NEW ALGORITHM FOR ATTITUDE DETERMINATION USING GPS SIGNALS

F. BERNELLI-ZAZZERA, M. MOLINA, M. VANOTTI, M. VASILE

Dipartimento di Ingegneria Aerospaziale, Politecnico di Milano, Milano

ABSTRACT

The paper presents methodology and results for an algorithm of integer ambiguity resolution. The aim is to validate a new concept of GPS attitude sensor, with every receiver independent from one other. The algorithm uses an instantaneous–static geometric inequality in order to reduce the integer search. A batch-loss function is evaluated for checking the remaining integers and finding the solution. The peculiarity of the algorithm is to find the right solution even for a coplanar antennas array. The experimental work is the proof of concept of the procedure for the University Microsatellite PalaMede, for which the foreseen baselines are 40 cm. The procedure is demonstrated using real data, collected from standard, not space-qualified, GPS receivers. The synchronisation between the receivers is reconstructed interpolating the acquired data.

1. INTRODUCTION

The Global Positioning System (GPS) constellation is widely used for determining the position of a user near the surface of the Earth. Over the two last decades the technology has highlighted the possibility of using the GPS signals for space applications, such as on-orbit autonomous navigation and attitude determination [1]. The key of the attitude measurement concept is the phase difference between signals received from different antennas [2]. Since carrier phase difference measurements are ambiguous due to the unknown number of integer cycles between two sightlines, the integer ambiguity resolution is a necessary task before determining the attitude. This work analyses the performance of the integer ambiguity resolution algorithm for a coplanar baseline array and performs a first attitude initialisation [3].

A motion-based algorithm has been implemented. The advantages over other methods include (I) the possibility of solving ambiguity without attitude knowledge, (II) avoiding large matrix inversion, (III) using a non-iterative procedure. First the algorithm introduces a geometric inequality to reduce the integer space search. Then a batch-type loss function is used to select a candidate solution. The final solution is validated by means of an integrity check on the error covariance matrix.
The system is verified with a GPS test facility consisting of a telescopic bar mounted on the mock-up of the satellite, to allow easy baseline variation. The bar attitude, and hence the satellite yaw, is estimated by the phase difference observed from two offset antennas, linked to a pair of independent receivers. As shown in Figure 1, the memorization of the raw-data on two different computers permits to perform off-line tests.

![Figure 1 Test facility description](image)

The synchronization of the two signals represents the crucial point. The attention will be pointed on an innovative GPS sensor design. The receivers are completely independent; each one maintains its own time reference with its original clock. The synchronisation is reconstructed interpolating the data of the acquisitions. This step highlights the performance degradation in the attitude estimation according to the baseline length.

The paper is organised as follows. Section 1 presents the PalAMede mission. Section 2 describes the basic concept of the attitude determination using GPS phase difference signals. Section 3 develops the algorithm for the integer ambiguity resolution for an array of coplanar baselines. Section 4 discusses the procedure for the attitude initialisation. Section 5 contains the validation of the algorithm presenting the performance of GPS commercial receivers and results of the tests. Finally the paper summarises the research now in progress.

### 2. PALAMEDE MISSION

The near-Earth three-axis stabilized satellite PalAMede is a basic platform entirely developed by the students of the Aerospace Engineering Department of Politecnico di Milano. The aims of this project are testing and developing standard technologies, and to get the know-how for the bus-design of small and cheap missions. The satellite will be placed in a sun-synchronous polar orbit at an altitude of 800 km, and currently launch is scheduled for 2003 [4].

The orbit selection will guarantee an interesting coverage of the globe at low cost and provides good lighting condition both for the target and the orbit. The former is of primary importance for the cameras, while the latter is related to the thermal and power subsystems. Power to all the subsystems will be provided by panels mounted on the lateral faces of the spacecraft, providing 40 W, and by a battery during eclipses. PalAMede will be Nadir pointing and equipped with two CCD camera as payload. One colour camera directed towards the Earth surface. Appendages outside main body (e.g. omni-directional antennas) will guarantee downlink communication.

The ADCS includes three magneto-torquers and a momentum wheel for the control. Non stringent pointing requirements allow integrating three-axis magnetometer and coarse sun sensors for primary attitude solution. A system of four GPS antennas for testing a new approach to the problem will be considered as secondary payload and connected to a GPS
receiver. For obvious visibility reasons the GPS antennas will be placed on the zenith face, toward the deep space, with a typical reference baseline of 40 cm.

3. GPS BACKGROUND

The attitude of a spacecraft is the orientation of a body fixed frame with respect to an external frame. The rotation between the two references is represented by the transformation matrix $A$. Performing an attitude solution from GPS carrier phase interferometry is essentially a three-step process: integer ambiguity resolution, attitude initialisation, attitude determination. Most attitude algorithms determine the attitude comparing a measured vector in the body frame with the true vector in the external frame. Cohen [2] applied this concept to the sub-centimetre precision of the GPS carrier phase. Due to the great distance between the GPS satellites and the receivers, the incoming wave front can be considered planar. The signal travelling at speed of light reaches the antennas of the baseline at slightly different times. The measured carrier phase difference between the antennas is used to determine their offset. Since the rotation about the unit line-of-sight vector can not be determined by a single GPS signal, the determination of the complete attitude requires at least an array of two baselines onboard and two GPS satellites available.

The GPS satellites generate a carrier signal in L1 band; its frequency is centred on 1575.42 MHz, corresponding to a wavelength of 19.03 cm. As illustrated, the phase difference $\Delta \varphi$ is related to the range difference $\Delta r$ between two antennas. The projection of the baseline $b_i$ into the line of sight direction to the GPS satellite and is obtained by:

$$\Delta r = b_i \cdot \cos \theta = \lambda (\Delta \varphi - n)$$  \hspace{1cm} (1)

where $b_i$ is the baseline length, $\theta$ is the front angle between the baseline and the line of sight unit vector, $\Delta \varphi$ is the fractional phase difference (in L1 cycles), $\lambda$ is the wavelength of the GPS signal and $n$ is the integer phase difference. The fractional single-phase difference $\Delta \varphi$, between master and slave antennas, is the fundamental measure and is expressed by:

$$\Delta \varphi = b_i^T A s_j + n$$  \hspace{1cm} (2)

where $s_j \in \mathbb{R}^3$ is the normalized line of sight vector to the GPS spacecraft in the external reference frame, $b_i \in \mathbb{R}^3$ is the baseline vector (in wavelengths) in the spacecraft body frame, $A \in \mathbb{R}^{3 \times 3}$ is the attitude orthogonal matrix representing the transformation between the two frames also known as the direction-cosine matrix. The complete observation equation of the phase difference measurement model can be written as:

$$\Delta \tilde{\varphi}_{ij} = b_i^T A s_j + n_{ij} + w_{ij}$$  \hspace{1cm} (3)

where the indices refer to the $i^{th}$ baseline and the $j^{th}$ sightlines, $w_{ij}$ indicates the zero mean Gaussian measurements error with standard deviation $\sigma=0.5/\lambda=0.026$. 
4. INTEGER AMBIGUITY RESOLUTION ALGORITHM

The integer ambiguity exists whenever a baseline is defined, hence for \( |b| > 0 \). Since the receivers only measure the fractional part of the carrier phase, the range difference is ambiguous until the integer ambiguity resolution is solved. The attitude-independent algorithm presented in this work refers to the method described by Crassidis, Lightsey and Markley [3]. It is based on geometric inequality constraints to reduce the integer space search and on a batch type loss-function to be minimized in order to determine the optimal solution. The procedure works in a couple of ways. The algorithm can use three sightlines and then consider one baseline at a time, or can use three baselines and then consider one sightline at a time. Since in this paper it is assumed that the baseline array is coplanar, due to PalaMede configuration, a set of non-coplanar sightlines must be available. \( \kappa \) are the integers associated with each baseline, where \( \kappa \) is twice the number of the wavelength contained in the baseline. For three sightlines, the search space would consist of \( \kappa^3 \) possible integer permutations. Applying the inequality to the \( i \)th baseline at the initial epoch, the number of combination falls down to the order of \( 3\kappa^2 \). The reduced subset consists of the integers that pass the following geometric inequality:

\[
\|b_i\|^2 > \|b_i\|^2 (s_i s_2)^2 + (\Delta \phi_{i1} - n_{i1})^2 - 2(\Delta \phi_{i1} - n_{i1}) (\Delta \phi_{i2} - n_{i2})(s_i s_2) + (\Delta \phi_{i2} - n_{i2})^2
\]

(4)

This equation reduces the search space, since for each baseline only two sightlines are considered simultaneously, instead of three. For a set of three coplanar baselines the problem is solved considering three non-coplanar sightlines. This approach requires that the selected GPS satellites remain available for the entire collection period. The integers to-be-determined remain constant throughout the entire data span. The solution is included in the integers that passed the geometric inequality. The integer search is based on an attitude-independent batch-type loss function:

\[
J(n_i) = \frac{1}{2} \sum_{k=1}^{\kappa} \left\{ \frac{1}{\sigma_i^2} \left[ \|S_i^\dagger(k) \Gamma_i(k) (\Phi_i(k) - n_i)\|^2 - \|b\|^2 + \text{trace}\{S_i^\dagger(k)\} \right]^2 + \log \sigma_i^2(k) \right\}
\]

(5)

\[
\sigma_i^2 = -\text{trace}\{S_i^\dagger(k)\} + (\Phi_i(k) - n_i)^\top \Gamma_i^\top(k)S_i^\dagger(k)\Gamma_i(k)(\Phi_i(k) - n_i)
\]

(6)

\[
\Gamma_i = \begin{bmatrix} \sigma_i^{-2} s_i & \sigma_i^{-2} s_2 & \sigma_i^{-2} s_3 \end{bmatrix}
\]

(7)

\[
n_i = \begin{bmatrix} n_{i1} \\ n_{i2} \\ n_{i3} \end{bmatrix}
\]

(8)

\[
\Phi_i = \begin{bmatrix} \Delta \phi_{i1} \\ \Delta \phi_{i2} \\ \Delta \phi_{i3} \end{bmatrix}
\]

(9)
\[ S_i = \sigma_i^2 S_i S_i^T + \sigma_i^2 S_2 S_2^T + \sigma_i^2 S_3 S_3^T \]  \hspace{1cm} (10)

where \( S_i \) and \( \Gamma_i \) are time dependent. It can be shown that the expression of \( J \) involves a scalar check on the norm vector residuals. For finding the solution, that minimizes Eq. (5), minimal motion is required. In order to ensure the candidate solution is optimum, the estimate error covariance is introduced:

\[
P_i = \sum_{k=1}^{4} \left( \frac{4}{\sigma_i^2(k)} \left[ \Phi_i(k) - n_i \right] \left[ \Phi_i(k) - n_i \right]^T \right)^{-1} \]

The selected integers-set is checked indirectly through the inequality on the diagonal elements of \( P \). The algorithm has the 0.0013 probability of selecting a wrong solution if three times the square root of every diagonal element of \( P_i \) is less than 0.5.

The idea of Eq. (4) is to create a reduced set of integers, using two sightlines at the first instant of the acquisition. The inequality is used strictly to reduce the search space. With fast processors it is possible to get a solution without involving the inequality, checking Eq. (5) for all possible integers. For short baselines this can be done very quickly.

5. ATTITUDE INITIALLISATION

Once the integer ambiguity is solved, the attitude can be determined. The computationally efficient TRIAD algorithm [5] provides three-axis attitude determination from two vector observations. Because of its simplicity this non optimal (i.e. deterministic) method has become one of the most popular for determining first three-axis attitude information. The aim is to find the attitude direction-cosine matrix \( A \), which satisfies the relation:

\[
A \hat{V}_i = \hat{W}_i \quad \left( i = 1...n \right) \quad \hspace{1cm} (12)
\]

where \( \hat{V}_1,..., \hat{V}_n \) is a set of unit vectors expressed in an external reference system toward \( n \) known directions and \( \hat{W}_1,..., \hat{W}_n \) are the same \( n \) unit vectors expressed in the body-observation system. Once two non-parallel reference unit vectors \( \hat{V}_1, \hat{V}_2 \) and the corresponding \( \hat{W}_1, \hat{W}_2 \) have been selected, the goal is finding the attitude matrix \( A \) which satisfies:

\[
A \hat{V}_1 = \hat{W}_1 \quad \hspace{1cm} A \hat{V}_2 = \hat{W}_2 \quad \hspace{1cm} (13)
\]

From the equations above, matrix \( A \) is over-determined. The first step consists of constructing two orthonormal triads by means of reference and body unit vectors. According to the notation used for the integer ambiguity resolution the triads are given by the following equations:

\[
\hat{s}_1 = \hat{V}_1 \quad \hat{s}_2 = \left( \hat{V}_1 \times \hat{V}_2 \right) / \left| \hat{V}_1 \times \hat{V}_2 \right| \quad \hat{s}_3 = \left( \hat{V}_1 \times \left( \hat{V}_1 \times \hat{V}_2 \right) \right) / \left| \hat{V}_1 \times \hat{V}_2 \right| \quad \hspace{1cm} (14)
\]
There is a unique orthonormal direction-cosine matrix \( A \) which satisfies the relation given by:

\[
A = M_{\text{obs}} M_{\text{ref}}^T
\]  

(16)

where \( M_{\text{ref}} \) and \( M_{\text{obs}} \) are \([3x3]\) matrices, each defined by the three unit coordinate vectors of the triad:

\[
M_{\text{obs}} = [\hat{\mathbf{r}}_1 \cap \hat{\mathbf{r}}_2 \cap \hat{\mathbf{r}}_3] \quad M_{\text{ref}} = [\hat{\mathbf{s}}_1 \cap \hat{\mathbf{s}}_2 \cap \hat{\mathbf{s}}_3]
\]  

(17)

Equations (16) and (17) define the solution of the TRIAD algorithm. Due to the orthogonality of the direction-cosine matrix \( A \), a necessary and sufficient condition is that the attitude solution given by Eq. (16) satisfies the following relation:

\[
\dot{\hat{\mathbf{r}}}_2 = \dot{\hat{\mathbf{r}}}_3 = \frac{\dot{\hat{\mathbf{r}}}_1 \times \dot{\hat{\mathbf{r}}}_2}{\| \dot{\hat{\mathbf{r}}}_1 \times \dot{\hat{\mathbf{r}}}_2 \|}
\]  

(15)

There is a unique orthonormal direction-cosine matrix \( A \) which satisfies the relation given by:

\[
\hat{r}_1 = \hat{W}_1 \quad \hat{r}_2 = \left( \hat{W}_1 \times \hat{W}_2 \right)/\| \hat{W}_1 \times \hat{W}_2 \| \quad \hat{r}_3 = \left( \hat{W}_1 \times \left( \hat{W}_1 \times \hat{W}_2 \right) \right)/\| \hat{W}_1 \times \hat{W}_2 \|
\]

(15)

The algorithm also develops a simple analytical expression for the covariance matrix of the TRIAD attitude solution. The conventional attitude covariance matrix is defined as the covariance matrix of the Euler parameterisation angles. This is very heavy to calculate and sometimes less useful than the covariance matrix referred to a set of angles in the body fixed system. The procedure shows the formalism for the body-referenced covariance matrix and the correlation to the covariance matrix with Euler angles. The error angle vector is a set of angles, transforming the real attitude into the measured attitude, defined as:

\[
\delta \theta = [\delta \theta_1 \delta \theta_2 \delta \theta_3]^T
\]  

(19)

thus, assuming the error in the angle to be small, for first order linearisation:

\[
A = \left[ \begin{array}{ccc} 1 & \delta \theta_3 & -\delta \theta_2 \\ -\delta \theta_3 & 1 & \delta \theta_1 \\ \delta \theta_2 & -\delta \theta_1 & 1 \end{array} \right] < A >
\]

(20)

where \(< A >\) indicates the expected value. The covariance matrix associated with the attitude matrix \( A \) is:

\[
P = < \delta A \delta A^T >
\]

(21)

where the error on the attitude matrix can be expressed as:

\[
\delta A = A - < A >
\]

(22)

It can be shown that the Cartesian attitude covariance matrix is given by:

\[
P_{\delta \theta} = \left( \frac{1}{2} \text{trace}(P) \right) I - P
\]

(23)
\[ P = P_{\text{obs}} + A P_{\text{ref}} A^T \] (24)

The desired expression of the Cartesian attitude covariance matrix, in terms of the observation vectors, is:

\[
P_{\vartheta \vartheta} = \sigma_1^2 I + \frac{1}{|\hat{W}_1 \times \hat{W}_2|^2} \left[ \left( \sigma_1^2 - \sigma_2^2 \right) \hat{W}_1 \hat{W}_1^T + \sigma_1^2 \left( \hat{W}_1 \cdot \hat{W}_2 \right) \left( \hat{W}_1 \hat{W}_2^T + \hat{W}_2 \hat{W}_1^T \right) \right] \] (25)

\[
\sigma_1^2 = \sigma_{v_1}^2 + \sigma_{w_1}^2 \quad \sigma_2^2 = \sigma_{v_2}^2 + \sigma_{w_2}^2
\] (26)

where \( \sigma_2^2 \) is the variance of a component of \( \hat{X} \) along the component normal to \( \langle \hat{X} \rangle \). It should be highlighted that even if usually the covariance matrix is computed in terms of Euler angles, the expression of the Cartesian body-referenced error covariance \( P_{\vartheta \vartheta} \) is more interesting, in terms of evaluating the reliability of the three-axis attitude determination.

6. RESULTS

A test facility has been set up to evaluate the algorithm performance. Different lengths have been considered in order to evaluate its consistency relating to multipath effects. The separation between the antennas was respectively 160 cm, 100 cm, and 40 cm (Figure 2). In the first two cases, the antennas are mounted on external appendages, in the last one they are directly placed on the mock-up of the satellite.

![Figure 2 Baseline](image)

The receivers used were GARMIN GPS 25 LP-Series with 12 channels. The real radiated GPS signals provided from the receivers have been recorded and used for off-line simulations. The raw data from the receivers are sampled at 1 Hz. In Figure 3 the dispersions of the point made by the receiver for the master and slave antennas are presented. That gives an indication on the performance of the receiver employed during the simulations.

In order to run the algorithm, at least three sightlines are required: this condition is verified by the GPS constellation geometry, that ensures a minimum of four GPS is in sight at any time. Moreover test conditions require that the same three GPS sightlines are visible for the whole duration of the acquisition, until the solution is found. During the collection period the software should keep track of the cycle, in order to avoid cycle slip. In this manner the integers to be determined remain constant throughout the entire data span.
For a set of coplanar baselines, the algorithm considers three sightlines and a baseline at a time. The integer computation for each baseline is completely independent from the others. Table 1 summarizes the fundamental characteristics of each test. As seen, the geometric inequality represented by Equation (4) is used to significantly reduce the search space. The table refers to a single sightline and shows the dimension of the reduced integers subsets, obtained applying the inequality to the sightlines $s_1$ and $s_2$. This space can be further reduced considering also the third sightline $s_3$. While executing, the algorithm sums the loss function in Equation (5) over time for a period corresponding to L epochs.

<table>
<thead>
<tr>
<th>Test</th>
<th>Length [cm]</th>
<th>Integer Range</th>
<th>Full Search Combinations: $k^3$</th>
<th>Reduced Subset</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>160</td>
<td>-8 to 8</td>
<td>4096</td>
<td>2096</td>
<td>48.8</td>
</tr>
<tr>
<td>B</td>
<td>100</td>
<td>-5 to 5</td>
<td>1000</td>
<td>460</td>
<td>54</td>
</tr>
<tr>
<td>C</td>
<td>40</td>
<td>-2 to 2</td>
<td>64</td>
<td>16</td>
<td>75</td>
</tr>
</tbody>
</table>

Table 1 Integer Ambiguity Resolution results associated with different baseline lengths

The procedure converges when three times the square root of every diagonal element of the covariance error matrix P in Equation (11) is less than 0.5. The use of standard GPS receivers has highlighted the limitations of the system. Since the standard software provided with the receiver outputs the acquisition time with a resolution of $10^{-8}$ seconds, being the period of the signal $T=10^{-10}$ seconds, the synchronization between the two receivers is not feasible with the required precision. This means that a loss in cycles acquisition occurs. The consequence is that the actual recorded data can be used to run the geometric inequality to show that an effective reduction of the search space can take place. On the other hand the integer ambiguity resolution is so strongly affected by the not synchronous phase acquisition, that the attained results are not reliable for practical attitude determination.

As shown in Table 2, different values of L have been considered during the test campaign. The second column describes the convergence step; the third column contains the number of CPU flops required to run the algorithm.
<table>
<thead>
<tr>
<th>L [sec]</th>
<th>Convergence Step</th>
<th>Flops \cdot 10^6</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>6</td>
<td>11.299</td>
</tr>
<tr>
<td>30</td>
<td>2</td>
<td>22.164</td>
</tr>
<tr>
<td>60</td>
<td>1</td>
<td>43.893</td>
</tr>
<tr>
<td>100</td>
<td>1</td>
<td>72.893</td>
</tr>
</tbody>
</table>

Table 2 Convergence performance

7. CONCLUSIONS

This work addressed the first two steps of the attitude determination problem and attitude
initialisation from vector observation. The integer ambiguity resolution algorithm includes the
advantages of both instantaneous and motion-based technique. First it uses an instantaneous
static approach to reduce the search space. Then a candidate solution has been found
associated with the minimum value of a batch-type loss function. The result is with an
integrity check on the error covariance matrix. Several are the advantages of the algorithm.
First, it works even for coplanar baselines. Second, the procedure is attitude-independent.
Third, the algorithm avoids large matrix inversion. Finally, it uses a non-iterative procedure
and provides an expression for the error covariance matrix. The procedure is strictly
correlated to the relative position of the sightlines used; due to the geometric constraints it is
not always possible to find a solution. Once the integers for three sightlines and two baselines
are solved, the attitude can be determined and used to find the integers associated with the
remaining sightlines.

Test sessions have been performed on the real spacecraft model. The analysis has highlighted
a strong correlation between the solution and the clock synchronization. The number of flops
has been shown, in order to evaluate the computational cost of the procedure.

The results will be the starting point for the attitude determination algorithm applied to the
coplanar baseline array [6]. The deal is to get reliable attitude information from a set of
antennas linked to only one receiver by means of a dedicated splitter. This step will highlight
the performance degradation of the system related to the baseline length, limits in terms of
sample frequency and synchronization problems.

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