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Trajectory optimization and real-time simulation for robotics applications

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The dynamic equations

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The multi-body equations of the robot are:

$$\boldsymbol{F}(\boldsymbol{x}(t), \dot{\boldsymbol{x}}(t), \boldsymbol{z}(t), \boldsymbol{u}(t), \boldsymbol{p}) = 0$$
$$\boldsymbol{G}(\boldsymbol{x}(t), \boldsymbol{p}) = 0$$

where:

- $\boldsymbol{x}(t)$ and $\boldsymbol{z}(t)$ are the states
- $\boldsymbol{u}(t)$ and $\boldsymbol{p}$ are the system inputs

The trajectory optimization problem

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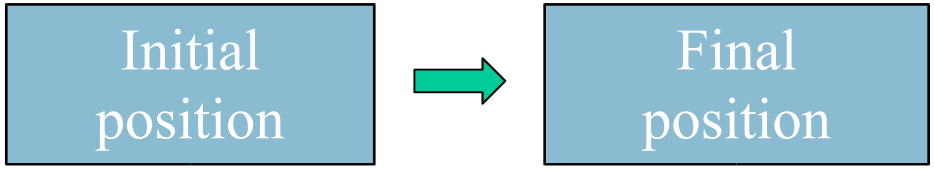
Find a $\boldsymbol{u}(t)$ control input and a $\boldsymbol{p}$ value in order to move the robot :



- Minimizing a suitable cost function.
- Satisfying some constraints over the trajectory:

$$\boldsymbol{C}(\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{p}) \geqslant 0$$

The trajectory optimization problem

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The cost function has the form:

$$f(\boldsymbol{u}(t), T) = \int_0^T (d_1 + d_2 \boldsymbol{v}(t)^T \boldsymbol{v}(t))\, dt$$

where

$$\boldsymbol{v}(t) = \int_0^t w(t-\tau)\boldsymbol{u}(\tau)\, dt$$

and $w(t)$ is the impulse response of a suitable filter $W(s)$.

In this work, a high pass filter is used, in order to have a control law with reduced energy at high frequency.

From the continuous to the discrete problem

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A direct method is used:

- The system input $\boldsymbol{u}(t)$ must be discretized over a time grid:
  - The discrete values become unknowns of the optimization problem
  - The continuous behavior is obtained by polynomial interpolation. The interpolation order can change at each time interval.
- The dynamics equations are integrated by MBDyn multi-body solver through a “shooting” procedure
- The constraints $\boldsymbol{C}(\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{p}) \geqslant 0$ are sampled:
  - Any continuous constraint generates many discrete constraints at different times.
  - Not all the integration times can be used because the problem becomes too large.
  - An heuristic algorithm selects only those times where the constraints are about to be violated.

From the continuous to the discrete problem

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The problem unknowns are:

- the final time T
- the discrete values of the j-th control input $\boldsymbol{u}_j$
- the constant system input **p**

$$\boldsymbol{y} = \begin{Bmatrix} T \\ \vdots \\ \boldsymbol{u}_j \\ \vdots \\ \boldsymbol{p} \end{Bmatrix}$$

From the continuous to the discrete problem

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An SQP algorithm is used to solve the optimal problem:

- Harwell VF13 solver
- Iterative solution
- The following differential quantities are needed:
  - ∇f
  - the constraints Jacobian $\boldsymbol{J}$

The derivatives are computed through central finite difference:

- 2n+1 dynamic equations integrations are needed at each optimization step, where n is the unknowns number.

The optimization algorithm

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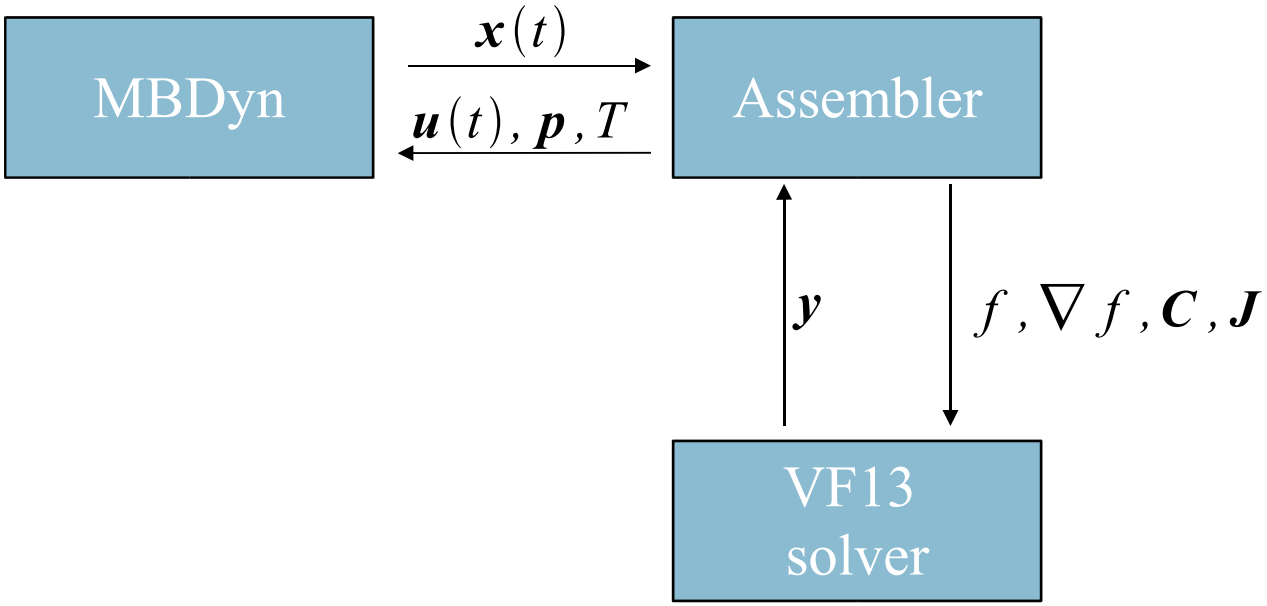

The optimization algorithm

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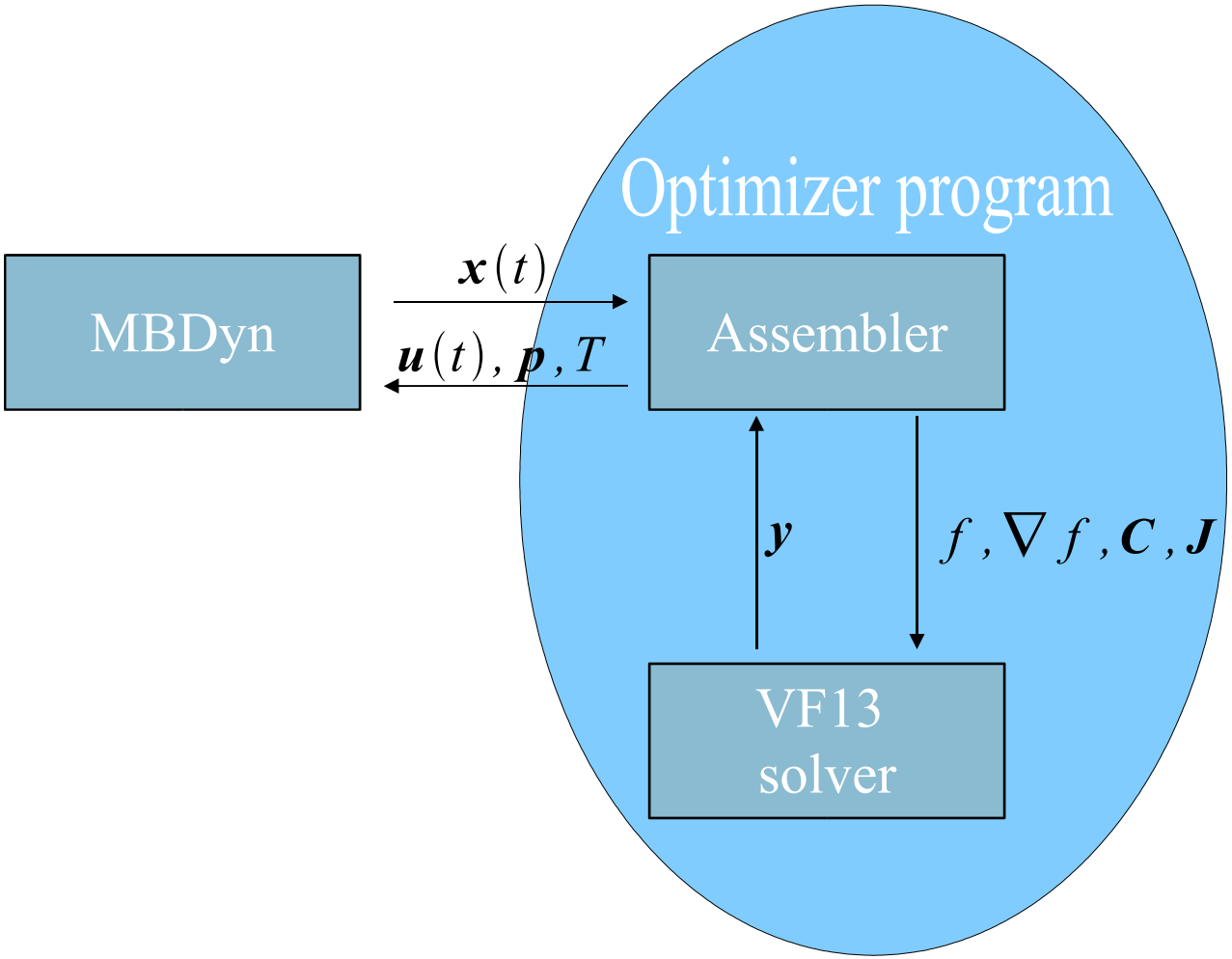

Problem adaptation

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When an optimal solution is found, a problem adaptation can be performed:

The control $\boldsymbol{u}(t)$ are adapted:

- some interpolation points are inserted or deleted in the discretization grid
- the most appropriate polynomial interpolation function is selected in a given interval of the analysis time

A new optimization can be performed starting from the former solution

MBDyn Overview

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Index 2/3 Differential-Algebraic, Initial-Value problem solver

$$
\begin{aligned}
M\,\dot{x} - \beta &= 0 \\
\dot{\beta} - F(x, \dot{x}, t) + \lambda &= 0 \\
\Phi(x, \dot{x}, t) &= 0
\end{aligned}
$$

Algebraic kinematic constraints; e.g.

- revolute joints

Multidisciplinary problems capability:

- aeroelasticity
- electric components
- hydraulic components

Implicit integration by means of second-order A/L stable scheme

MBDyn Overview

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Free software project:

http://www.aero.polimi.it/~mbdyn/

Developed at the “Dipartimento di Ingegneria Aerospaziale” of the University “Politecnico di Milano”

General purpose:

- parallel/multithread linear/nonlinear solvers
- distributed Real-Time enabled by RTAI/RT-Net/RTAILab
- interface to arbitrary CFD solvers for aeroelastic analysis

Algorithm validation

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The validation has been made by means of problems with analytical solutions:

- Material point linear movement thrust through a force, with various constraints.
- The found solutions agree with the analytical ones

Application:
Two arms robot

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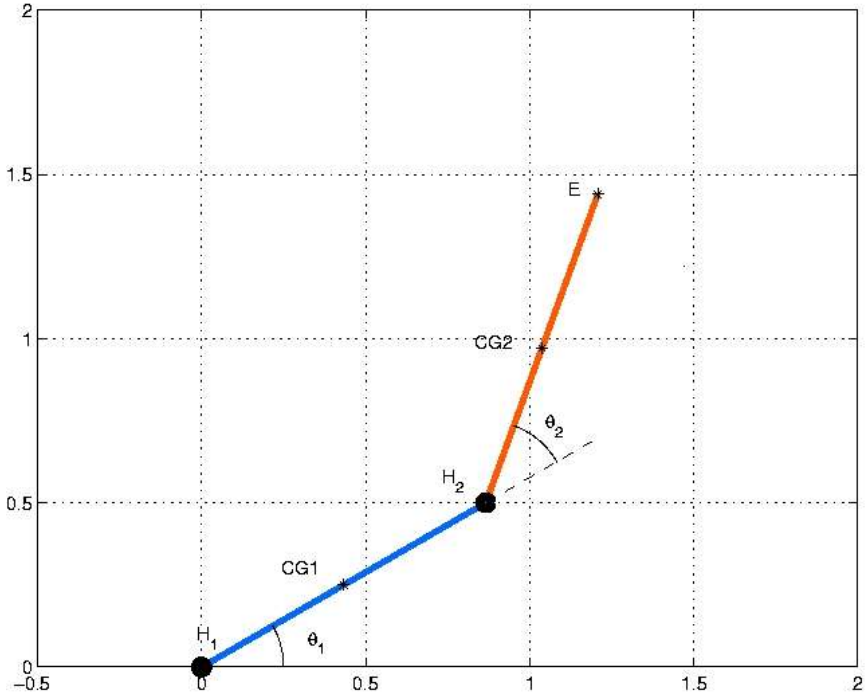
The robotic arm is a two degree of freedom planar robot

The system inputs are the couples C_1 , C_2 applied at the two hinges.

The couples can vary between -1 and 1 whereas the hinge angles θ_1 , θ_2 between -135 and 135 degrees



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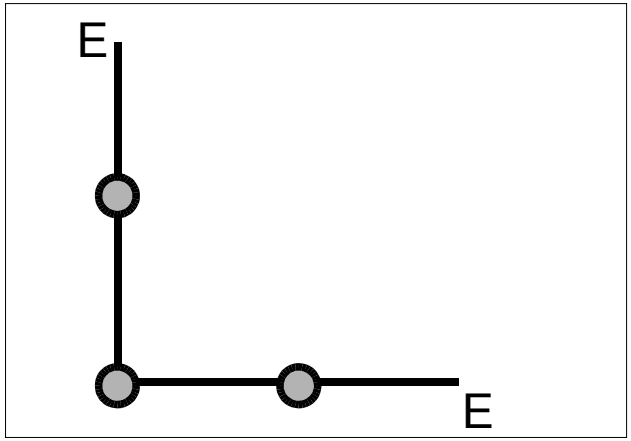

Robot geometry and mass:

- Link 1:

  mass 3.9 kg

  length 1.0 m

- Link 2:

  mass 0.685 kg

  length 1.0 m

Path conditions:

- The end effector initial position is $x_e = 2, y_e = 0$
- The final position is $x_e = 0, y_e = 2$
- At the initial and final position the robot is motionless.

The cut-off frequency of the filter is 8Hz

Application:
Two arms robot

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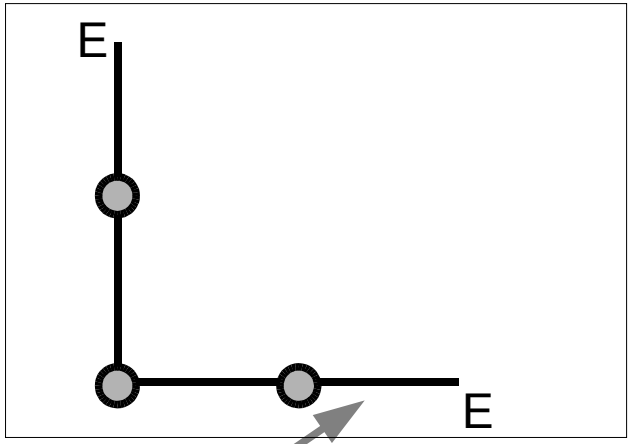

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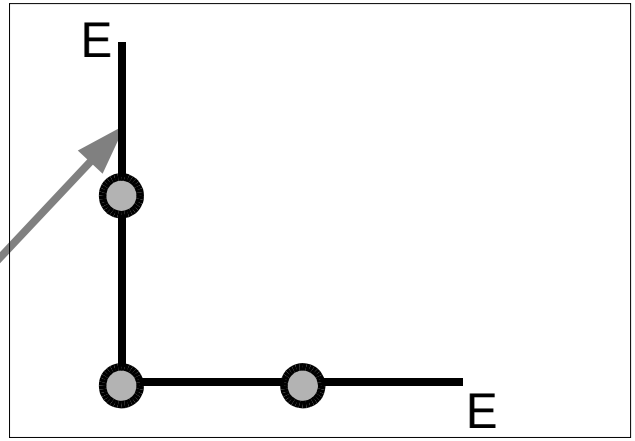

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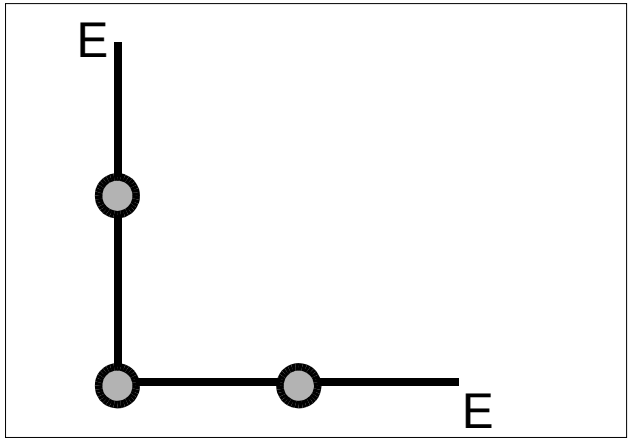

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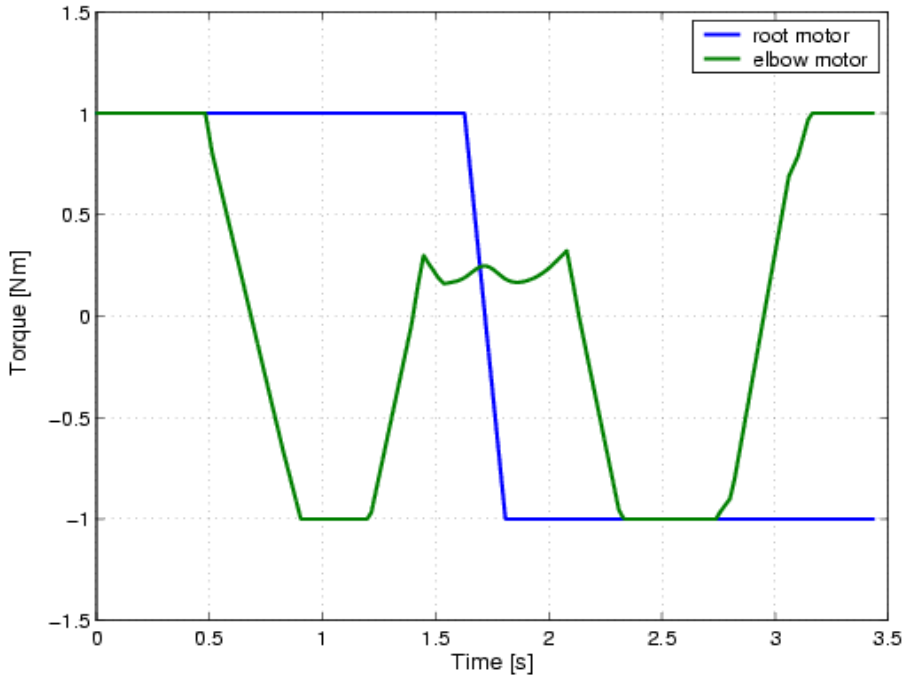
Results:

- The optimal solution is found after:
  - 233 iterations
  - 16633 MBDyn runs
  - 5 control adaptations
- The traveling time is 3.436s



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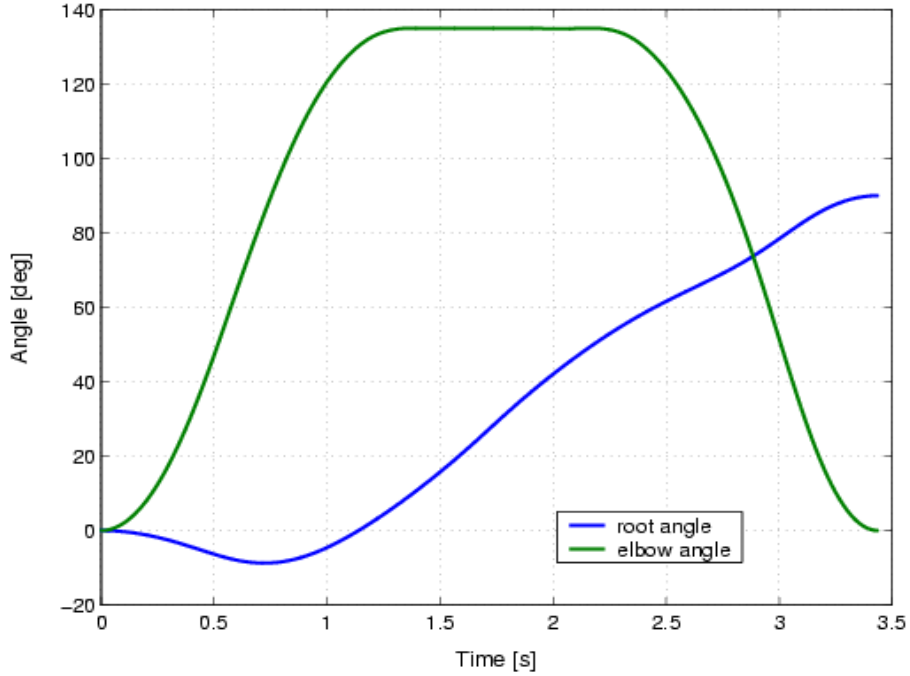
The hinge angles are:



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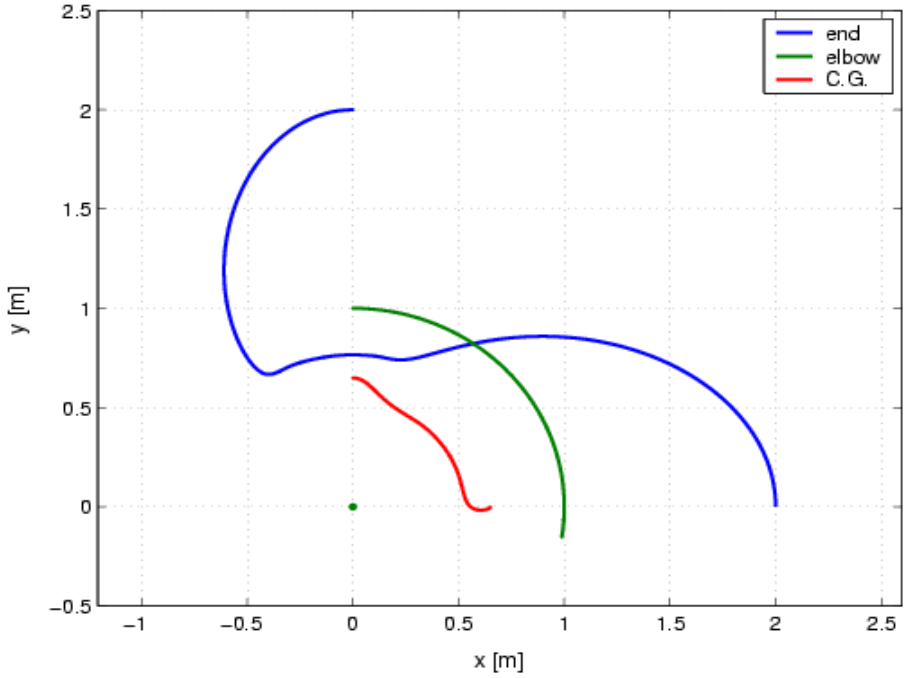
... and the resulting path is:



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Then the optimal solution is proved with a more sophisticated model:

- The two link are flexible:
  - First link: iron, thickness 5.0mm
  - Second link: aluminium alloy, thickness 2.5mm
- A feedback control is needed:
  - The torque applied at each hinge becomes the sum of the feedback control output and the optimal solution

Application:
Two arms robot

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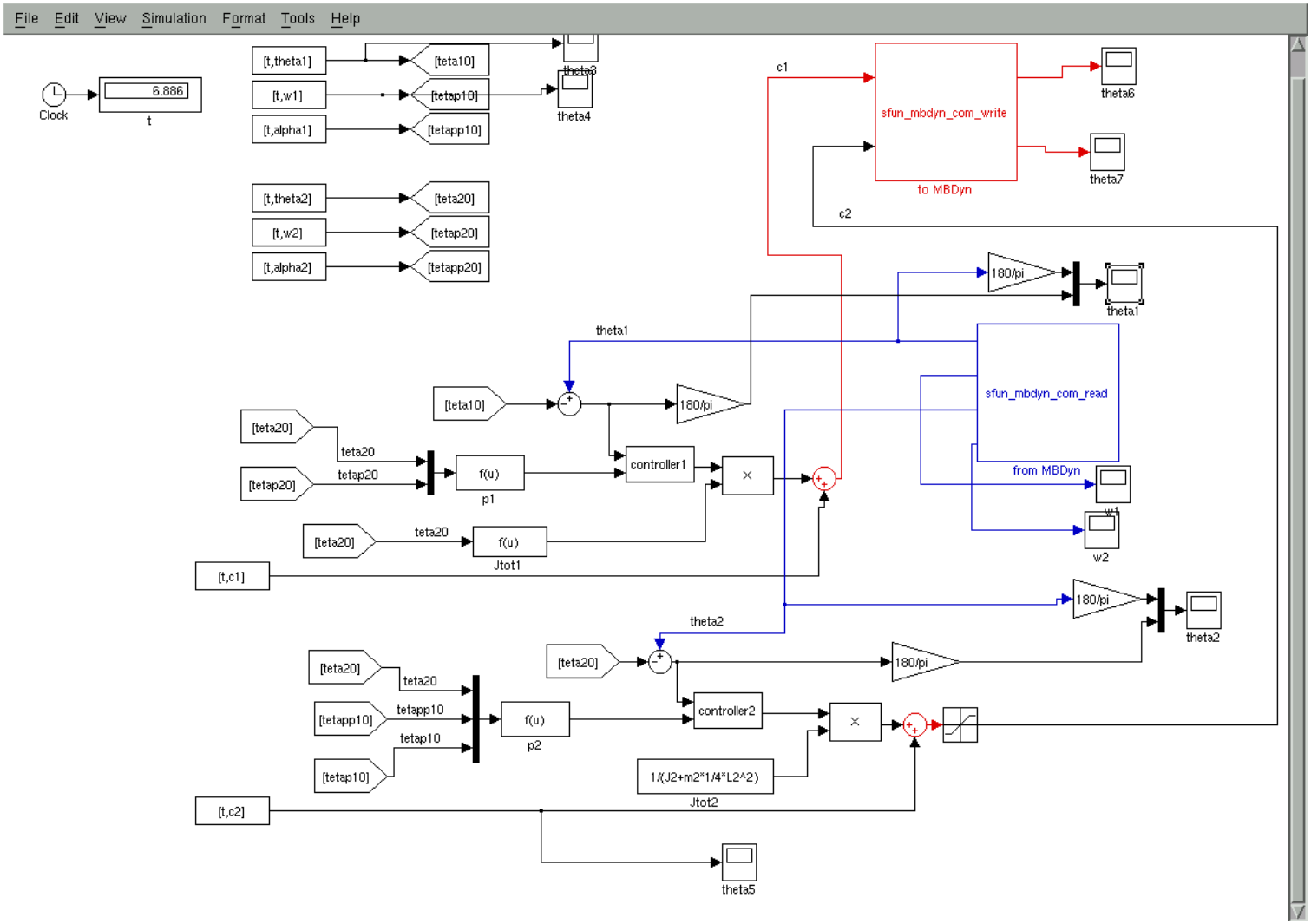

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Application:
Two arms robot

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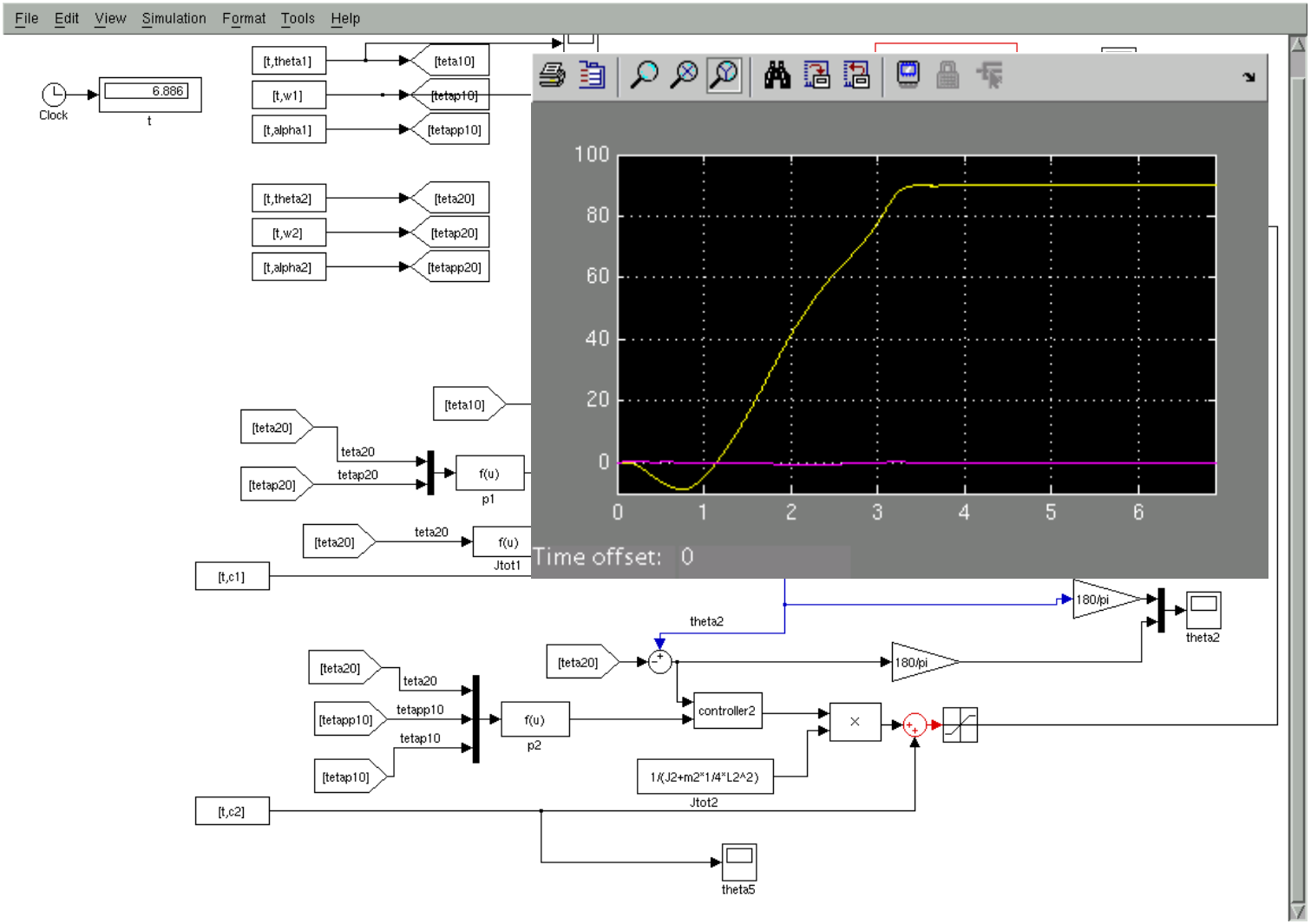

Application:
Two arms robot

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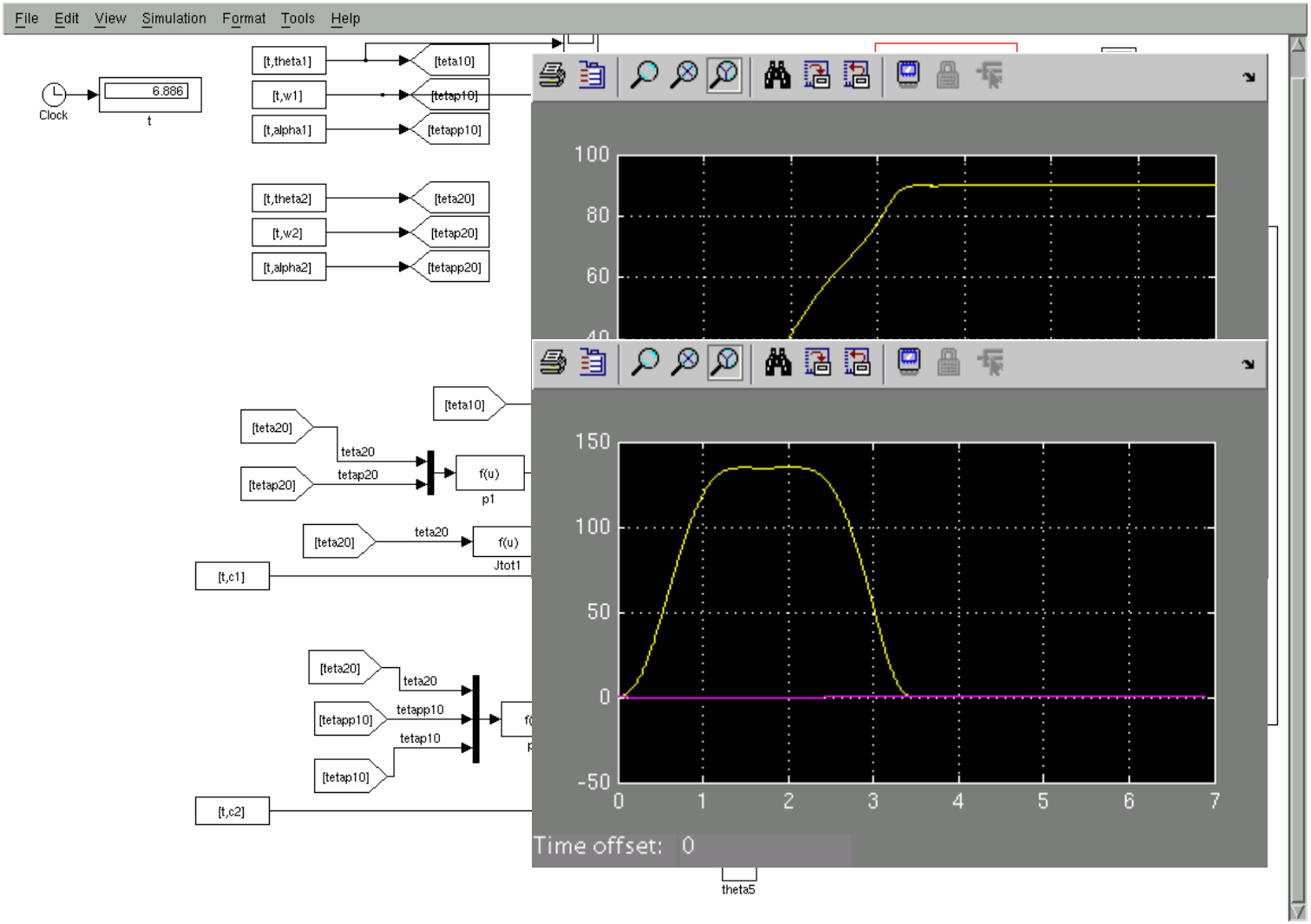

Application:
Two arms robot

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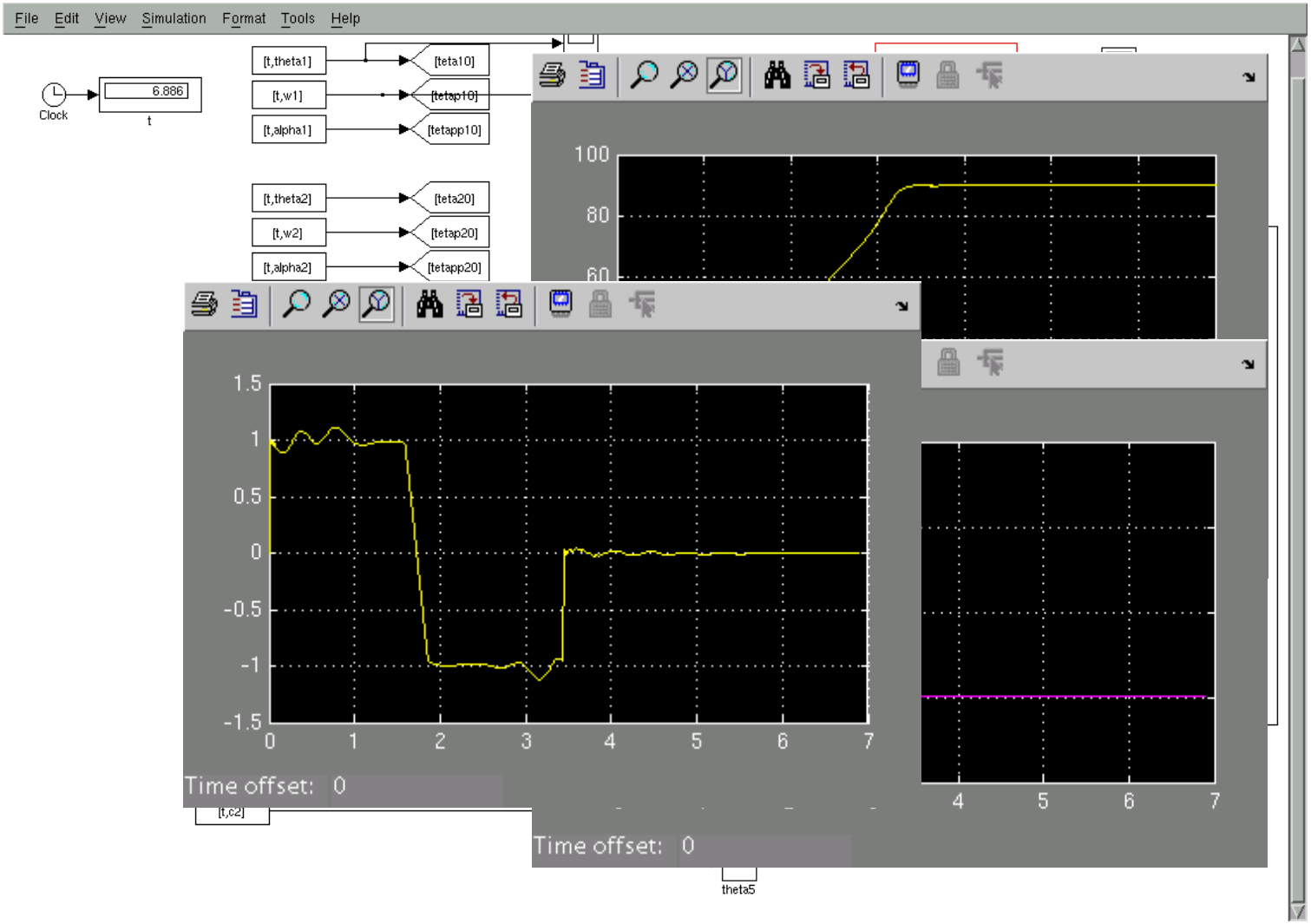

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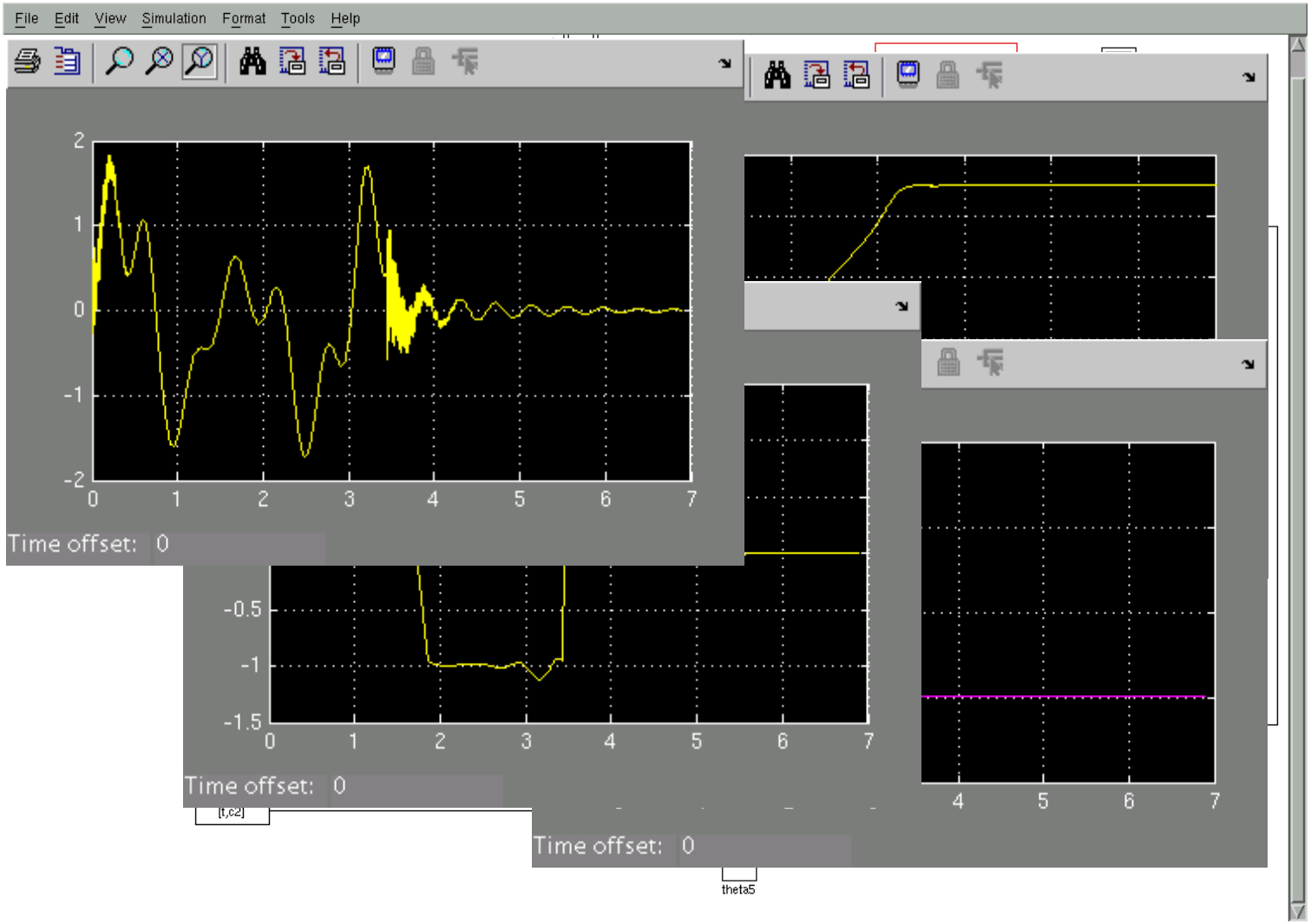

Real-Time Simulation

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MBDyn allows real-time simulation under Linux

Real-Time Application Interface (RTAI) http://www.rtai.org/

Advantages:

- same software for rather different purposes
- same models/model components; no modeling limitations

Drawbacks:

- “large” models (redundant set) => sample rate limitations
- no theoretical guaranteed upper bound to worst case time

Good performances obtained so far

- 6 dof robot with friction, 120 eq.: >2 kHz on Athlon 2.4 GHz

Real-Time Simulation (cont.)

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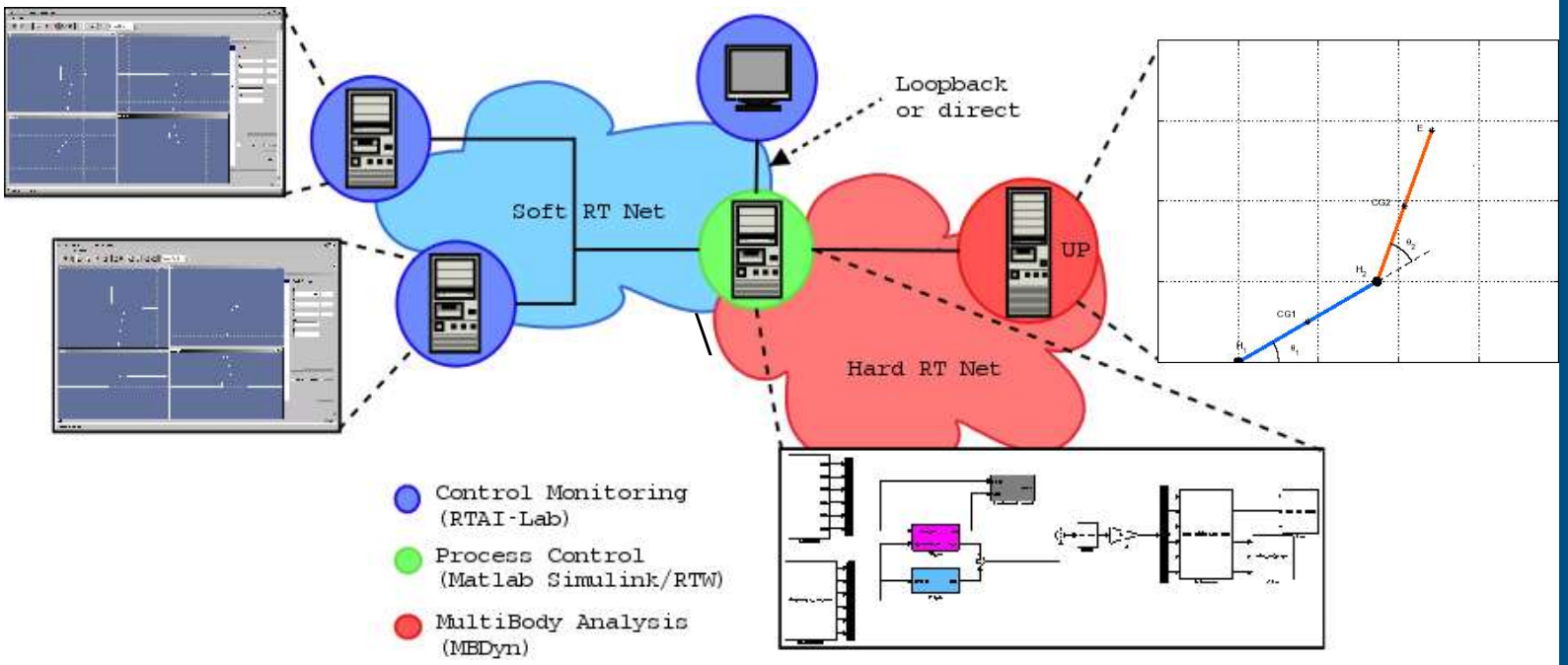
Distributed Real-Time simulation:

- Multibody Analysis
- Control
- Monitoring



Conclusions

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The optimization using a MBDyn software is a powerful and versatile tool.

It is possible to verify the optimal solution with the same model in a complete control scheme

Further development:

- Optimization with flexible models