Trajectory optimization and real-time simulation for robotics applications

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1. The trajectory optimization problem.


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4. The real-time simulation.
The dynamic equations

The multi-body equations of the robot are:

\[
F(x(t), \dot{x}(t), z(t), u(t), p) = 0
\]
\[
G(x(t), p) = 0
\]

where:

\begin{itemize}
  \item \( x(t) \) and \( z(t) \) are the states
  \item \( u(t) \) and \( p \) are the system inputs
\end{itemize}
The trajectory optimization problem

Find a $u(t)$ control input and a $p$ value in order to move the robot:

- Minimizing a suitable cost function.
- Satisfying some constraints over the trajectory:

$$C(x(t), u(t), p) \geq 0$$
The trajectory optimization problem

The cost function has the form:

\[
f(u(t), T) = \int_0^T (d_1 + d_2 v(t)^T v(t)) \, dt
\]

where

\[
v(t) = \int_0^t W(t - \tau) u(\tau) \, d\tau
\]

and \( W(t) \) is the impulse response of a suitable filter \( W(s) \).

In this work, a high pass filter is used, in order to have a control law with reduced energy at high frequency.
From the continuous to the discrete problem

A direct method is used:

- The system input $u(t)$ must be discretized over a time grid:
  - The discrete values become unknowns of the optimization problem
  - The continuous behavior is obtained by polynomial interpolation. The interpolation order can change at each time interval.
- The dynamics equations are integrated by MBDyn multi-body solver through a “shooting” procedure
- The constraints $C(x(t), u(t), p) \geq 0$ are sampled:
  - Any continuous constraint generates many discrete constraints at different times.
  - Not all the integration times can be used because the problem becomes too large.
  - An heuristic algorithm selects only those times where the constraints are about to be violated.
From the continuous to the discrete problem

The problem unknowns are:

- the final time $T$
- the discrete values of the $j$-th control input $u_j$
- the constant system input $p$

$$y = \begin{pmatrix} T \\ \vdots \\ u_j \\ \vdots \\ p \end{pmatrix}$$
From the continuous to the discrete problem

An SQP algorithm is used to solve the optimal problem:

- Harwell VF13 solver
- Iterative solution
- The following differential quantities are needed:
  - $\nabla f$
  - the constraints Jacobian $J$

The derivatives are computed through central finite difference:

- $2n+1$ dynamic equations integrations are needed at each optimization step, where $n$ is the unknowns number.
The optimization algorithm

MBDyn → Assembler

\[ x(t), u(t), p, T \]

VF13 solver

\[ f, \nabla f, C, J \]
The optimization algorithm

MBDyn

Optimizer program

Assembler

VF13 solver

\[ f, \nabla f, C, J \]

\[ x(t), u(t), p, T \]
Problem adaptation

When an optimal solution is found, a problem adaptation can be performed:

The control \( u(t) \) are adapted:

- some interpolation points are inserted or deleted in the discretization grid
- the most appropriate polynomial interpolation function is selected in a given interval of the analysis time

A new optimization can be performed starting from the former solution
MBDyn Overview

Index 2/3 Differential-Algebraic, Initial-Value problem solver

\[
M \dot{x} - \beta = 0 \\
\dot{\beta} - F(x, \dot{x}, t) + \lambda = 0 \\
\Phi(x, \dot{x}, t) = 0
\]

Algebraic kinematic constraints; e.g.
- revolute joints

Multidisciplinary problems capability:
- aeroelasticity
- electric components
- hydraulic components

Implicit integration by means of second-order A/L stable scheme
MBDyn Overview

Free software project:

http://www.aero.polimi.it/~mbdyn/

Developed at the “Dipartimento di Ingegneria Aerospaziale” of the University “Politecnico di Milano”

General purpose:

• parallel/multithread linear/nonlinear solvers
• distributed Real-Time enabled by RTAI/RT-Net/RTAILab
• interface to arbitrary CFD solvers for aeroelastic analysis
Algorithm validation

The validation has been made by means of problems with analytical solutions:

- Material point linear movement thrust through a force, with various constraints.

- The found solutions agree with the analytical ones
Application: Two arms robot

The robotic arm is a two degree of freedom planar robot.

The system inputs are the couples $C_1$, $C_2$ applied at the two hinges.

The couples can vary between -1 and 1 whereas the hinge angles $\theta_1$, $\theta_2$ between -135 and 135 degrees.
Application:
Two arms robot

Robot geometry and mass:
- Link 1:
  - mass 3.9 kg
  - length 1.0 m
- Link 2:
  - mass 0.685 kg
  - length 1.0 m

Path conditions:
- The end effector initial position is $x_e = 2, y_e = 0$
- The final position is $x_e = 0, y_e = 2$
- At the initial and final position the robot is motionless.

The cut-off frequency of the filter is 8Hz
Application: Two arms robot

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Results:

- The optimal solution is found after:
  - 233 iterations
  - 16633 MBDyn runs
  - 5 control adaptations
- The traveling time is 3.436s
Application:
Two arms robot

The hinge angles are:
Application:
Two arms robot

... and the resulting path is:
Application: Two arms robot

Then the optimal solution is proved with a more sophisticated model:

- The two links are flexible:
  - First link: iron, thickness 5.0mm
  - Second link: aluminium alloy, thickness 2.5mm
- A feedback control is needed:
  - The torque applied at each hinge becomes the sum of the feedback control output and the optimal solution
Application:
Two arms robot
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Real-Time Simulation

MBDyn allows real-time simulation under Linux
Real-Time Application Interface (RTAI) http://www.rtai.org/

Advantages:
- same software for rather different purposes
- same models/model components; no modeling limitations

Drawbacks:
- “large” models (redundant set) => sample rate limitations
- no theoretical guaranteed upper bound to worst case time

Good performances obtained so far
- 6 dof robot with friction, 120 eq.: >2 kHz on Athlon 2.4 GHz
Real-Time Simulation (cont.)

Distributed Real-Time simulation:
- Multibody Analysis
- Control
- Monitoring
Conclusions

The optimization using a MBDyn software is a powerful and versatile tool.

It is possible to verify the optimal solution with the same model in a complete control scheme.

Further development:

• Optimization with flexible models