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Advances in Turbulence IX
Proceedings of the Ninth European Turbulence Conference
I.P. Castro, P.E. Hancock & T.G.Thomas (Eds.)

The linear response of a turbulent channel flow

M. Quadrio[1] and P. Luchini[2]

[1]Dept. Aerospace Engineering, Politecnico di Milano
v.La Masa 34, Milano 20132, Italy.
[2]Dept. Mechanical Engineering, University of Salerno
84084 Fisciano, Italy.

Contact address: *maurizio.quadrio@polimi.it*

1 Introduction

The dynamical behavior of a turbulent flow can be studied by means of the linear response of the velocity field to external perturbations. The knowledge of such response is useful both for a physical understanding of turbulence and for the setup of flow control strategies for technologically important problems, such as turbulent drag reduction or turbulent mixing enhancement. In view of the flow-control possibilities offered by the recent technological breakthroughs in the construction of micron-sized electro-mechanical sensors and actuators (MEMSs), many emerging issues related to the design of the actuator could benefit from the knowledge of the response function. Such a knowledge can answer the basic question of which effects are to be felt *here* if an actuator has been moved *there*.

A precise definition of the response function can be achieved through the concept of phase-locked average: after adding a known external forcing to the flow, we can separate the deterministic effect of the forcing from the random background turbulence, even when the latter is bigger in amplitude, by extracting the component of the signal that remains in phase with the forcing and averaging over a long enough time. Since with phase-locking the response to an arbitrarily small external forcing is well defined (within practical limits), we can introduce an average linear response function, and characterize it either in the frequency domain as the response to sinusoidal forcing of varying frequency, or in the time domain as the response to a Dirac δ function, the former being the Fourier transform of the latter.

A first experimental work in this sense has been that of Hussain [2]. In the recent literature some more experimental results have appeared: for example Camussi *et al.* [1] examined the response of a jet flow to a step-function perturbation, and Tardu [4] measured the response of wall turbulence to a sinusoidal wall-normal velocity perturbation applied at the wall. The complete spatio-temporal shape of the response function is however far from being determined.

In this paper we use the Direct Numerical Simulation (DNS) of turbulence (time-resolved numerical solution of the Navier–Stokes equations) to compute the average linear response of a turbulent plane channel flow to perturbations applied at the wall.

2 Calculation of the impulse response

The calculations are performed with the pseudo-spectral DNS code recently developed by Quadrio & Luchini and described in [3], which solves the Navier–Stokes equations employing a mixed discretization: Fourier expansions are used in the homogeneous x and z directions, while y derivatives are discretized through fourth-order accurate, compact finite differences over a variable-spacing mesh. Time advancement is effected by the traditional partially implicit approach. The code is capable to efficiently run in parallel on both SMP shared-memory architectures and/or a number of distributed-memory computers, and is used on a purposely built cluster of 8 commodity dual-CPU PC.

Our aim is the calculation of the response function $\mathcal{H}$ of a turbulent plane channel flow to a perturbation of the wall-normal velocity applied at the wall. If $\mathcal{O}(x,y,z,t)$ is the output of the system, and the input $\mathcal{I}(x,z,t)$ is applied at the channel wall, in a linear system the response function $\mathcal{H}$ relates input and output according to the convolution integral:

$$\mathcal{O}(x,y,z,t) = \int_{-\infty}^{+\infty} \mathcal{H}(x-x',y,z-z',t-t')\ \mathcal{I}(x,z,t)\ \mathrm{d}x'\ \mathrm{d}z'\ \mathrm{d}t' \qquad (1)$$

$\mathcal{H}$ is in fact the response of the channel to a Dirac δ in both space and time, and in the absence of fluctuations could have been computed, within the limits of the space-time discretization, by just applying such a δ function for the wall-normal component v of velocity at the wall, i.e. $\mathcal{I}_v(x,z,t) = \delta(x)\delta(z)\delta(t)$. Here, however, in order to separate the linear response to a small external disturbance from the random turbulent fluctuations, a phase average must be introduced. The computational challenge is then that, if the amplitude of the forcing is small enough for the response to be linear, a large averaging time becomes necessary.

Our first approach at the calculation of $\mathcal{H}$ was similar to that employed by most experimentalists, i.e. working in the frequency domain: a DNS is performed with a wall-normal velocity which varies sinusoidally in time with a suitably small amplitude and a given frequency, and the deterministic effect of the perturbation is separated from the turbulent noise by phase-averaging over a number of periods. However we soon realized that the repetition of the computation for a suitably large number of frequencies to yield a reasonably complete characterization of the response function would have been impractical. We turned then our attention to the direct use of a Dirac δ as a boundary condition for v: now phase averages have to be taken over the periodical repetition of impulses, but the complete response function is obtained at once. We realized early that this too was going to be an unaffordable simulation: whereas impulse forcing provides in one shot the same amount of information as many sinusoidal simulation, it does so at the expense of larger nonlinear effects, and a correspondingly smaller allowed forcing amplitude and longer averaging time.

We eventually realized that the best of both worlds could be obtained by adopting an externally generated random signal as our forcing. A standard asset of signal theory is that, when a white noise is passed through a linear filter, the correlation R_{io} between input and output is proportional to the impulse response of the system. Therefore the impulse response can be obtained in one shot by just computing such a correlation. If the applied random signal is uncorrelated with the turbulent fluctuations, the latter will be averaged out just as in phase-locking. At the same time, forcing power is uniformly distributed over time, and the amplitude can be as large as in the case of sinusoidal forcing.

3 Results

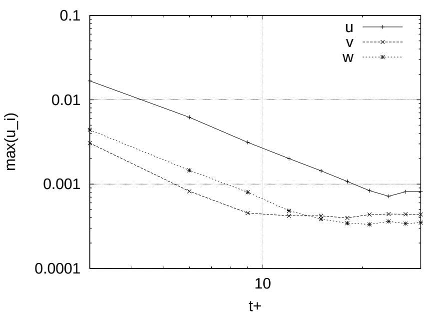

Figure 1: Decrease with time of the maximum value of $\mathcal{H}$.

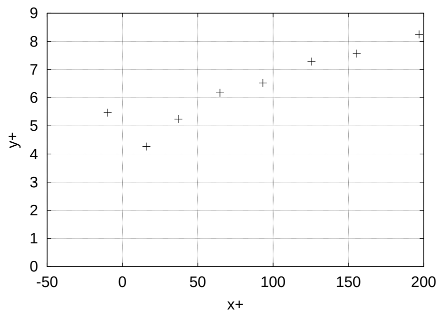

Figure 2: Spatial location of maxima of $\mathcal{H}_u$ as function of time.

We have performed a preliminary calculation for a turbulent channel flow at Reynolds number $Re_\tau = 180$ (based on the friction velocity u_τ and half the distance $2h$ between the two walls). The Fourier-series periods in the streamwise and spanwise directions are $L_x = 2\pi h$ and $L_z = \pi h$ respectively. 128 points are used in the wall-normal direction, and 129 Fourier modes discretize each homogeneous direction. The response $\mathcal{H}$ has been computed of all three velocity components in a whole half-channel, and stored at 11 values of the time delay separated by approximately 3 viscous time units from one another.

Figure 1 reports the time evolution of the maximum (computed over the whole half-channel) of the function $\mathcal{H}$ for the 3 velocity components. Turbulent diffusion causes an exponential decrease of the effect of the perturbation, until it dissolves in the residual turbulent noise. The decrease rate of the three components is comparable, but the u component has the greatest response.

Figure 2 shows, for the response of the u component, how the position where the response is maximum in absolute value changes with time. The effect of the perturbation is felt almost immediately at 4 wall units above the wall, then the y position of the maximum keeps slowly increasing, and a downstream convection at an approximately constant velocity is evident.

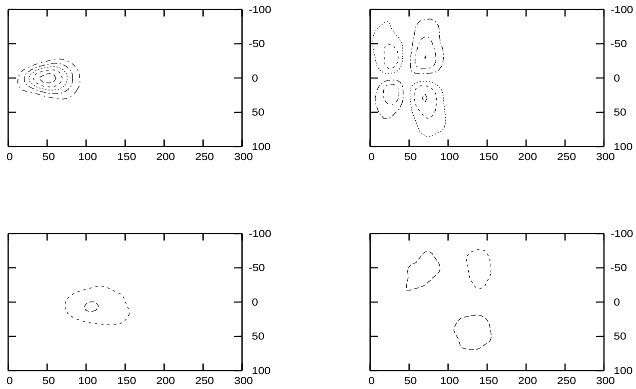

Figure 3: Contours of the response of the streamwise (left) and spanwise (right) velocity components at a time delay of $t^+ = 10$ (top) and $t^+ = 20$ (bottom).

In figure 3 we see the spatial distribution of the response of the u and w components, in a plane at a distance of $y^+ \sim 10$ from the channel wall and at two time delays of $t^+ \sim 9$ and $t^+ \sim 18$. The phase-averaged effect of the wall perturbation shows up primarily as a pocket of negative u-velocity perturbation. Note that the response of the spanwise component w appears to move downstream faster and to have a greater lateral spreading than the u component.

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