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Technical Brief

On the Accuracy of Wall Similarity Methods in Determining Friction Velocity Over Smooth and Ribletted Surfaces

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1 Introduction

An accurate determination of friction velocity u_τ or, alternatively, of wall shear stress $\tau_w = \rho\, u_\tau^2$ is of paramount importance in many turbulent boundary layer problems. In applications concerning drag reducing flows, moreover, the wall shear stress is required to be known with high accuracy, since the amount of drag reduction achievable with passive drag reducing techniques is usually of the order of a few percent.

A direct evaluation of the gradient $\partial U/\partial y$ at the wall can but seldom be achieved because of experimental problems. Integral momentum methods for the evaluation of friction velocity are often too inaccurate and direct shear stress measurement techniques are either inaccurate or expensive. This is why to date the most widely used techniques for determining u_τ are indirect, and rely on the well-known law of the wall. Such techniques are known as wall similarity methods.

The universality of the law of the wall, which can be deduced from simple dimensional considerations, is, however, subject to many important assumptions and limitations, in particular, the boundary layer must be fully developed and must flow, in equilibrium conditions, over smooth and flat walls, with no longitudinal pressure gradient. Provided these conditions are fully satisfied, the so-called logarithmic region of the mean velocity profile (i.e., that portion where the coordinate normal to the wall is between 30 ν/u_τ and 0.15δ, δ being the boundary layer thickness) can be expressed in dimensionless form as a function of the friction velocity u_τ (Coles, 1956):

$$\frac{U(y)}{u_\tau} = A \ln\left(\frac{y u_\tau}{\nu}\right) + B \tag{1}$$

where y is the coordinate normal to the wall, $U(y)$ is the local mean streamwise velocity component, A and B are, respectively, the slope and the intercept of the law of the wall, which is linear when plotted in semilogarithmic coordinates. In the present work, the classical values $A = 2.44$ and $B = 5.45$ will be adopted, following the early suggestion of Patel (1965).

Besides the basic boundary layer in zero pressure gradient, there are several flows where the law of the wall, even if strictly speaking not applicable, gives nevertheless satisfactory results. This is the case, for example, of flows with mild (positive or negative) streamwise pressure gradient, and flows over ribletted surfaces. For such flows also the law of the wall can be used, provided its intercept is allowed to assume a value different from the "universal" one by a quantity ΔB (Acharya and Escudier, 1984).

In the following, four of the most widely used wall similarity methods are considered, with special emphasis on their limitations and problems that are necessarily encountered in their application to real velocity profiles, i.e., profiles affected by unavoidable scattering and experimental errors. One of these methods, the so-called slope method, is selected on the basis of its capability to treat boundary layers over both smooth and rough walls and is then modified to reduce its inherent margins of subjectivity.

It must be clear that the present work is not intended to design and propose new and better wall similarity techniques. Rather, the limits and potentialities of some widely used techniques are discussed and a modified method is proposed which has the property of greatly reducing such arbitrary choices.

2 Wall Similarity Methods

Wall similarity methods can be considered a means for calculating the friction velocity u_τ that gives the best fit between a measured velocity profile and the law of the wall (Kline et al., 1967). These methods work in the linear portion of the velocity profile plotted in semilogarithmic coordinates.

Standard wall similarity methods can be divided into two main groups, depending on the constant B in Eq. (1) being educed from the method itself or being fixed a priori. In the following, two classical methods for each group are considered: the Clauser chart method (CCM), the standard (SBM) and modified (MBM) Bradshaw methods, and the standard slope method (SSM).

CCM (Clauser, 1954) is quite independet from local measurement errors and scattering, because of their statistical compensation operated by the fitting procedure. However, this method explicitely uses the law of the wall equation, and therefore requires that a value for the constant B to be assigned a priori. SBM (Bradshaw, 1959) again requires a value of B to be assigned, and is extremely sensitive to measurement errors.

With MBM one is able to compute the values of B and u_τ at the same time, hence handling flows over rough surfaces. Last, SSM does not require any value to be imposed a priori for B, and with its linear fitting guarantees a statistical compensation for the scattering of the experimental data. The slope of the velocity profile is not affected by a possibly inaccurate evalua-

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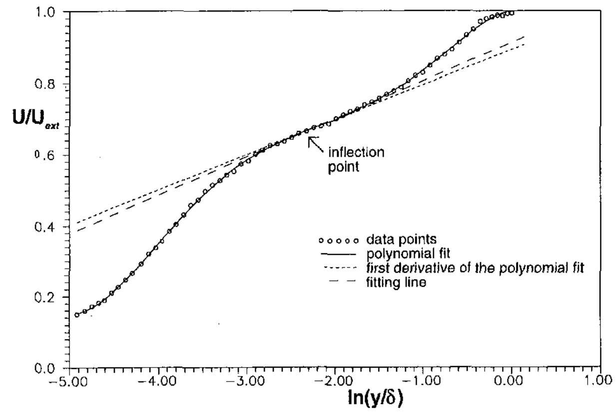

Fig. 1 Interpolation of the experimental velocity distribution with an 8th degree polynomial. The slope of the experimental profile does not coincide with the slope of the polynomial in the point of zero curvature.

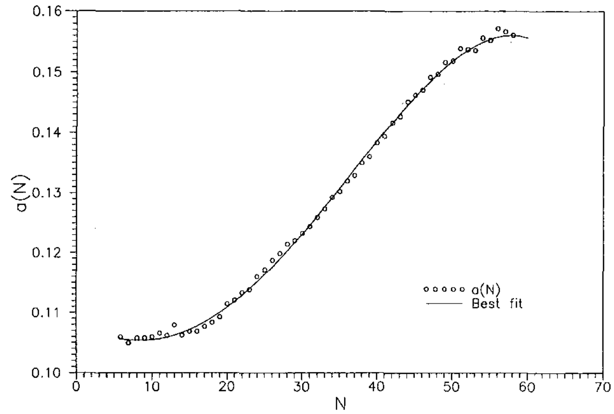

Fig. 2 Behavior of the slope a of the velocity profile in outer coordinates, versus the number of points N used for the linear regression. The center of the linear region is the inflexion point evidenced in Fig. 1. The solid line shows a polynomial fit.

tion of δ. The central problem encountereed in the use of this method on a real velocity profile is the strong lack of repeatability associated with the subjective identification of the location and width of the linear region of the fit, i.e., where and how the linear region is selected.

3 A Suggested Procedure: A Modification of the Slope Method

The classical slope method presents the advantage of wide applicability and low sensitivity to the scattering of the data; its main drawback seems to be a clear lack of repeatability. In the following, a procedure is proposed which is aimed at determining the linear region in the mean velocity profile in an automatic and repeatable way. The procedure only applies to turbulent boundary layers, and addresses the problem in two steps.

In the first step, the center of the linear region is determined. This point must coincide with a point where the curvature of the velocity profile changes its sign. The fitting of the experimental curve with an 8th degree polynomial, and the analytical calculation of the point of zero curvature, have been shown to be a dependable way for identifying the center of the linear region. The use of higher-degree polynomials changes neither the results nor the quality of the fit.

If the tangent to the polynomial at the point of zero curvature were coincident with the slope of the linear part of the velocity distribution, the entire problem could be easily solved. Unfortunately the scattering of the experimental data can be such that the local slope of the experimental distribution is often quite different from the tangent, as evident from Fig. 1 (the data set used in Figs. 1 and 2 is the same as Figs. 3 and 4). The second step, therefore, consists in determining the number of experimental points that have to be used for the linear regression.

A reliable solution to this problem can be found by plotting the behavior of the slope a of the fitting line (the same result holds for the intercept) versus the number N of points used for the linear regression analysis (Fig. 2). With the exception of the first values, obtained with too few points and consequently highly irregular because of their scattering, the slope remains quite constant (as it should be in absence of scattering), until points outside the linear region are considered and the values of the slope start getting higher and higher. A polynomial fit may be used to determine the first point of stationarity of the curve $a(N)$ versus N, and therefore to select in an exact and objective way the number of points to be used for the linear regression.

It has to be said that, unless the computed number of points is even, the choice has to be made on how to distribute them around the central one. This leads, even for the modified slope method, to some variation in the predicted values of u_τ and B.

Once the central point of the linear region and its width have been determined, it is possible to use a linear regression analysis for calculating the values of the slope a and the intercept b in the equation:

$$U/U_{\text{ext}} = a \ln y/\delta + b$$

Finally, once a and b are known, the friction velocity u_τ and the intercept B of the law of the wall are computed; all interventions, judgments, or personal visual interpretation of the experimenter have been avoided, so that repeatability of the procedure is achieved.

4 Validation

In Fig. 3, an evaluation is reported of the sensitivity of the results of the various examined wall similarity techniques to

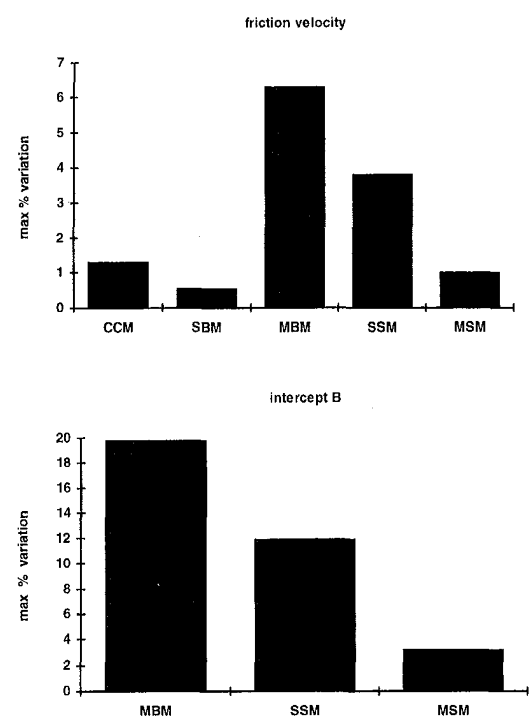

Fig. 3 Evaluation of the sensitivity of the friction velocity u_τ (left) and of the intercept B of the law of the wall (right) to the various wall similarity methods

the free choices that the experimenter has to perform during the overall data reduction process.

In the Clauser chart method (CCM), the best-fitting value of C_f has been varied in a range of ± 0.001. In the standard Bradshaw method (SBM), the height K of the point of intersection has been moved from 30 to 90 wall units (extrema of the logarithmic portion of the velocity profile). In the modified Bradshaw method (MBM) the heights K_1 and K_2 have been varied independently in the range between 30 to 60 wall units, and 60 to 90 wall units, respectively. In the standard slope method (SSM), the width of the region used for the regression analysis has been varied inside the range determinable by visual observation.

The results obtained by varying the above-mentioned free parameters in their range have been computed, and their scatter in terms of maximum percentage variation around the mean value has been reported in Fig. 3. Also computed are the predictions obtained with the modified slope method (MSM), in which, as mentioned above, a small variation is present, associated to an odd number of points to be used for the linear regression.

It can be seen that, as far as the friction velocity is concerned, CCM and SBM, which require that a value for the intercept B be assigned, and have therefore a single degree of freedom, show quite better performances, in terms of percentage variations, with respect to MBM and SSM, which have two degrees of freedom. Their results are comparable with that of MSM. On the other hand, the value of B, which is predictable only by using this last group of wall similarity methods, appears to be reliable only when educed from a procedure, such as the proposed one, which removes the influence of the choices of the experimenter.

In Fig. 4, a typical application is shown, concerning the comparison between the mean velocity distributions in law of the wall form for turbulent flows over a flat and a ribletted plate. The measurements have been performed by the authors in a low speed wind tunnel at the von Kàrmàn Institute for Fluid Dynamics. The experiment is described in detail in Baron and Quadrio (1993a).

The virtual origin of the mean velocity profile for the riblet case has been set according to the concept of longitudinal protrusion height (Bechert et al., 1989; Luchini et al., 1991) and taking into account the actual geometry of the riblet contour (Baron et al., 1993b).

The friction velocity necessary for plotting the velocity profiles in law of the wall coordinates has been computed according to the modified slope procedure (MSM) illustrated in the preceding paragraph. The slope of the linear part, as expected, remains unchanged, while the intercept B increases from 5.38 to 5.95 for the riblettted one. This upward shift ΔB, the amount of which is equal to 0.57, is in good agreement with published experimental results (e.g., those reported by Choi, 1989), and corresponds to a reduction in the friction coefficient of approximately 3–4 percent, which is consistent with the theoretical prediction one could obtain on the basis of the computed value of the protrusion height of the real geometry of the ribs.

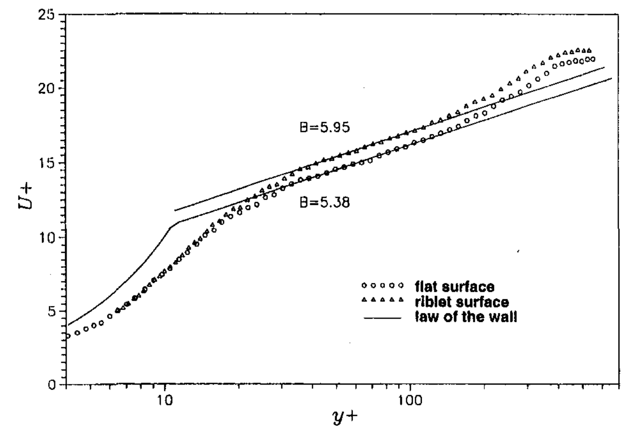

Fig. 4 Application of the modified slope method (MSM) to a real case of turbulent flow over flat and ribletted surfaces

5 Conclusions

An analysis has been performed of four widely used wall similarity methods. Particular attention has been devoted to the so-called standard slope method (SSM), because of its ability to educe the value of the constant B and, consequently, to also deal with drag reducing flows while preserving the capability of statistically compensating the scattering of the experimental data.

However, the standard slope method also presents an intrinsic limitation concerning the subjectivity in the determination of the amplitude of the linear region and the location of its center. This led to the formulation of a procedure, based on some relevant properties in the velocity profile when interpolated with a polynomial, which has been eventually applied to a data set from previous experiments.

This procedure can be employed also to estimate the upward shift ΔB in the law of the wall, which can be extremely small in flows where drag reduction is achieved by passive techniques.

Acknowledgments

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