Reduction of turbulent friction by spanwise wall oscillations

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Abstract

Direct numerical simulations of the Navier–Stokes equations for a turbulent channel flow subject to lateral oscillations of the walls are employed to study the turbulent wall-shear stress, both to the aim of investigating the transient bahaviour of this quantity to the steady state regime and of establishing a documented database of numerically computed friction reductions, with an estimated accuracy of 1%.

The analysis of the transient regime reveals that the spanwise flow fully adapts to the new forcing after about one period of oscillation, while the duration of the transient for the longitudinal wall shear-stress does not depend on the oscillation period, but it is notably related to the maximum wall velocity.

After the initial transient has elapsed, we compute a maximum drag reduction of 45%, and we assess the possibility for the power saved to be higher than the power spent for the movement of the walls (when mechanical losses are neglected), with a maximum net energy saving of 11%. Furthermore, a parameter, which depends on both the maximum wall velocity and the period of the oscillation, is found to be linearly related to the amount of drag reduction. The correlation holds as long as the half-period of the oscillation is shorter than a typical life time of the turbulent near-wall structures.

1 Introduction

A number of recently published research papers has shown that wall-bounded turbulent flows, both in the planar and cylindrical geometry, exhibit interesting modifications when cyclic surface motions are imposed in the spanwise direction, or when an oscillating spanwise pressure gradient is applied. The most interesting and practically appealing amongst these effects is perhaps the significant reduction of the streamwise turbulent friction. This emerges from numerical studies based on the direct numerical simulation (DNS) of the incompressible Navier–Stokes equations, as well as from investigations of experimental nature. In the first group of papers, we can mention the first research work on the subject by Jung *et al.* [7], and the extension of the results from the plane channel flow to the pipe flow by Quadrio & Sibilla [16]. Amongst the experimental studies, we recall the first investigation by Laadhari *et al.* [10] and the recent study by Choi [3].

First, we devote our attention to the initial, transient period which immediately follows the start-up of the oscillations. Very limited information regarding the initial transient of a turbulent channel flow modified by spanwise wall oscillations can be found in the numerical study by Jung *et al.* [7], who reported the behaviour of streamwise wall shear-stress as a function of time from the beginning of the wall movement, but did not concentrate on the very first instants of motion. The purpose of this part of the analysis is to gain a thorough understanding of the flow mechanisms induced by the initial movements of the walls and investigate how this modified flow attains its final new quasi-equilibrium character. The complete analysis of the transient regime can be found in the recent work by Quadrio & Ricco [15].

The investigation then focusses on the character of the flow when the temporal transient has completed. Despite the number of available studies, there is still a number of open issues related to the drag reduction properties of the oscillating wall that have not yet received a definite answer. First of all, the exact value of the maximum drag reduction attainable by spanwise wall oscillations has not been assessed yet. Some researchers (Baron & Quadrio [1], Quadrio & Sibilla [16] and Il-Choi *et al.* [2], for example) have reported reductions of the order of 40%. On the other hand, Laadhari *et al.* [10], Trujillo *et al.* [17] and Choi & Graham [4] have indicated significantly lower maximum drag reductions.

A second fundamental open issue concerns the global energy balance given by the oscillating wall. Since an amount of external power is required to move the wall against the viscous resistance of the fluid, it is important to determine the parameters of the oscillation yielding to the best *overall* performances, considering both energetic costs and benefits, and to verify whether this technique can lead to a net gain, at least in an idealized situation when mechanical losses are neglected. Preliminary computational studies by Baron & Quadrio [1] and by Quadrio & Sibilla [16] are encouranging since they indicate that the energy input and the energy savings might be of the same order of magnitude, but no information is provided about the optimal choice of oscillation parameters and on the actual possibility of achieving such an important energy saving. In this part of our investigation, we greatly capitalize on the fact that our computational system allows us to calculate amounts of drag reduction with an accuracy estimated within 1%, whereas the uncertainty of the previously-mentioned studies is higher thus precluding a precise determination of the net energy saving in the oscillatory parameters space.

Agreement is also lacking on the scaling of drag reduction, namely on the existence of a quantity, function of the oscillation parameters, to which the amount of drag reduction uniquely relates. A few investigators, (Trujillo *et al.* [17] and Karniadakis & Choi [8], for instance) suggested that drag reduction scales with the maximum wall velocity of the oscillation, $W_m = D\pi/T$, where D is the maximum diplacement and T is the period. On the contrary, Jung *et al.* [7], Baron & Quadrio [1], Dhanak & Si [5], Quadrio & Sibilla [16] and Nikitin [11] determined an optimal period at any fixed W_m. Only Il-Choi *at al.* [2] seem to have found a quantity which shows some correlation with drag reduction, but nonetheless the issue of the drag reduction scaling is still source of debate.

In this work, we use a pseudo-spectral Navier–Stokes solver to perform a large number of direct numerical simulations (DNS) of a turbulent channel flow over spanwise-oscillating walls. The main objective is to produce a well documented and reliable database, presenting a complete map of drag reduction data versus the parameters defining the oscillation of the wall, namely W_m, T and D. Emphasis is placed on computational procedures and error analysis, in order to give a reasonable estimate for the percentage error of the measurements. We then address the transient bahaviour from the beginning of the oscillations and the three open questions illustrated above.

The layout of the paper is as follows. All the numerical issues, error analysis and procedures will be discussed in the following Section. Section §3 then presents the results of the simulations with particular attention in presenting them in view of the previous investigations on the subject. Lastly, §4 is devoted to conclusions.

2 Numerical method and procedures

We solve the Navier-Stokes equations for the flow in a plane channel with DNS. A pseudo-spectral solver for an incompressible fluid written in cartesian coordinates, recently developed by Quadrio & Luchini [13], is employed. At the wall, the usual no-slip and no-penetration conditions are applied. Fourier expansions are used for the homogeneous directions, whereas fourth-order accurate, compact finite-differences schemes over a variable-spacing mesh discretize the differential operators for the wall-normal coordinate.

The boundary condition for the spanwise (z) component of velocity at the walls is:

$$W = W_m \sin\left(\frac{2\pi}{T} t\right),$$

i.e. the two walls move in phase with a spanwise velocity W with period T and amplitude W_m. The Reynolds number is $Re_\tau = 200$, being Re_τ based on u_τ, the friction velocity in the uncontrolled case, and on h, half the distance between the channel walls.

The streamwise (x) length of the computational domain is discretized with 321 Fourier modes, and 129 of them are used for the spanwise direction; the number of collocation points in the wall-normal (y) direction is 129. The dimensions of the computational box are $L_x = 21h$ and $L_z = 4.2h$, so that the spatial resolution in the reference case is $\Delta x^+ = 13.1$, $\Delta z^+ = 6.5$ and $\Delta y^+ = 0.8 - 5.4$ (the $+$ superscript indicates quantities made dimensionless with inner variables, i.e. with the friction velocity u_τ of the reference case and the kinematic viscosity ν of the fluid). The time integration interval is 1000 h/U_P, where U_P is the centerline velocity of a laminar Poiseuille flow with the same flow rate. This time interval corresponds in viscous units to $t^+ \approx t u_\tau^2/\nu \approx 8400$, and to $\approx 1250 h/U_c$, where U_c is the centerline velocity of the turbulent flow.

The accurate determination of the duration of the initial transient phase is crucial for a reliable measurement of drag reduction in numerical simulations: the time averaging procedure can be meaningfully started only after this initial transient has disappeared. In the present study, based on a visual case-by-case observation of the the time history, the instant t_i where the statistical analysis is started is identified when a time interval significantly longer than the initial transient has elapsed.

The computed value of C_f in the reference simulation at $Re_\tau = 200$ is $7.928 \cdot 10^{-3}$. It compares very well (within a 0.2% difference) with the value reported by [9], after rescaling to take into account their different Reynolds number ($Re_\tau = 180$), and by assuming $C_f \sim Re^{-0.25}$. We have tested the sensitivity of the absolute value of the friction coefficient to various discretization parameters. We have performed a simulation with less spatial resolution, in a smaller computational domain and for a shorter time interval. This run with $N_x = 161$, $N_y = 129$ and $N_z = 129$, with a reduced length of $L_x = 4\pi h$ and a reduced integration time of 500 h/U_P, has given a friction coefficient of $7.884 \cdot 10^{-3}$, with a difference of 0.6%. Similar tests with the moving walls also showed similar differences, so that we can conclude that the error in the percentage change in C_f due to the oscillation of the wall should be below the 1% range.

3 Results and discussion

3.1 Transient from beginning of oscillations

This section outlines the bahaviour of the modified turbulent flow immediately following the spanwise oscillations of the walls. The interested reader should refer to Quadrio & Ricco [15] for more details. Our analysis shows that the drag-

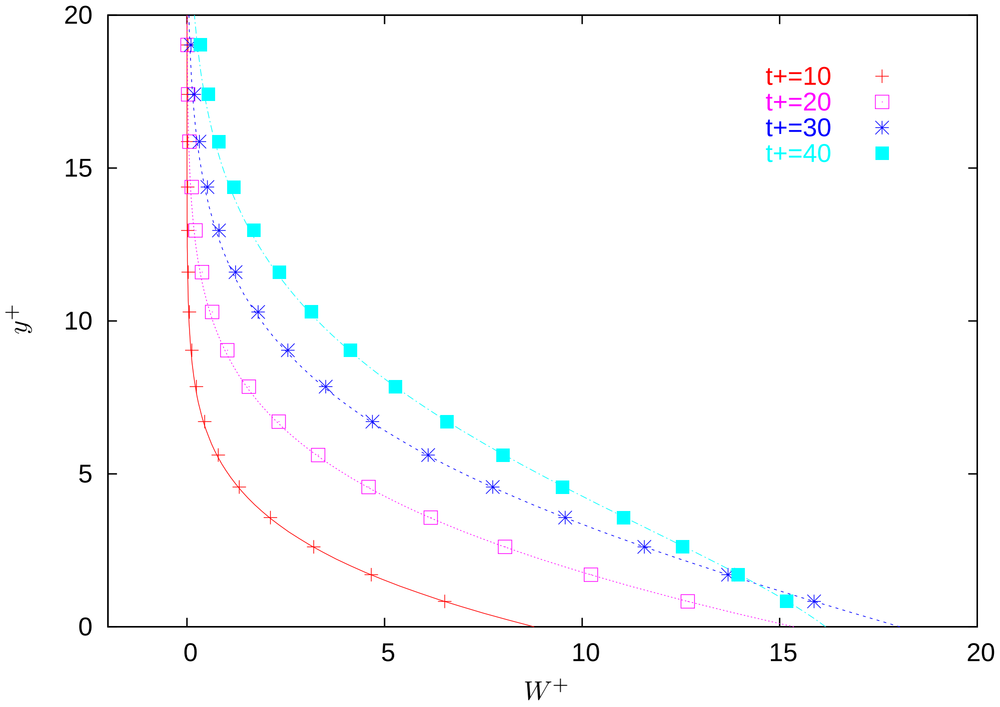

Figure 1: Spanwise velocity profiles at different instants during the start-up of the oscillations. Comparison between the analytical laminar solution (continuous lines) and DNS data (symbols). The oscillating conditions are $W_m^+ = 18$ and $T^+ = 125$.

reducing effect of the oscillation is not felt during the very first instants of the oscillation ($t^+ < 5$). This is because the vertical distance to which the wall motion extends its influence is still smaller than the average height of the low-speed streaks, i.e. near-wall structures elongated in the streamwise direction, existing approximately at a distance from the solid wall between $y^+ = 3$ and $y^+ = 10$. Spanwise shear stresses generated at the wall take some time to diffuse to the interior and to significantly drag laterally the low-speed streaks, so disrupting their spatial coherence with the overriding quasi-streamwise vortical structures, which typically exist at $y^+ > 20$, as shown for example by Jeong *et al.* [6]. This is consistent with the suggestion, put forward by Quadrio & Sibilla [16], that the relative lateral displacement of low-speed near-wall regions and quasi-streamwise vortices is related to the reduction of friction drag.

The space-averaged spanwise velocity profile is now compared with the analytical solution of the transient of Stokes second problem Let us first rescale the time and wall-normal coordinates as:

$$\tau = \frac{2\pi}{T} t; \qquad \eta = \frac{y}{\sqrt{\nu T/(2\pi)}}$$

so that the wall movement is described by $W(\tau) = W_m \sin\tau$. The well-known steady-state solution of this laminar flow expresses the laminar spanwise velocity profile as function of the similarity variables η and τ as:

$$\frac{w(\eta,\tau)}{W_m} = e^{-\eta/\sqrt{2}} \sin\left(\tau - \frac{\eta}{\sqrt{2}}\right) \tag{1}$$

As reported for example in [12], when the wall starts moving from rest, the solution can be expressed as the steady-state term (1) corrected by an additional transient term, which must vanish as $\tau \to \infty$ (i.e. when $t \to \infty$):

$$\frac{w(\eta,\tau)}{W_m} = e^{-\eta/\sqrt{2}} \sin\left(\tau - \frac{\eta}{\sqrt{2}}\right) +$$

$$+\frac{1}{\sqrt{4\pi\tau}} \int_0^\infty f(y') \left(e^{-(\eta-y')^2/4\tau} - e^{-(\eta+y')^2/4\tau}\right) dy' \tag{2}$$

In (2), the function $f(y')$ equals the value of the integral at $\tau = 0$, and is given by:

$$f(y') = e^{-y'/\sqrt{2}} \sin\left(\frac{y'}{\sqrt{2}}\right)$$

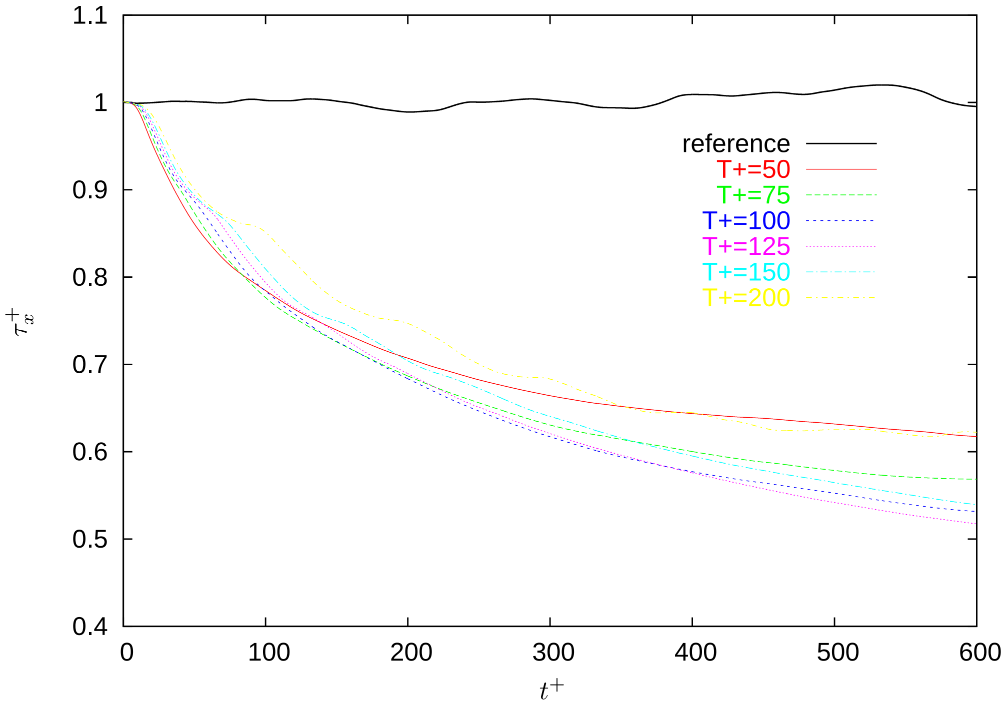

Figure 2: Late time history (ensemble average over the two walls) of the longitudinal wall shear-stress τ_x^+ compared to the stationary case (black line), at different periods. The maximum wall velocity is $W_m^+ = 18$.

The above expression can be obtained by setting $\tau = 0$ in equation (1) and by noting that the transient part must be opposite to the steady-state solution at the beginning of the oscillation.

Figure 1 clearly illustrates how the transient laminar solution essentially coincides with the spatially averaged spanwise flow in the turbulent case. It is remarkable how the laminar profile maintains itself, notwithstanding the presence of significant fluctuations of the spanwise velocity component.

In figure 2, where the oscillation period is varied and the maximum wall velocity is fixed, the decrease in drag only presents a slight dependence on T for $t^+ < 300$; at later times the curves asymptotically adjust to different long-term drag reduction values. While the oscillation period has no remarkable effect on the time history for the streamwise friction, the maximum wall velocity strongly affects its transient behaviour. This is clearly illustrated in figure 3, where the oscillation period is kept constant and W_m is varied. The time needed by the flow to adapt to its modified state appears to become significantly longer for higher W_m.

3.2 Absolute drag reduction

The absolute drag reduction P_{sav} is expressed as percentage of the friction power in the unmanipulated reference case, and is simply given by:

$$\%P_{sav} = \frac{C_{f,0} - C_f}{C_{f,0}}. \tag{3}$$

Figure 4 is a graphical representation of (most of) the computed drag reduction data. This plot reveals that the highest drag reductions can be attained by keeping T^+ in the $100 - 125$ range, and that drag reduction appears to be monotonically increasing with W_m, for a fixed period of oscillation. The maximum drag reduction is about 45% and it was found for $W_m^+ = 27$ and $T^+ = 125$. It appears clear that drag reduction does not scale with W_m, as proposed by a few investigators (see, for example, Trujillo *et al.* [17], Choi [3] and Karniadakis & Choi [8]).

None of the experimental investigators were able to measure drag reduction amounts of the order of 40% because the wall was not oscillated at D^+ values as high as 1000. Moreover, the experimental evidence does not show an optimum period of oscillation at constant D since the surface did not oscillate at periods lower than $T^+ = 50$. Experimental campaigns at very low oscillation periods bear however difficulty due to limitations of the mechanical system.

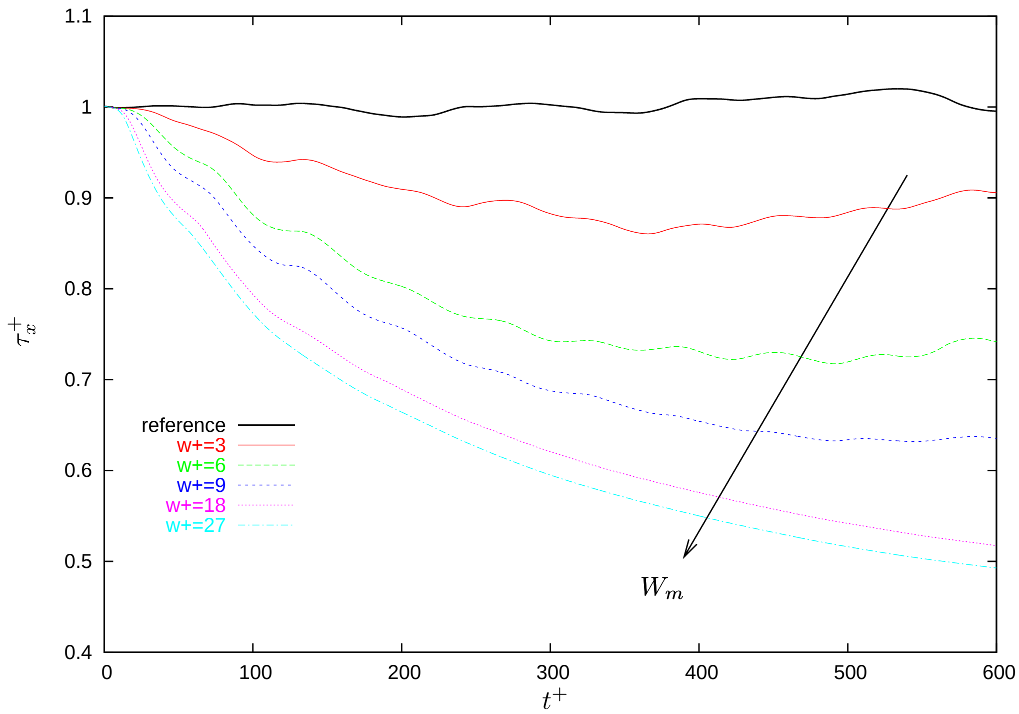

Figure 3: Late time history (ensemble average over the two walls) of the longitudinal wall shear-stress τ_x^+ compared to the stationary case (black line), at different maximum wall velocities. The period is $T^+ = 125$.

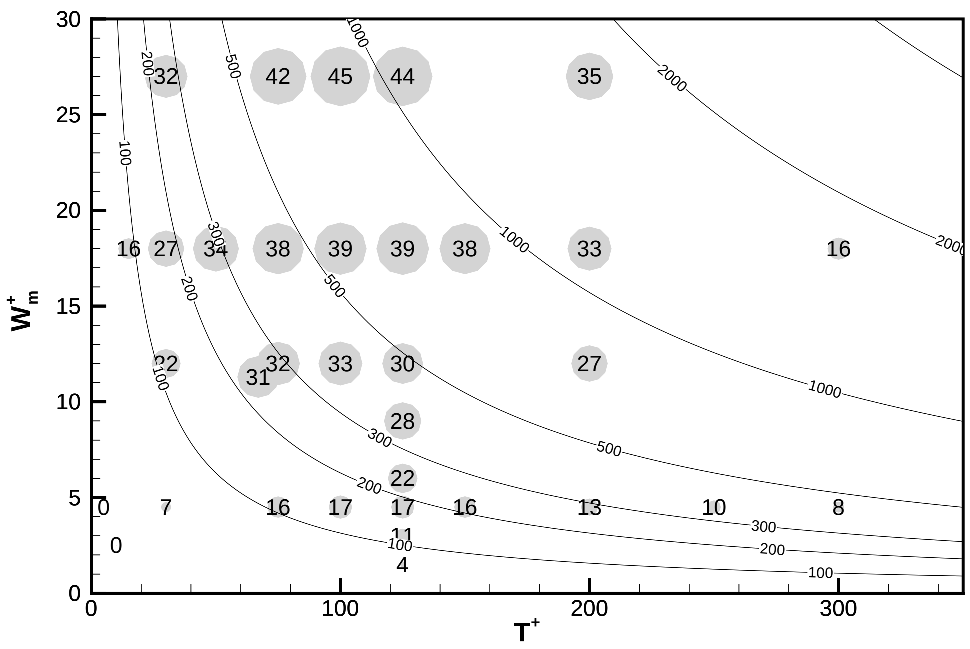

Figure 4: Three-dimensional plot of percentage drag reduction versus T^+ and W_m^+. The area of the circles is proportional to the percentage drag reduction, the numerical value of which is reported inside. Hyperbolae are curves of constant maximum displacement of the wall D^+. Note that a few points at high T^+ are not shown in the plot.

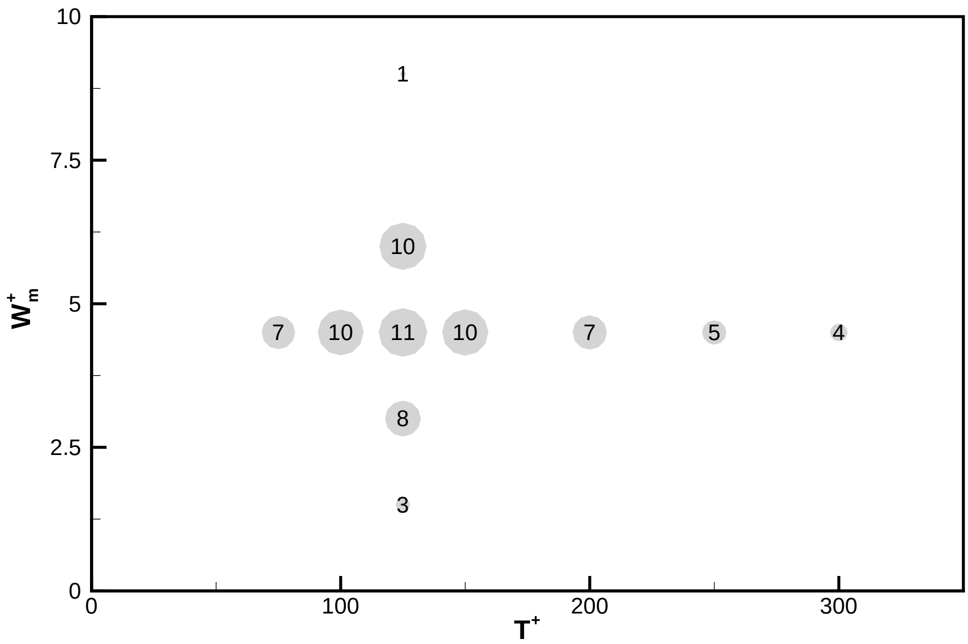

Figure 5: Three-dimensional plot of net energy saving versus T^+ and W_m^+. The area of the circles is proportional to the percentage net energy saving, the numerical value of which is reported inside. Only points with positive gains are reported.

3.3 Net energy savings

The net energy saving is computed by subtracting the power spent to move the wall P_{req} from the absolute drag reduction. P_{req} is computed as follows:

$$P_{req} = -\frac{L_x L_z}{t_f - t_i} \int_{t_i}^{t_f} (\tau_z^{(\ell)} + \tau_z^{(u)}) W \mathrm{d}t, \tag{4}$$

where $W = W(t)$ is the velocity of the two walls and τ_z represents the spanwise component of the shear stress at the wall. It can also be expressed as percentage of the friction power loss in the unmanipulated reference case. Only a limited region of the $T^+ - W_m^+$ plane is shown in figure 5, where the budget is positive: this occurs for low values of W_m. The optimum is located around the optimum value of the period (i.e. $T^+ \approx 125$) and at small values of maximum wall velocity, around $W_m^+ = 4.5$. The present results confirm and extend the findings of Baron & Quadrio [1] and Quadrio & Sibilla [16], whose preliminary analysis shows comparable positive net power balances for similar values of the oscillatory parameters.

3.4 Scaling of drag reduction

The first investigation on the scaling of drag reduction on the parameters of the oscillation is the one by Il-Choi *et al* [2], who found a reasonably good correlation of the drag reduction data with the following quantity S^+:

$$S^+ = 2\sqrt{\frac{\pi}{T^+}} \ln\left(\frac{W_m^+}{W_{th}^+}\right) \mathrm{e}^{-\overline{y}^+\sqrt{\pi/T^+}} \tag{5}$$

where, W_{th} is a threshold velocity representing a characteristic value of the turbulent fluctuations and $\overline{y}$ is a typical distance at which the wall oscillation affects the turbulent structures. S^+ can be viewed as the product of the maximum local acceleration of the laminar Stokes layer and the critical distance $\overline{y}$. In [2] the two free parameters $\overline{y}$ and W_{th} are determined empirically, and the available data fall with a reasonable scatter on a quadratic curve in S^+.

In the present work, more accurate data are available, and the two free parameters are determined in such a way as to maximize the correlation between the measured drag reduction data and the quantity S^+. The resulting values of $W_{th} = 1$ and $\overline{y}^+ = 6.25$ are quite similar to those employed in [2], and bear a clear physical meaning. The present dataset is able to highlight the fact that, as seen in figure 6, a significant portion of the data show a *linear* correlation to S^+, and the correlation is much stronger than the nonlinear (and scattered) one reported by Il-Choi *et al.* in [2]. On the other hand,

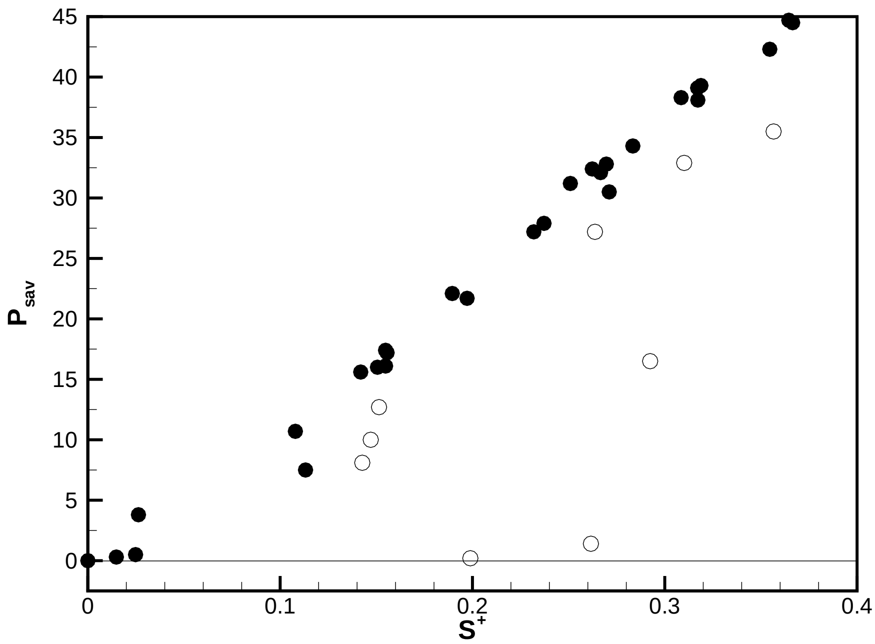

Figure 6: Correlation between P_{sav} and the parameter S^+, computed with $\overline{y}^+ = 6.25$ and $W_{th}^+ = 1.2$.

the empty circles, representing data with $T^+ > 150$, are much less correlated. This is because at high T the oscillation of the wall tends to become uncoupled from the near-wall turbulence dynamics. In their work on the integral space- and time-scales in wall turbulence, Quadrio & Luchini [14] have shown that a typical pseudo-Lagrangian time scale can be computed for the near-wall turbulent structures, based on spatio-temporal correlation data. This characteristic time scale can be taken as a typical survival time of the longest-lived and statistically significant turbulent structures, and, when computed for the longitudinal velocity fluctuations, it is about 60 viscous time units for a value of the Reynolds number similar to the present one. The new scaling parameter then is linearly related to this quantity as long as the typical interaction time between the oscillating wall and the near-wall turbulent structure, namely $T^+/2$, is shorter than the typical longitudinal lifetime of the structures themselves. When T is too high, the structures have enough time to develop their inner dynamics between successive sweeps of the transversal Stokes layer, and enough time is allowed for the near-wall turbulence to readapt to its natural equilibrium state, thus restoring the unperturbed value of friction drag.

4 Conclusions

We have addressed some open issues related to the drag reduction properties of a turbulent channel flow modified by wall oscillations, by carrying out a number of direct numerical simulations of the Navier–Stokes equations, and building a database of changes in the friction coefficient with an estimated accuracy within 1%.

It is found that the spanwise transient flow is composed by the turbulent flow superimposed on a laminar flow which is coincident with the analytical solution of the boundary layer equations for the case of a wall suddenly set in oscillatory motion. The time scales on which the spanwise and the longitudinal flows adapt to the new condition and reach a new equilibrium state are well separated; the former only depends on the period of the wall oscillation, while the latter varies both with the period of the oscillation and with the maximum wall velocity.

Amounts of drag reduction as high as 45% have been computed and previous numerical results have been confirmed. The drag reduction properties of the flow have been observed to depend on both the maximum wall velocity and the period of oscillation, in contrast with the majority of experimental data, which support the idea that W_m is the correct scaling quantity.

The possibility of achieving an overall positive energy balance has been assessed by showing that net energy savings higher than 10% can be attained for $4 < W_m^+ < 6$ and $100 < T^+ < 150$, and the range $0 < W_m^+ < 10$ and $T^+ > 50$ guarantees positive net balances.

A parameter has been proposed, function of both the maximum wall velocity and the period of the oscillation, which shows a high linear correlation with drag reduction data. The linear correlation holds as long as the half-period of the oscillation is lower than a typical time scale of the flow, related to the survival time of the statistically significant near-wall turbulent structures.

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