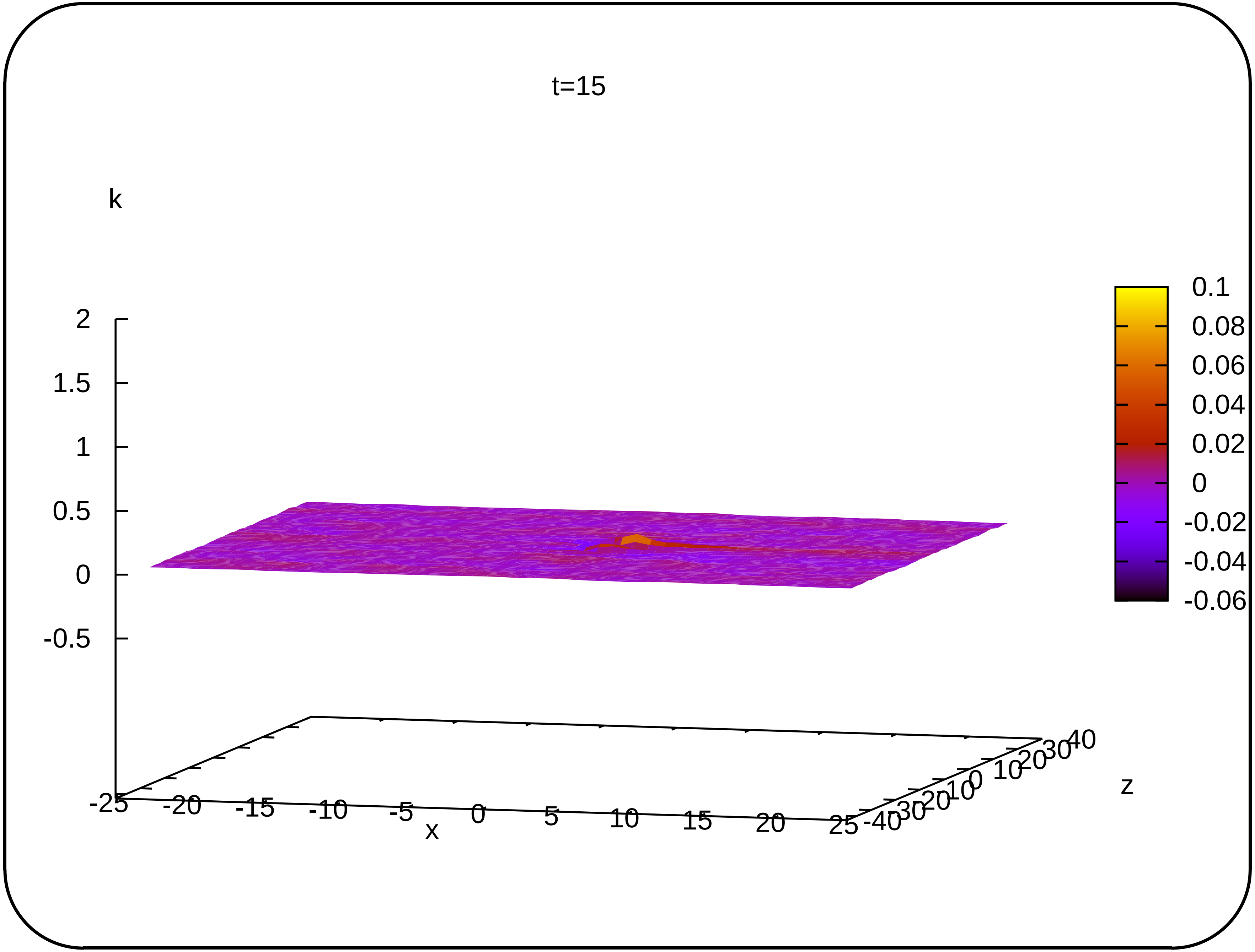
z
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06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0.5
0
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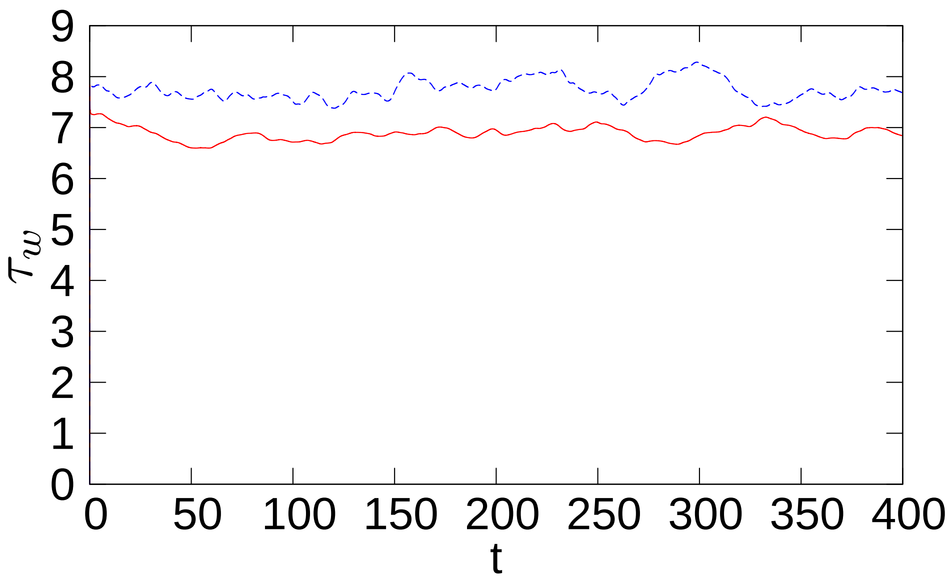
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-25 -20 -15 -10 -5 0 5 10 15 20 25
x
-40 -30 -20 -10 0 10 20 30 40
z

Conclusion: state of the project

- First test of an **optimal-control method** based on the **Wiener-filter** idea has been presented. It enables us to compute the optimal feedback kernel based on the mean linear response of the turbulent flow to external disturbances and on its actual turbulence statistics.
- Wiener-based LQ optimization actually seems to be effective in the presence of **noisy** response-function and correlation data.
- **Bottomline**: exciting new developments in the optimal design of a dynamic feedback controller for active drag reduction are going on in Salerno, Milano and San Diego.
- **Announcement**: even in the complete Navier-Stokes problem it works! We had our first DNS demonstration of nontrivial drag reduction a few days ago.

Test run

$$4\pi \times 2 \times 4\pi/3, \quad Re_\tau = 180$$



Drag reduction: 12 %

Future

- Data refinement and parameter-space exploration has just begun: obtaining the linear-response and correlation data from the DNS is akin to an experiment. We still do not know the **true potential** of our technique.
- **Nonlinear optimization** is possible: once the linear procedure is oiled and working smoothly, it can be used as the linearized step of a Newton algorithm to optimize the feedback kernel without any small-perturbation hypothesis.

Potential for real-world experiments

- **Construction of the real-world controller**: if anybody has the (MEMS) technology to give it a try, we can provide the kernels!
- **High-Reynolds number controller design**: the basic ingredients for our design, two-point correlations between wall stress and internal velocity with and without external forcing, can only be “measured” from a DNS at moderate Reynolds number. Actual laboratory measurements would extend the range considerably.