SECTION B3

SPRINGS

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B 3.0.0 SPRINGS

B 3.1.0 Helical Springs

B 3.1.1 Helical Compression Springs

Most compression springs are open-coil, helical springs which offer resistance to loads acting to reduce the length of the spring. The longitudinal deflection of springs produces shearing stresses in the spring wire. Where particular load-deflection characteristics are desired, springs with varying pitch diameters may be used. These springs may have any number of configurations, including cone, barrel, and **hourglass**.

Round-Wire Springs

The relation between the applied load and the shearing stress for helical springs formed from round wire is

$$f_s = \frac{8PD}{\pi d^3} \quad \dots\dots\dots \quad (1)$$

where

- f_s = shearing stress in pounds per sq. inch. (not corrected for curvature)
- P = axial load in pounds
- D = mean diameter of the spring coil (Outside diameter minus wire diameter or inside diameter plus wire diameter)
- d = diameter of the wire in inches.

Equation (1) is based on the assumption that the magnitude of the stress varies directly with the distance from the center of the wire; but, actually, the stress is greater on the inside of the cross section due to the curvature. The stress correction factor (k) used to determine the maximum shearing stress for static loads is found in Fig. B 3.1.1-1. This correction factor gives the effect of both torsion and direct shear. The equation for the maximum stress is

$$f_{max} = k\, f_s = k \frac{8PD}{\pi d^3} \quad \dots\dots\dots \quad (2)$$

B 3.1.1 Helical Compression Springs (Cont'd)

where

$k = k_c + \frac{0.615}{C_1}$	Stress concentration factor plus the effect of direct shear
$k_c = \frac{4C_1 - 1}{4C_1 - 4}$	Stress concentration factor due to curvature
$C_1 = \frac{D}{d}$	Ratio of mean diameter of helix to the diameter of the bar or wire

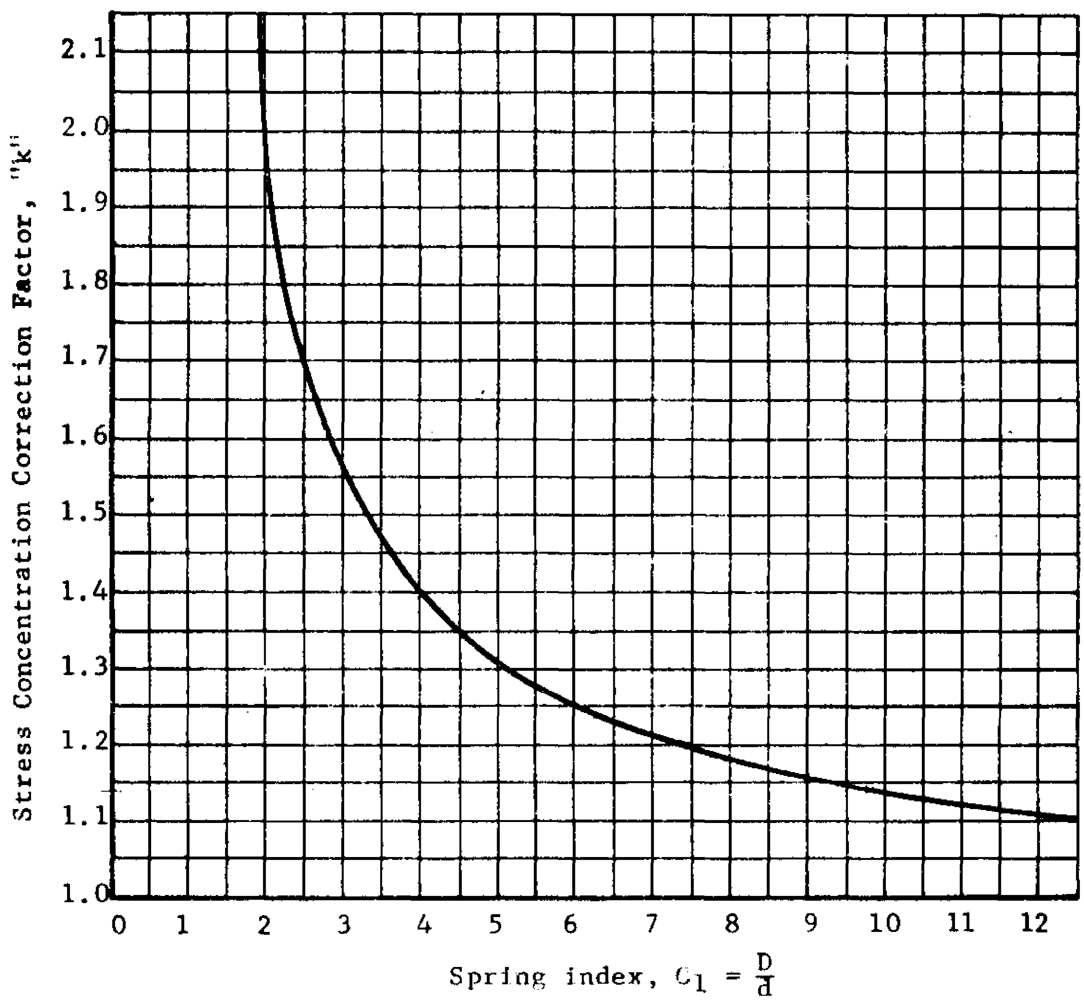


Fig. B 3.1.1-1

B 3.1.1 Helical Compression Springs (Cont'd)

Stress correction for temperature.

Corrections must be made to account for changes in strength and in elastic properties of spring materials at elevated temperatures. This correction is made to the allowable stress of the spring material. Values for various materials at various temperatures may be obtained from Section 3.0.0 of the Design Manual.

Deflection

The formula for the relation between deflection and load when using round wire in helical springs is

$$\delta = \frac{8NPD^3}{Gd^4} \quad \ldots\ldots \quad (3)$$

where

δ = total deflection
N = number of coils
G = modulus of rigidity

The deflection may also be given in terms of the shearing stress by combining Eqs. (1) and (3). Stress concentration usually does not affect deflection to an appreciable degree, and no adjustment is needed in Eq. (1). The expression for the deflection is

$$\delta = \frac{N f_s \pi D^2}{Gd} \quad \ldots\ldots \quad (4)$$

Spring rate

The spring rate (K) is defined as the amount of force required to deflect the spring a unit length. By proper substitution of the previous equations, the spring rate may be shown to be

$$K = \frac{d^4 G}{8ND^3} \quad \ldots\ldots \quad (5)$$

Buckling of Compression Springs

A compression spring which is long compared to its diameter will buckle under relatively low loads in the same manner as a column. However, the problem of buckling is of little consequence if the compression spring operates inside a cylinder or over a rod.

B 3.1.1 Helical Compression Springs (Cont'd)

As the critical buckling load of a column is dependent upon the end fixity at the supports, so is the critical buckling load of a spring dependent upon the fixity of the ends. In general, a compression spring with ends squared, ground, and compressed between two parallel surfaces can be considered a fixed-end spring. The following formula gives the critical buckling load.

$$P_c = JKL \qquad (6)$$

where

J = factor from Fig. B 3.1.1-2 = $\frac{\text{Deflection}}{\text{Free Length}}$
L = free length of spring
K = spring rate (See Eq. 5)

Fig. B 3.1.1-2 (curve 1) is for squared and ground springs with one end on a flat surface and the other on a ball. Curve 2 indicates buckling for a squared and ground spring both ends of which are compressed against parallel plates. This is the most common condition with which the user must contend.

Helical Springs of Rectangular Wire.

When rectangular wire is used for helical springs, the value of the shearing stress can be found by use of the equations for rectangular shafts. A stress concentration factor is applied in the usual way to compensate for the effect of curvature and direct shear. For the springs in Fig. B 3.1.1-3 (a) and (b) the stresses at points A_1 and A_2 are as follows:

$$f_s = \frac{kPR}{\alpha_1 bc^2} \quad \text{for point } A_1 \qquad (7)$$

$$f_s = \frac{kPR}{\alpha_2 bc^2} \quad \text{for point } A_2 \qquad (8)$$

values for α_1 and α_2 for various b/c ratios are found in Table B 3.1.1-1.

The stress concentration factor should be applied for point A_1 in Fig. B 3.1.1-3(a) and to point A_2 in Fig. B 3.1.1-3 (b). The stress concentration factors of Fig. B 3.1.1-1 may be used as an approximate value for rectangular wire.

B 3.1.1 Helical Compression Springs (Cont'd)

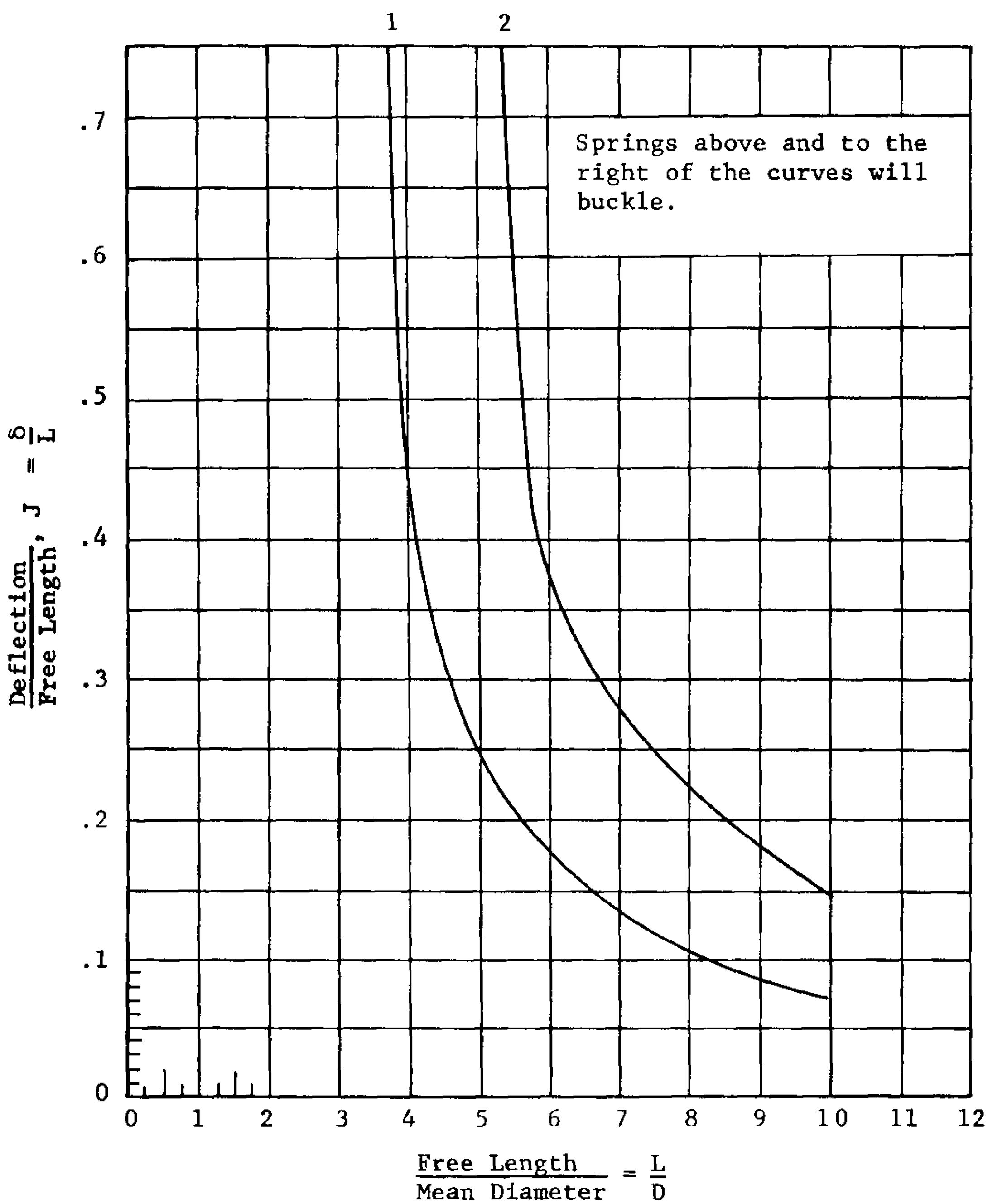
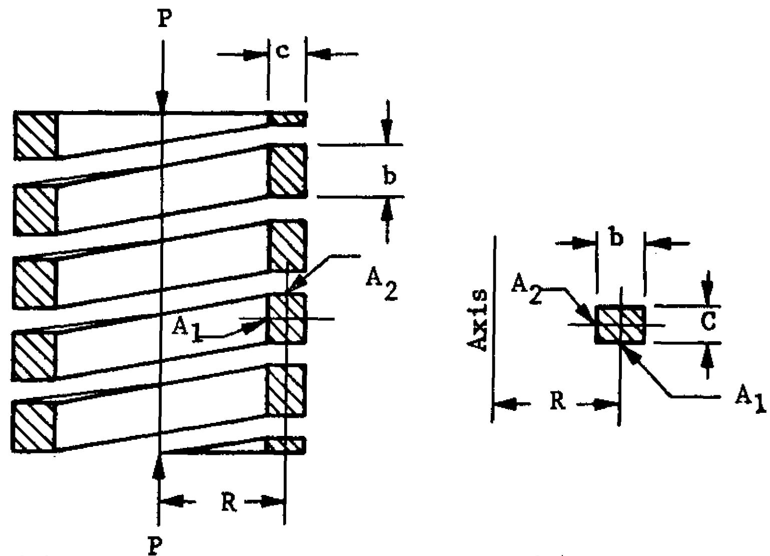


Fig. B 3.1.1-2



(a) Spring index, $C_1 = \frac{2R}{c}$

(b) Spring index, $C_1 = \frac{2R}{b}$

Fig. B 3.1.1-3

b/c	1.00	1.20	1.50	1.75	2.00	2.50	3.00	4.00	5.00	6.00	8.00	10.00	∞
α_1	.208	.219	.231	.239	.246	.258	.267	.282	.291	.299	.307	.312	.333
α_2	.208	.235	.269	.291	.309	.336	.355	.378	.392	.402	.414	.421	--
β	.1406	.166	.196	.214	.229	.249	.263	.281	.291	.299	.307	.312	.333

Table B 3.1.1-1

The equation for the relation between the load (P) and the deflection (δ) is

$$\delta = \frac{2\pi PR^3N}{\beta Gbc^3} \qquad (9)$$

where:

β is obtained from Table B 3.1.1-1
b and c are as shown in Fig. B 3.1.1-3
R is the mean radius of the spring
N is the number of coils
G is the modulus of rigidity
P is the axial load

B 3.1.2 Helical Extension Springs

Helical extension springs differ from helical compression springs only in that they are usually closely coiled helices with ends formed to permit their use in applications requiring resistance to tensile forces. It is also possible for the spring to be wound so that it is preloaded, that is, the spring is capable of resisting an initial tensile load before the coils separate. This initial tensile load does not affect the spring rate. See Figure B 3.1.2-1 for the load-deflection relationship of a preloaded helical extension spring.

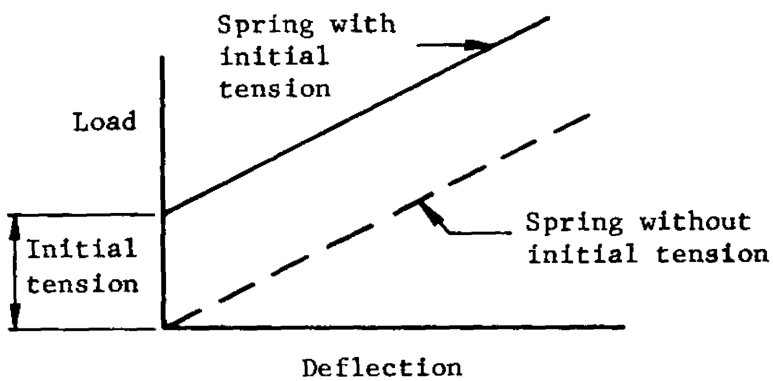


Fig. B 3.1.2-1

Stresses and Deflection

In helical extension springs, the shape of the hook or end turns for applying the load must be designed so that the stress concentration effects caused by the presence of sharp bends are decreased as much as possible. This problem is covered in the next article.

If the extension spring is designed with initial tension, formulas (1) through (9) from Section B 3.1.1 are valid, but must be applied with some understanding of the nature of the forces involved.

B 3.1.2 Helical Extension Springs (Cont'd)

Stress concentration in hooks on extension springs.

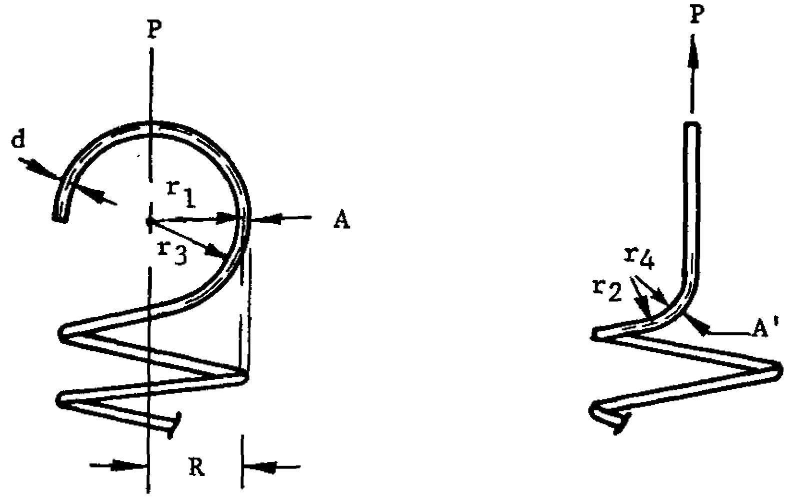


Fig. B 3.1.2-2

a. Bending Stress

Fig. B 3.1.2-2(a) illustrates the bending moment (PR) due to the load (P). The bending stress at point A is:

$$f_b = \frac{32PR}{\pi d^3} k + \frac{4P}{\pi d^2} \quad \text{.................................} \quad (10)$$

where k is the correction factor obtained from Fig. B 3.1.2-3, using the ratio $2r_1/d$.

r_1 = radius of center line of maximum curvature.

A simplified equation is:

$$f_b = \frac{32PR}{\pi d^3} \left(\frac{r_1}{r_3}\right) \quad \text{.................................} \quad (11)$$

r_3 = inside radius of bend.

The maximum bending stress obtained by this simplified form will always be on the safe side and, under normal conditions, only slightly higher than the true stress.

B 3.1.2 Helical Extension Springs (Cont'd)

b. Torsional Stress

At point A', Fig. B 3.1.2-2(b), where the bend joins the helical portion of the spring, the stress condition is primarily torsion. The maximum torsional shear stress due to the moment (PR) is

$$f_s = \frac{16PR}{\pi d^3}\left(\frac{4C_1 - 1}{4C_1 - 4}\right) \quad \text{.......................... (12)}$$

$$C_1 = \frac{2r_2}{d}$$

A simplified form similar to the one for bending is

$$f_s = \frac{16PR}{\pi d^3}\left(\frac{r_2}{r_4}\right) \quad \text{.............................. (13)}$$

This will also give safe results.

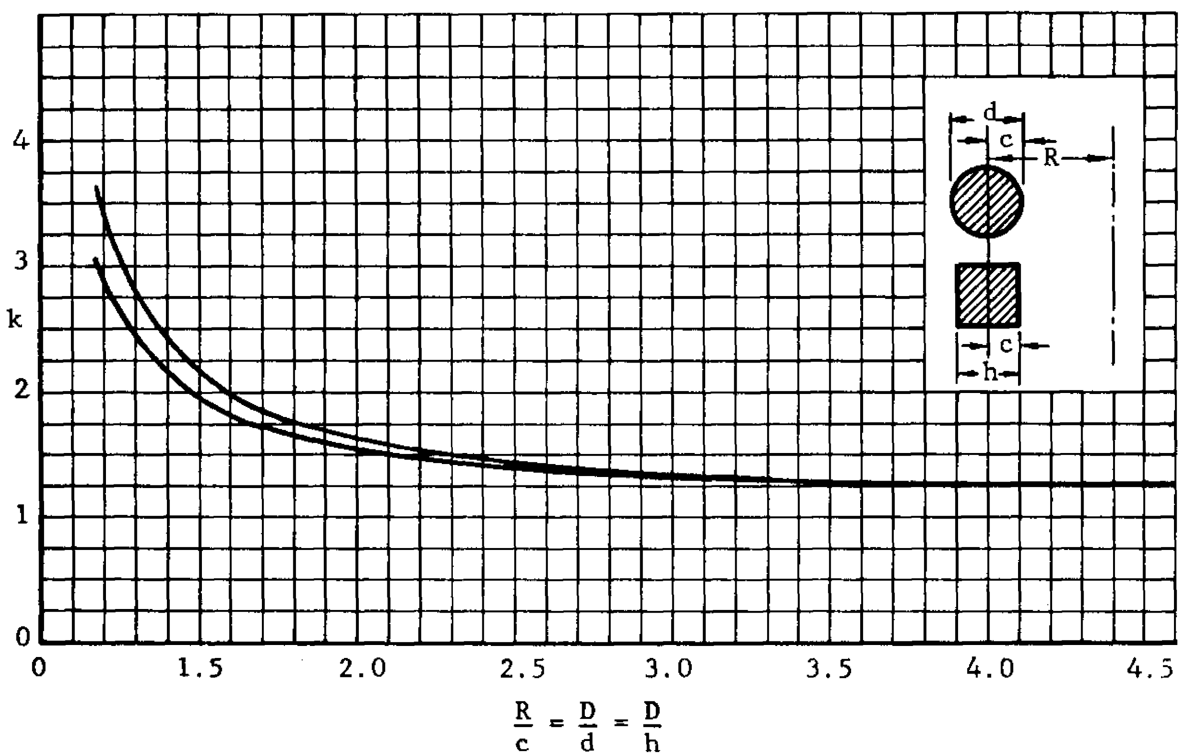


Fig. B 3.1.2-3

B 3.1.3 Helical Springs with Torsional Loading

A helical spring can be loaded by a torque about the axis of the helix. Such loading, as shown in Fig. B 3.1.3-1(a), is similar to the torsional loading of a shaft. The torque about the axis of the helix acts as a bending moment on each section of the wire as shown in Fig. B 3.1.3-1(b). The stress is then

$$f_b = k_n \frac{Mc}{I} \quad \dots\dots\dots\dots\dots\dots\dots \quad (14)$$

where the stress concentration factor, K_n, is given as

$$k_1 = \frac{3C_1^2 - C_1 - 0.8}{3C_1(C_1 - 1)} \quad \text{inner edge}$$

$$k_2 = \frac{3C_1^2 + C_1 - 0.8}{3C_1(C_1 - 1)} \quad \text{outer edge}$$

(Rectangular cross section wire)

$C_1 = \frac{2R}{h}$; "h" is the depth of section perpendicular to the axis.

$$k_3 = \frac{4C_1^2 - C_1 - 1}{4C_1(C_1 - 1)} \quad \text{inner edge}$$

$$k_4 = \frac{4C_1^2 + C_1 - 1}{4C_1(C_1 + 1)} \quad \text{outer edge}$$

(Round cross section wire)

$$C_1 = \frac{2R}{d}$$

Angular deformation

The deformation of the wire in the spring is the same as for a straight bar of the same length "S". The total angular deformation θ between tangents drawn at the ends of the bar is:

$$\theta = \frac{MS}{EI} \quad \dots\dots\dots\dots\dots\dots\dots \quad (15)$$

Angle θ in some cases may amount to a number of revolutions.

B 3.1.3 Helical Springs with Torsional Loading (Cont'd)

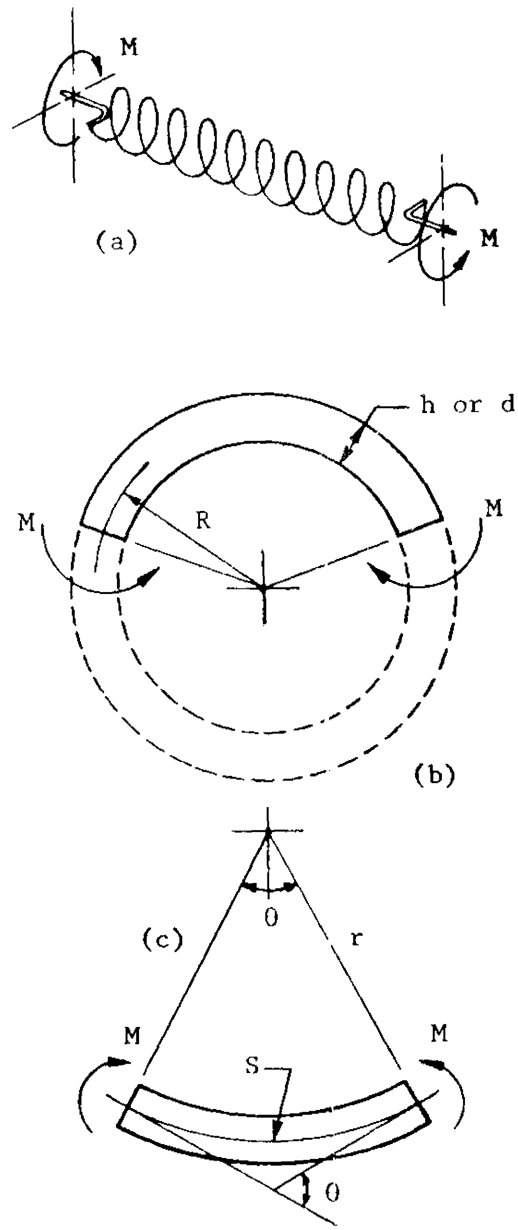


Fig. B 3.1.3-1

B 3.1.4 Analysis of Helical Springs by Use of Nomograph

The procedure for using the nomographs (Fig. B 3.1.4-1 and Fig. B 3.1.4-2) for helical springs are as follows:

1. Set the appropriate wire diam. on the "d" scale.

2. Set the appropriate mean diam. on the "D" scale.

3. Connect the two points and read the curvature correction factor:

   a. For tension and compression springs read the "y" scale on Fig. B 3.1.4-1.

   b. For torsion springs read the "k" scale on Fig. 3.1.4-2.

4. Set the correction factor, obtained in step 3, on the appropriate "y" or "k" scale to the right of Fig. B 3.1.4-1 and Fig. B 3.1.4-2.

5. Set the "calculated" (Eq. 1 or Eq. 14 with $k_n = 1$) stress on the "f" scale.

6. Connect the two points, from steps 4 and 5, and read the corrected stress on the (f') scale.

B 3.1.4 Analysis of Helical Springs by Use of Nomograph (Cont'd)

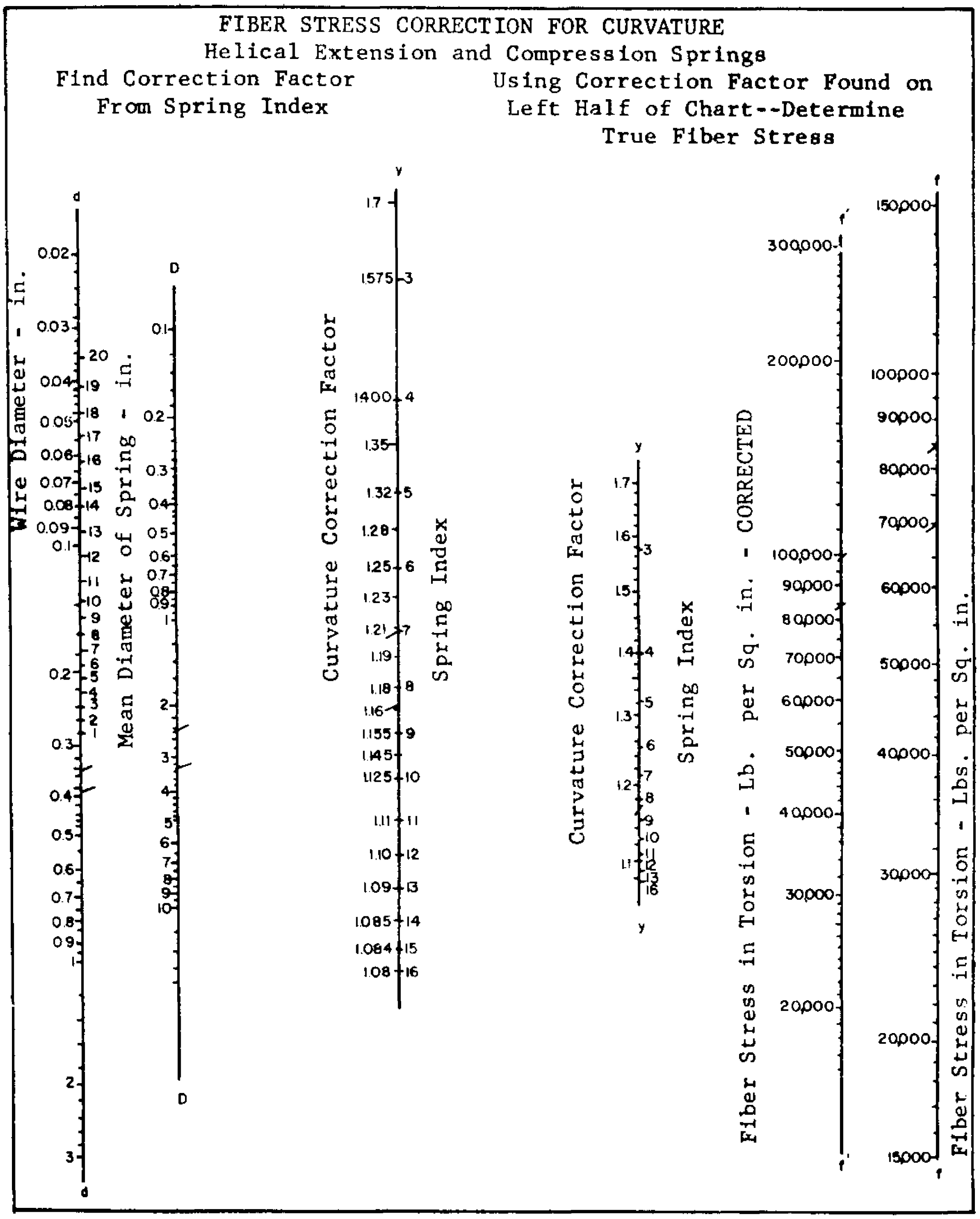


Fig. B 3.1.4-1

B 3.1.4 Analysis of Helical Springs by Use of Nomograph (Cont'd)

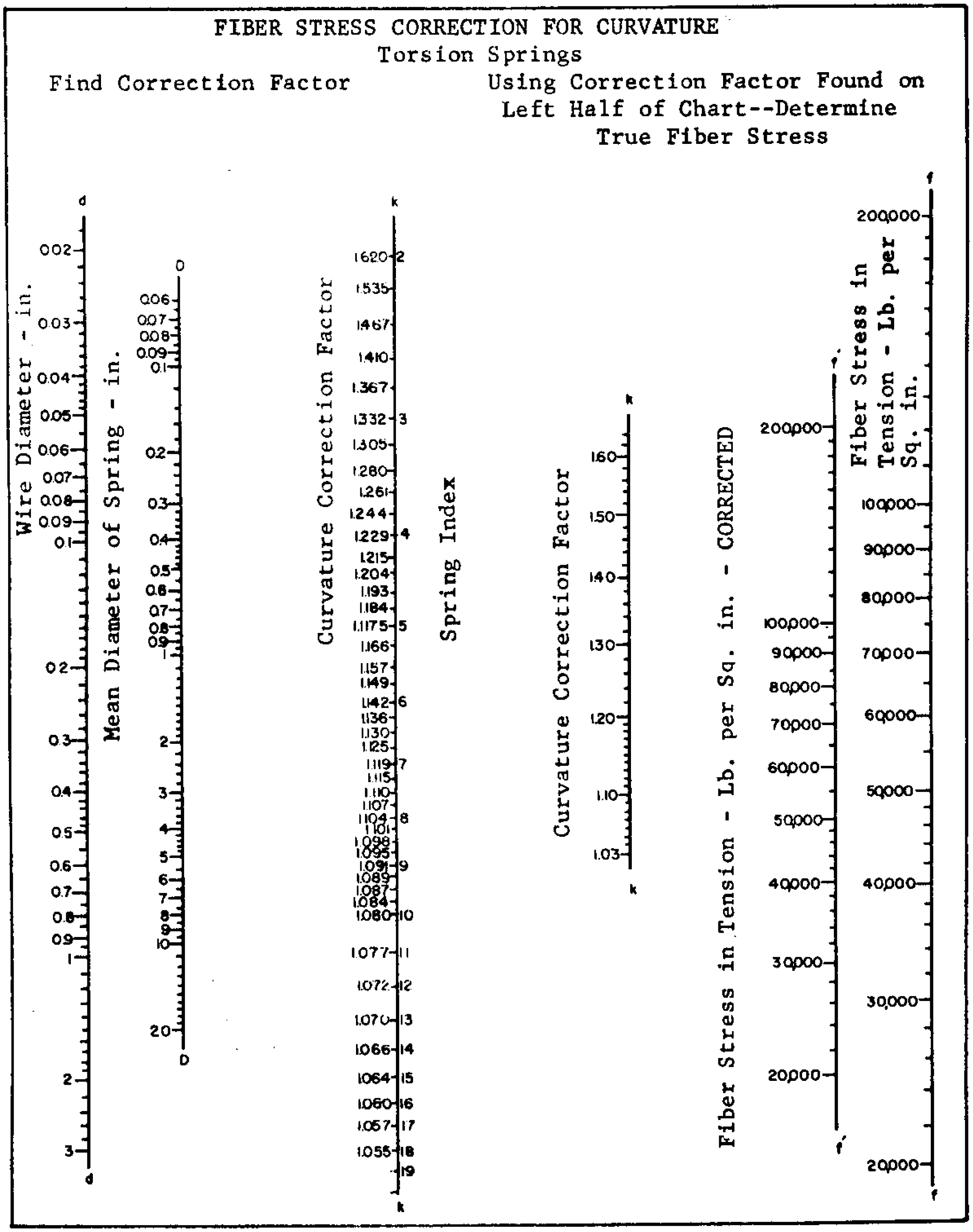


Fig. B 3.1.4-2

B 3.1.5 Maximum Design Stress for Various Spring Materials

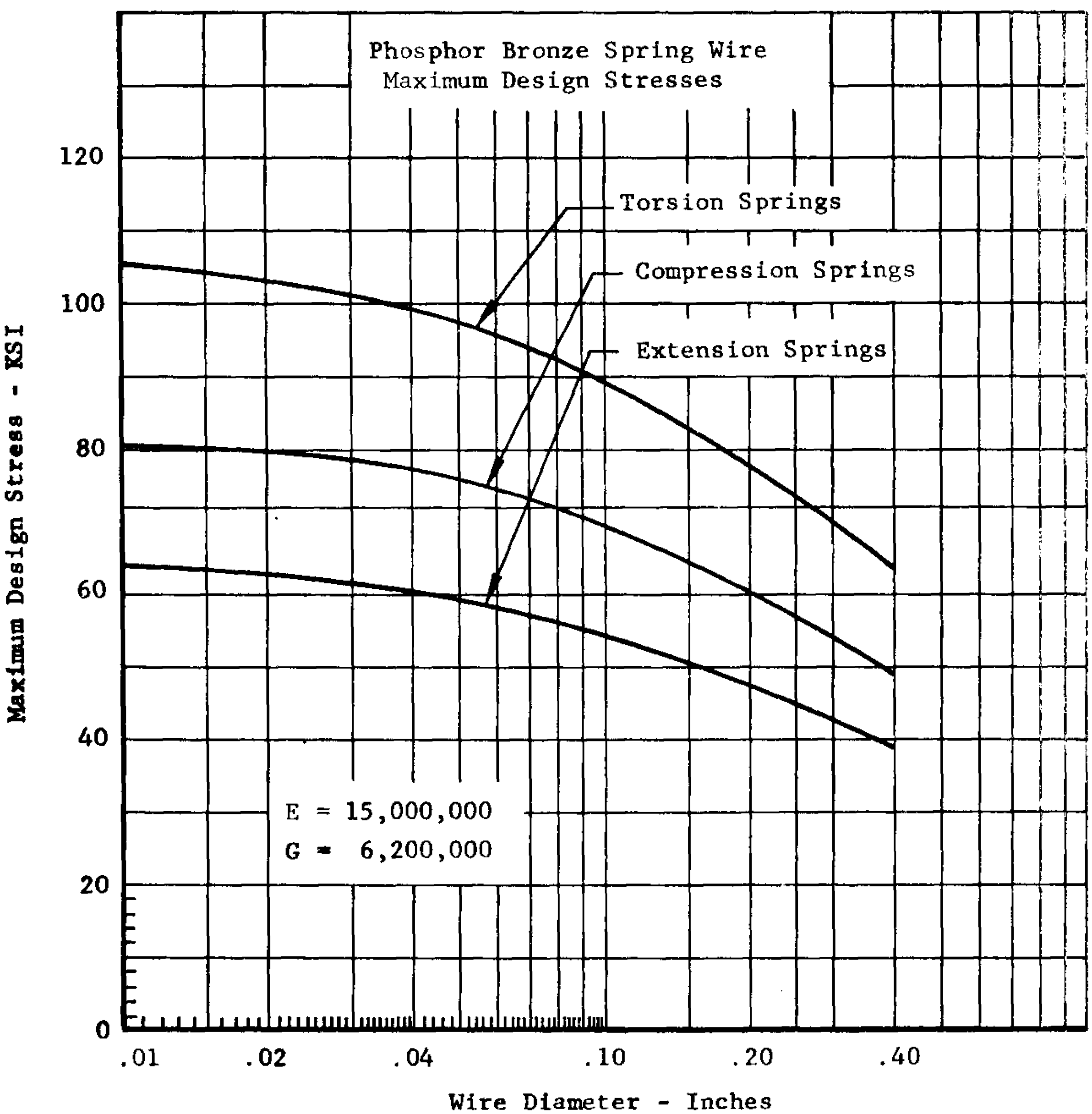


Fig. B 3.1.5-1

B 3.1.5 Maximum Design Stresses for Various Spring Materials (Cont'd)

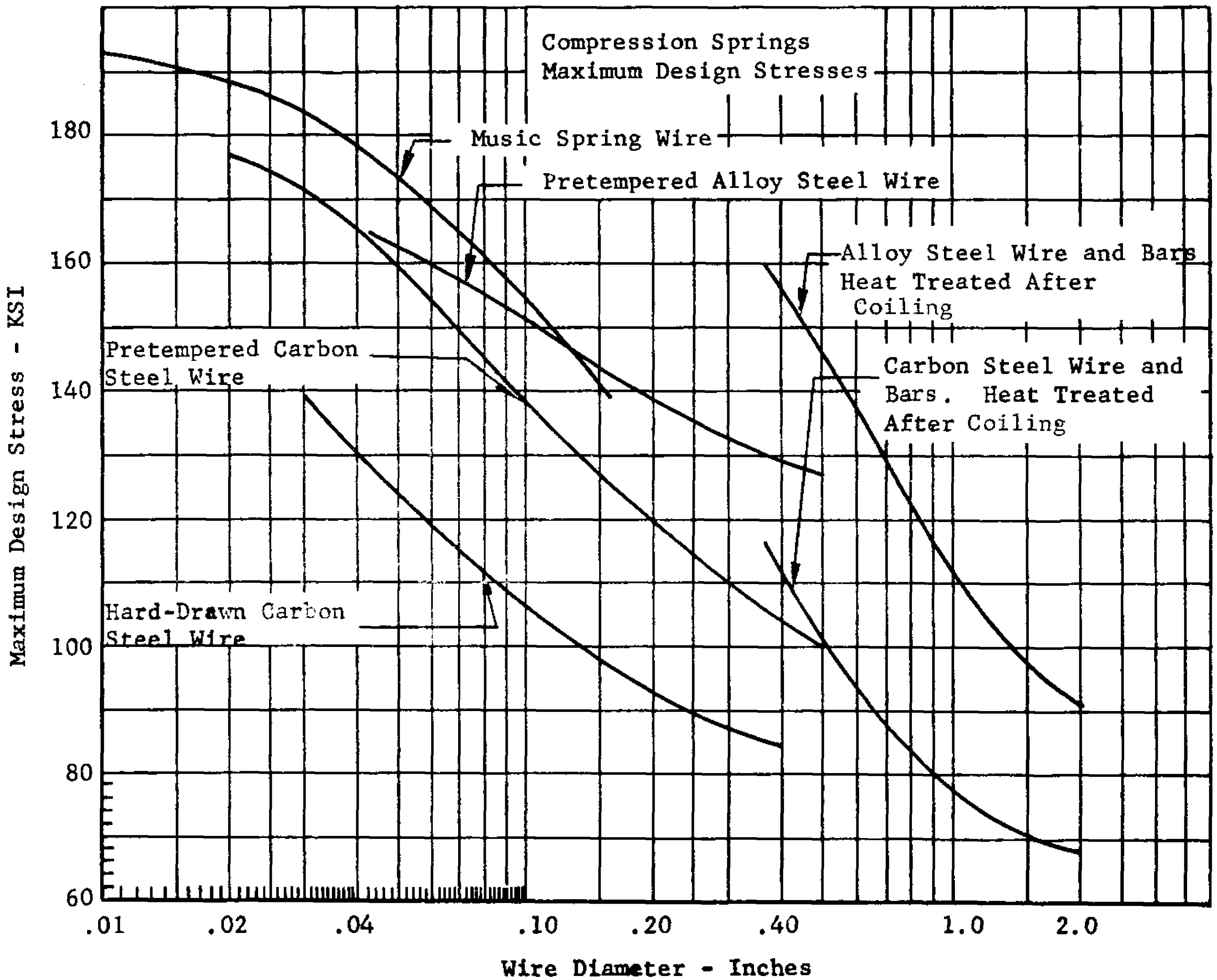


Fig. B 3.1.5-2

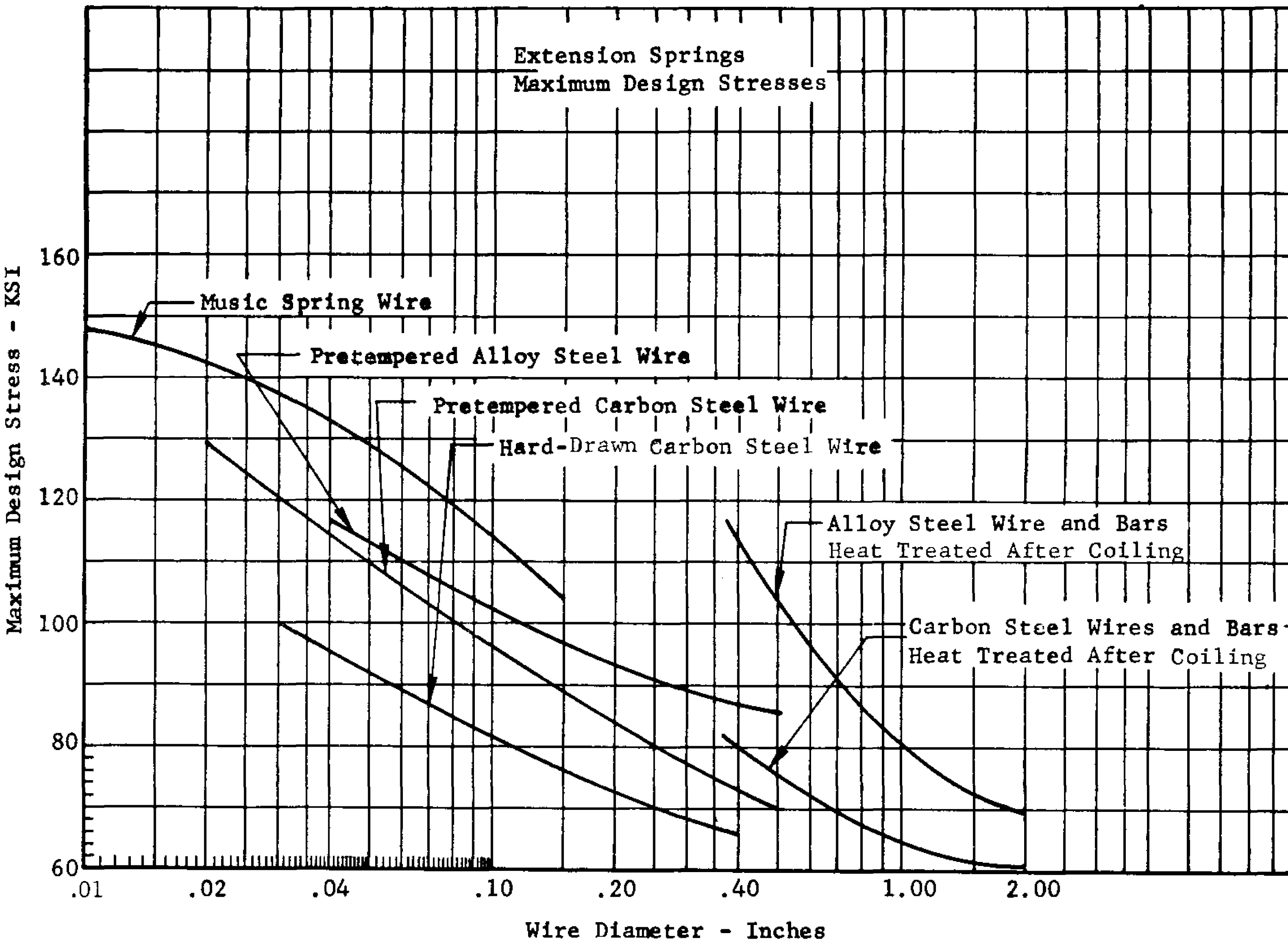

Extension Springs
Maximum Design Stresses
Music Spring Wire
Pretempered Alloy Steel Wire
Pretempered Carbon Steel Wire
Hard-Drawn Carbon Steel Wire
Alloy Steel Wire and Bars
Heat Treated After Coiling
Carbon Steel Wires and Bars
Heat Treated After Coiling
160
140
120
100
80
60
.01
.02
.04
.10
.20
.40
1.00
2.00
Maximum Design Stress - KSI
Wire Diameter - Inches

Fig. B 3.1.5-3

B 3.1.5 Maximum Design Stresses for Various Spring Materials (Cont'd)

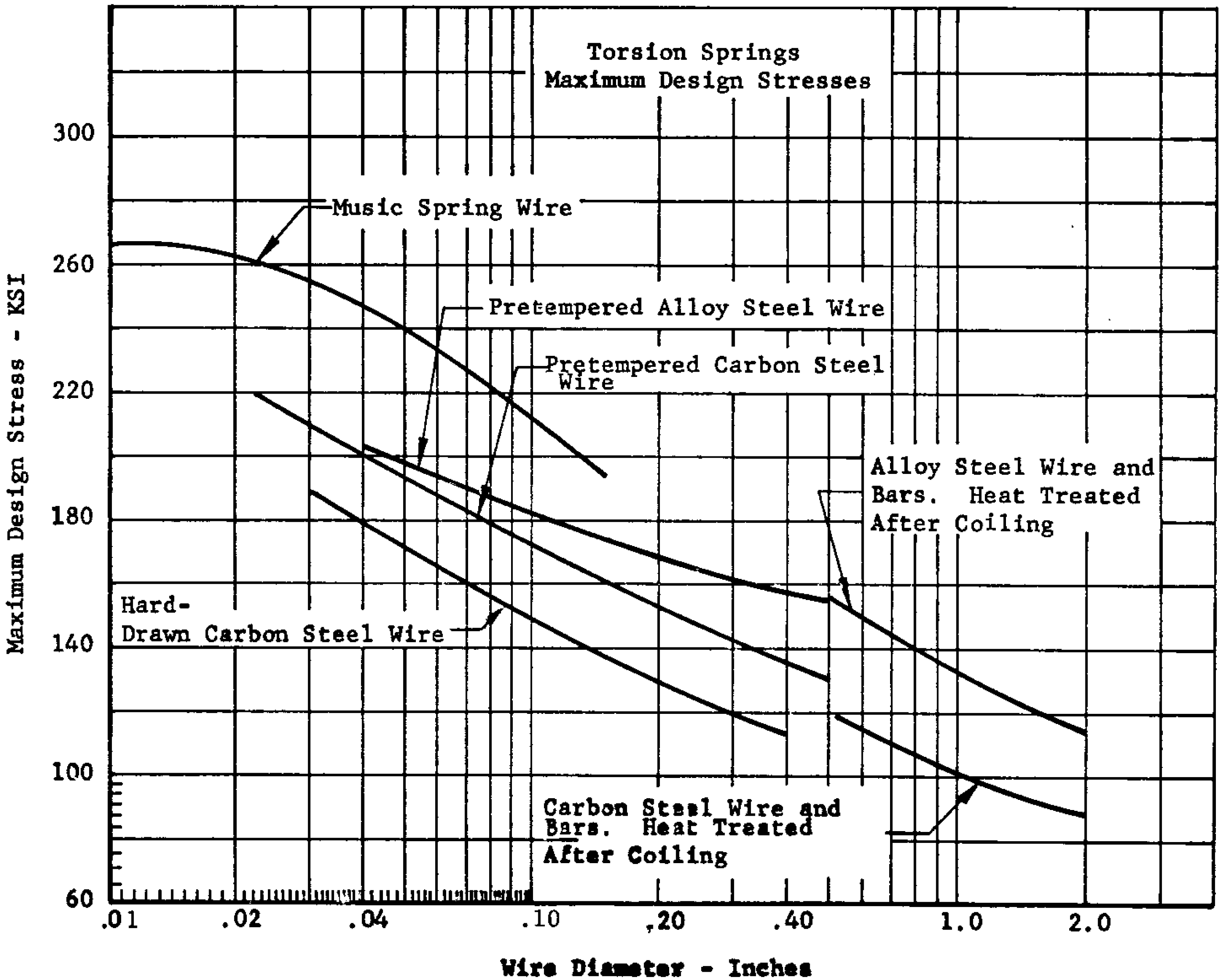


Fig. B 3.1.5-4

B 3.1.6 Dynamic or Suddenly Applied Spring Loading

A freely falling weight, or moving body, that strikes a structure delivers a dynamic or impact load, or force. Problems involving such forces may be analyzed on the basis of the following idealizing assumptions:

1. Materials behave elastically, and no dissipation of energy takes place at the point of impact or at the supports owing to local inelastic deformation of materials.

2. The deflection of a system is directly proportional to the magnitude of the dynamically or statically applied force.

Then, on the basis of the principle of conservation of energy, if it may be further assumed that at the instant a moving body is stopped, its kinetic energy is completely transformed into the internal strain energy of the resisting system, the following formulas will apply:

(a) For very slowly applied loads

$$\delta = \frac{P}{K} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (16)$$

(b) For loads suddenly applied

$$\delta = \frac{2P}{K} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (17)$$

(c) For loads dropped from a given height

$$\delta^2 = \frac{2P(S + \delta)}{K} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (18)$$

where:

δ = Total deflection
K = Spring rate
P = Load on spring
S = Height load is dropped.

The following problems will illustrate such conditions and their solutions:

Problem 1.

Given a spring which compresses one inch for each pound of load, determine the maximum load and deflection resulting from a 4 lb. weight.

B 3.1.6 Dynamic or Suddenly Applied Spring Loading (Cont'd)

(a) Case 1:

The weight is laid gently on the spring.

$$\delta = \frac{P}{K} = \frac{4}{1} = 4 \text{ in. from Eq. 16}$$

since K = 1 maximum load = 4 lb.

(b) Case 2:

The weight is suddenly dropped on the spring from zero height.

$$\delta = \frac{2P}{K} = \frac{2(4)}{1} = 8 \text{ in. from Eq. 17}$$

maximum load = 8 lb.

(c) Case 3:

The weight is dropped on the spring from a height of 12 inches.

$$\delta^2 = \frac{2P(S + \delta)}{K} = \frac{2(4)}{1} (12 + \delta) \quad \text{from Eq. 18}$$

or

$$\delta^2 - 8\delta - 96 = 0$$

$$\delta = \frac{8 + \sqrt{8^2 + 4(96)}}{2} = 14.6 \text{ in.}$$

maximum load 14.6 lb.

From the maximum load produced, Eq. 2 section B 3.1.1 may be used to calculate the stress produced. This should be within the limits indicated on Fig. B 3.1.5-1, B 3.1.5-2, B 3.1.5-3 and B 3.1.5-4.

Problem 2.

For many uses it is necessary to know the return speed of a spring or the speed with which it will return a given weight. A typical example of this problem could be stated as follows: a spring made of 5/16 in. by 3/16 in. rectangular steel contains 4 3/8 total coils, 2 3/8 active coils, on a mean diameter of 1 5/16 in. The spring compresses 5/32 in. for 200 lb. load. If the spring is compressed and then instantaneously released, how fast will it be moving at its original free length position of 1 21/32 in.? The solution is as follows:

B 3.1.6 Dynamic or Suddenly Applied Spring Loading (Cont'd)

Weight per turn = π(1 5/16)(3/16)(5/16)(.283) lb. per cu. in.
= .0685 lb. of steel in each coil

.0685 (2 3/8) active coils (1/3) = .0542 lb (one-third of the weight of active spring material involved.)

To this .0542 lb. we add the dead coil at the end plus the moving weight, if any.

The equivalent total weight (.0542 + .0685) is 0.1227 lb.

The potential energy of a spring is equal to 1/2 the total load times the distance moved, or 1/2(200)(5/32) = 15.6 in. lb. The kinetic energy equals 1/2 Mv^2 wherein (M) is the mass and (v) is the velocity.

$$\text{Mass} = \frac{\text{Weight}}{g}$$ where g is the gravitational acceleration,

or 32.16 ft/sec./sec.

$$\text{Therefore } 15.6 = \frac{.1227v^2}{2(32.16)(12)} \quad \text{or } v = 314 \text{ in/sec}$$

Often springs are used to absorb energy of impact. In most such instances springs must be designed so that they will absorb the entire energy. In a few cases partial absorption is tolerated. A typical problem of this type follows.

Problem 3.

A 30 lb. weight has a velocity of 4 ft. per second. How far will a spring that has a spring rate of 10 lb/in. be compressed?

KINETIC ENERGY

$$\text{K.E.} = \frac{1}{2} Mv^2 = \frac{30(4)(4)}{2(32.16)} = 7.46 \text{ ft. lb.}$$

or

7.46(12) = 89.52 in.lb.

Spring energy = $\frac{1}{2}$ load times deflection

Load = rate per inch times deflection

Spring energy = $\frac{1}{2} K\delta^2$ = 89.52 in. lb.

B 3.1.6 Dynamic or Suddenly Applied Spring Loading (Cont'd)

$$\frac{1}{2}\,10\delta^2 = 89.52$$

$$\delta^2 = 17.90$$

deflection, $\delta = 4.23$ in.

If springs are used to propel a mass, a parallel attack using velocity and acceleration applies.

Problem 4.

Let it be required to find the spring load that will propel a 1-lb. ball 15 ft. vertically upward in 1/2 second. It is assumed that the spring can be compressed a distance of 1 ft.

In order to travel 15 ft. in 1/2 second the 1-lb. load must have a certain initial velocity. This can be found as follows:

$$v = \frac{h}{t} + \frac{gt}{2}$$

wherein: h = height
g = 32.16 ft. per sec^2
t = time

$$v = \frac{15}{\frac{1}{2}} + \frac{32.16\left(\frac{1}{2}\right)}{2} = 38.04 \text{ ft. per sec.}$$ Spring velocity at free height.

$$\text{Spring acceleration} = \frac{v^2}{2s} = \frac{(38.04)^2}{2(1)} = 723 \text{ ft/sec/sec}$$

Force equals mass times acceleration so

$$F = \frac{723(1)}{32.16} = 22.5 \text{ lb. avg.}$$

The average spring pressure is 1/2 the total load. Hence the spring will compress 1 ft. with 2(22.5) or 45-lb. of load. Often, it is desired to know how high the weight would be propelled. This can be determined by equating the work performed by the spring to the work of the falling weight; thus work equals force times distance.

In the spring we have $\frac{45}{2}$ (1) ft.

In the weight we have 1-lb. (h)

Hence $1(h) = \frac{45}{2}(1) = 22.5$ ft., the height to which the weight would be thrown.

B 3.1.6 Dynamic or Suddenly Applied Spring Loading (Cont'd)

If we were to apply the previous formula

$$\delta^2 = \frac{2P(S + \delta)}{K} \text{ to the springs,}$$

we must remember (S) is the height the load is dropped. The total distance traveled by the weight is $(S + \delta)$.

Therefore, $h = S + \delta$ in this case

Substituting $1 = \frac{2(1)h}{45}$

$$h = 22.5 \text{ ft.}$$

B 3.1.7 Working Stress for Springs

If the loading on the spring is continuously fluctuating, due allowance must be made in the design for fatigue and stress concentration. A method of determining the allowable or working stress for a particular spring is dependent on the application as well as the physical properties of the material.

B 3.2.0 Curved Springs

The analytical expression for determining stresses for curved springs is

$$f = \pm \frac{P}{A} \pm \frac{M}{AR}\left(1 + \frac{1}{Z}\,\frac{y}{R+y}\right) \quad \text{.........................} \quad (19)$$

in which the quantities have the same meaning defined in section B 4.3.1.

Displacement of curved springs is determined by use of Castigliano's theorem.

$$\delta_p = \frac{\partial v}{\partial P} = \int \frac{N}{EA}\,\frac{\partial N}{\partial P}\,ds + \int \frac{V}{GA}\,\frac{\partial V}{\partial P}\,ds + \int \frac{M}{EAy_0R}\,\frac{\partial M}{\partial P}\,ds$$

$$+ \int \frac{M}{EAR}\,\frac{\partial N}{\partial P}\,ds + \int \frac{N}{EAR}\,\frac{\partial M}{\partial P}\,ds \quad \text{..............} \quad (20)$$

in which

N = normal force
E = Modulus of Elasticity
G = Modulus of Rigidity
A = Cross-sectional area
R = Radius to centroid
ds = Incremental length
$y_0 = \frac{ZR}{Z+1}$
$Z = -\frac{1}{A}\int_A \frac{y}{R+y}\,dA$
y = is measured from the centroid

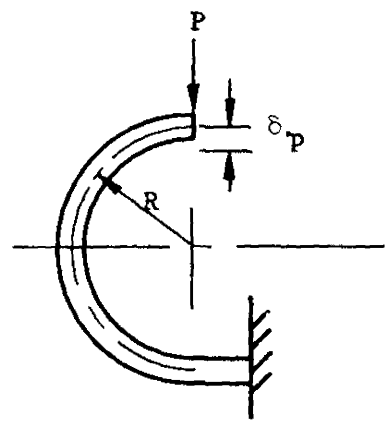


Fig. B 3.2.0-1

These expressions for stresses and displacements are quite cumbersome; therefore, correction factors are used to simplify the analysis. The correction factors (K) used to determine the stresses are given in section B 4.3.1. The expression for the stress is

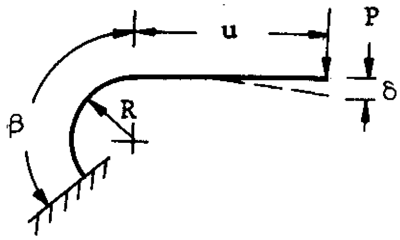
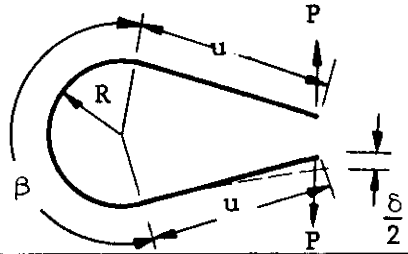
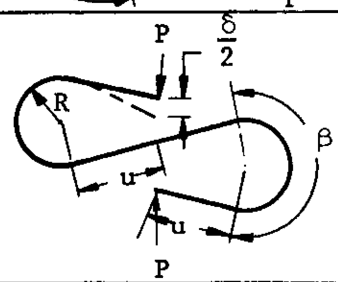
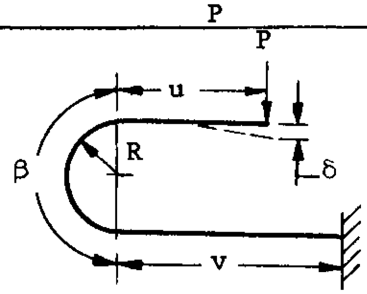
$$f = K\,\frac{Mc}{I} \quad \text{...} \quad (21)$$

See Table B 4.3.1-1 for values of correction factor K.

Deflection formulas for some basic types of curved springs are given in Table B 3.2.0-1. Complex spring shapes may be analyzed by combination of two or more basic types.

B 3.2.0 Curved Springs (Cont'd)

Table B 3.2.0-1

Spring types	Deflection
A 	$\delta = \frac{KPR^3}{3EI}(m + \beta)^3$ * where $\alpha = \beta$ for finding K
B 	$\delta = \frac{2KPR^3}{3EI}\left(m + \frac{\beta}{2}\right)^3$ where $\alpha = \frac{\beta}{2}$ for finding K
C 	$\delta = \frac{4KPR^3}{3EI}\left(m + \frac{\beta}{2}\right)^3$ where $\alpha = \frac{\beta}{2}$ for finding K
D 	$\delta = \frac{P}{3EI}\left[2KR^3\left(m + \frac{\beta}{2}\right)^3\left(v - u\right)^3\right]$ where $\alpha = \frac{\beta}{2}$ for finding K

$* \; m = \frac{u}{R}$

B 3.2.0 Curved Springs (Cont'd)

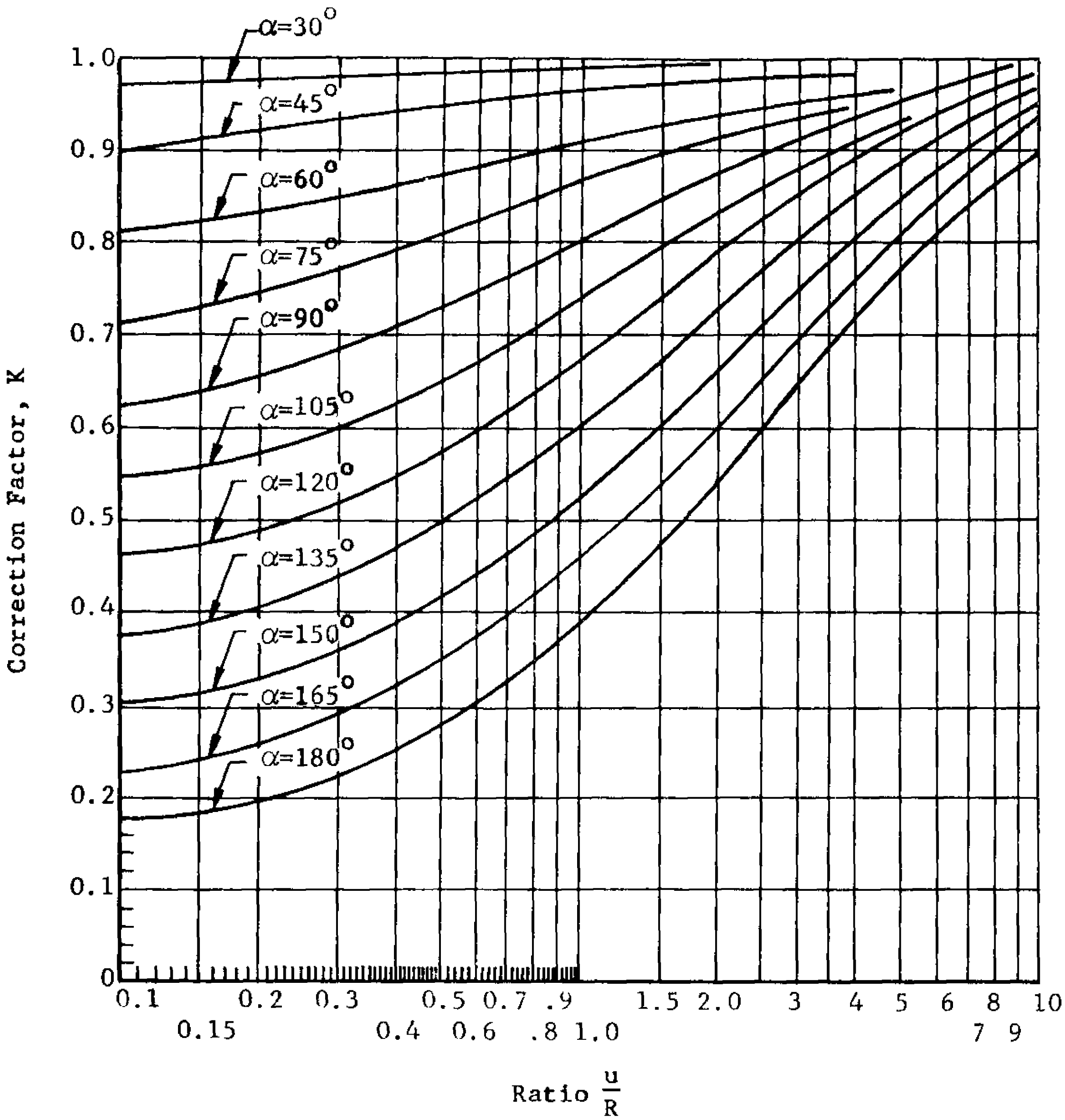


Fig. B 3.2.0-2

B 3.2.0 Curved Springs (Cont'd)

For close approximations, the following conditions should be met:

flat springs

$$\frac{\text{spring thickness}}{\text{radius of curvature}} = \frac{h}{R} < 0.6$$

round wire

$$\frac{\text{wire diameter}}{\text{radius of curvature}} = \frac{d}{R} < 0.6$$

Figure B 3.2.0-3 is a typical curved spring. The deflection of the spring at point A is calculated as follows:

Spring Characteristics

$\frac{h}{R} < 0.6$ $\quad u_2 = 2.5''$

$\beta_1 = \pi$ $\quad R_1 = R_2 = 1''$

$\beta_2 = \frac{\pi}{2}$

$u_1 = 1''$

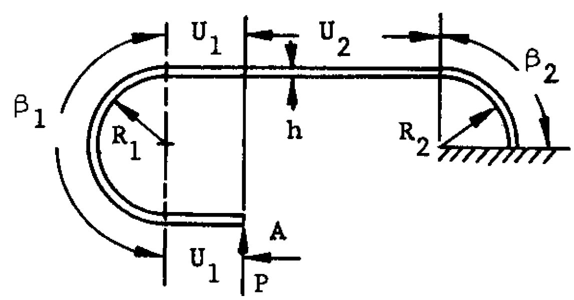


Fig. B 3.2.0-3

Solution -

The solution involves two basic types (type B and A of Table B 3.2.0-1). Type A solution is used for that portion of the spring denoted by subscript (2), and type B solution is used for that portion of the spring denoted by subscript (1).

Correction factor, K (from Fig. B 3.2.0-2)

$\frac{u_1}{r_1} = \frac{1}{1} = 1, \quad \beta_1 = 180^\circ, \quad K_1 = .80$

$\alpha_1 = 90^\circ$

$\frac{u_2}{r_2} = \frac{2.5}{1} = 2.5,$

$\beta_2 = 90^\circ, \quad K_2 = .86$

Deflection at point A

$$\delta_A = \frac{2K_1PR_1^{\,3}}{3EI}\left(m + \frac{\beta_1}{2}\right)^3 + \frac{K_2PR_2^{\,3}}{3EI}\left(m + \beta_2\right)^3$$

$$\delta_A = \frac{2(.8)\ P(1)^3}{3EI}\left(1 + \frac{\pi}{2}\right)^3 + \frac{.86P(1)^3}{3EI}\left(2.5 + \frac{\pi}{2}\right)^3$$

$$\delta_A = \frac{28.4P}{EI}$$

B 3.3.0 Belleville Springs or Washers

Belleville type springs are used where space requirements necessitate high stresses and short range of motion. A complete derivation of data that is presented in this section will be found in "Transactions of Amer. Soc. of Mech. Engineers", May 1936, Volume 58, No. 4, by Almen and Laszlo.

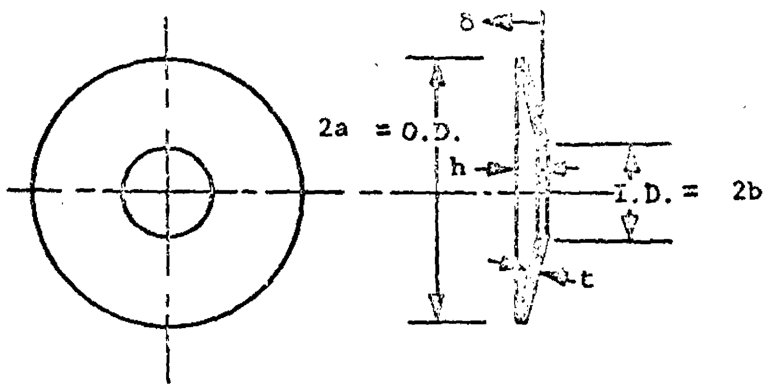


Fig. B 3.3.0-1

Symbols

- P = Load in pounds
- δ = Deflection in inches
- t = Thickness of material in inches
- h = Free height minus thickness in inches
- a = One-half outside diameter in inches
- E = Young's modulus
- f = Stress at inside circumference
- k = ratio of $\frac{O.D.}{I.D.} = \frac{a}{b}$
- ν = Poisson's ratio

M, C_1 and C_2 are constants which can be taken from the chart, Fig. B 3.3.0-4, or calculated from the formulas given.

The formulas are:

$$M = \frac{6}{\pi \log_e k} \frac{(k-1)^2}{k^2} \qquad \text{(22)}$$

$$C_1 = \frac{6}{\pi \log_e k} \left[\frac{(k-1)}{\log_e k} - 1 \right] \qquad \text{(23)}$$

$$C_2 = \frac{3}{\pi \log_e k} (k-1) \qquad \text{(24)}$$

B 3.3.0 Belleville Springs or Washers (Cont'd)

The deflection-load formula, using these constants is

$$P = \frac{E\delta}{(1 - \nu^2)Mk^2b^2}\left[\left(h - \frac{\delta}{2}\right)\left(h - \delta\right) t + t^3\right] \quad \text{.........} \quad (25)$$

The stress formula is as follows:

$$f = \frac{E\delta}{(1-\nu^2)Mk^2b^2}\left[C_1\left(h - \frac{\delta}{2}\right) + C_2t\right] \quad \text{...............} \quad (26)$$

Before using these formulas to calculate a sample problem, there are some facts which should be considered. In the stress formula it is possible for the term $(h - \delta/2)$ to become negative if (δ) is large. When this occurs, the term inside the brackets should be changed to read $C_1(h - \delta/2) - C_2t$. Such an occurrence means that the maximum stress is tensile.

For a spring life of less than one-half million stress cycles, a fiber stress of 200,000 p.s.i. can be substituted for f, even though this might be slightly beyond the elastic limit of the steel. This is because the stress is calculated at the point of greatest intensity, which is on an extremely small part of the disc. Immediately surrounding this area is a much lower-stressed portion which so supports the higher-loaded corner that very little setting results at atmospheric temperatures. For higher operating temperatures and longer spring life lower stresses must be employed.

Fig B 3.3.0-2 displays the load-deflection characteristics of a .040 in. thick washer for various h/t ratios.

It is noted (from Fig. B 3.3.0-2) that for ratios of (h/t) under 1.41 the load-deflection curve is somewhat similar to that of other conventional springs. As this ratio approaches the value of 1.41, the spring rate approaches zero (practically horizontal load-deflection curve) at the flat position. When the (h/t) ratio is 2.83 or over there is a portion of the curve where further deflection produces a lower load. This is illustrated in the curves for the washers having (h/t) ratios of 2.83 and 3.50. Such a spring, when deflected to a certain point, will snap through center and require a negative loading to return it to its original position.

The washers may sometimes be stacked so as to obtain the load-deflection characteristics desired. The accepted methods are illustrated in Fig. B 3.3.0-3.

B 3.3.0 Belleville Springs or Washers (Cont'd)

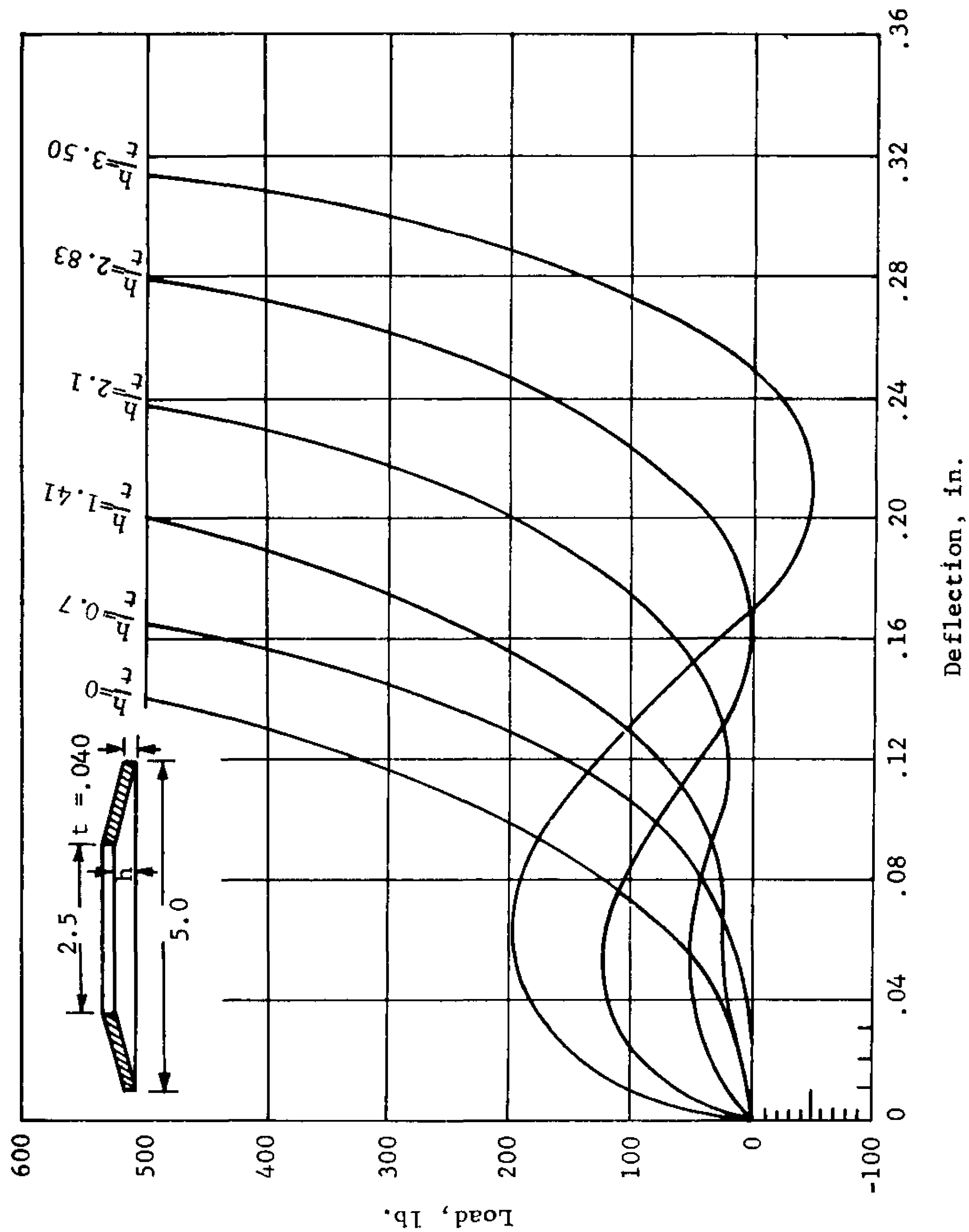


Fig. B 3.3.0-2

B 3.3.0 Belleville Springs or Washers (Cont'd)

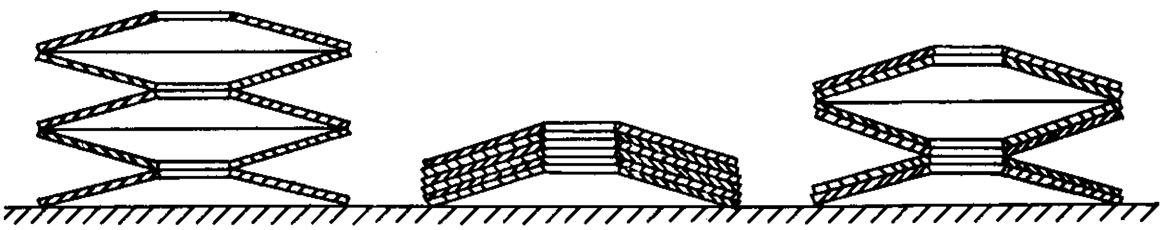


Fig. B 3.3.0-3

As the number of washers used increases, so does the friction in the stacks. This is not uniform and could result in spring units which are very erratic in their load-carrying capacities. Belleville springs, as a class, are one of the most difficult to hold to small load-limit tolerances.

B 3.3.0 Belleville Springs or Washer (Cont'd)

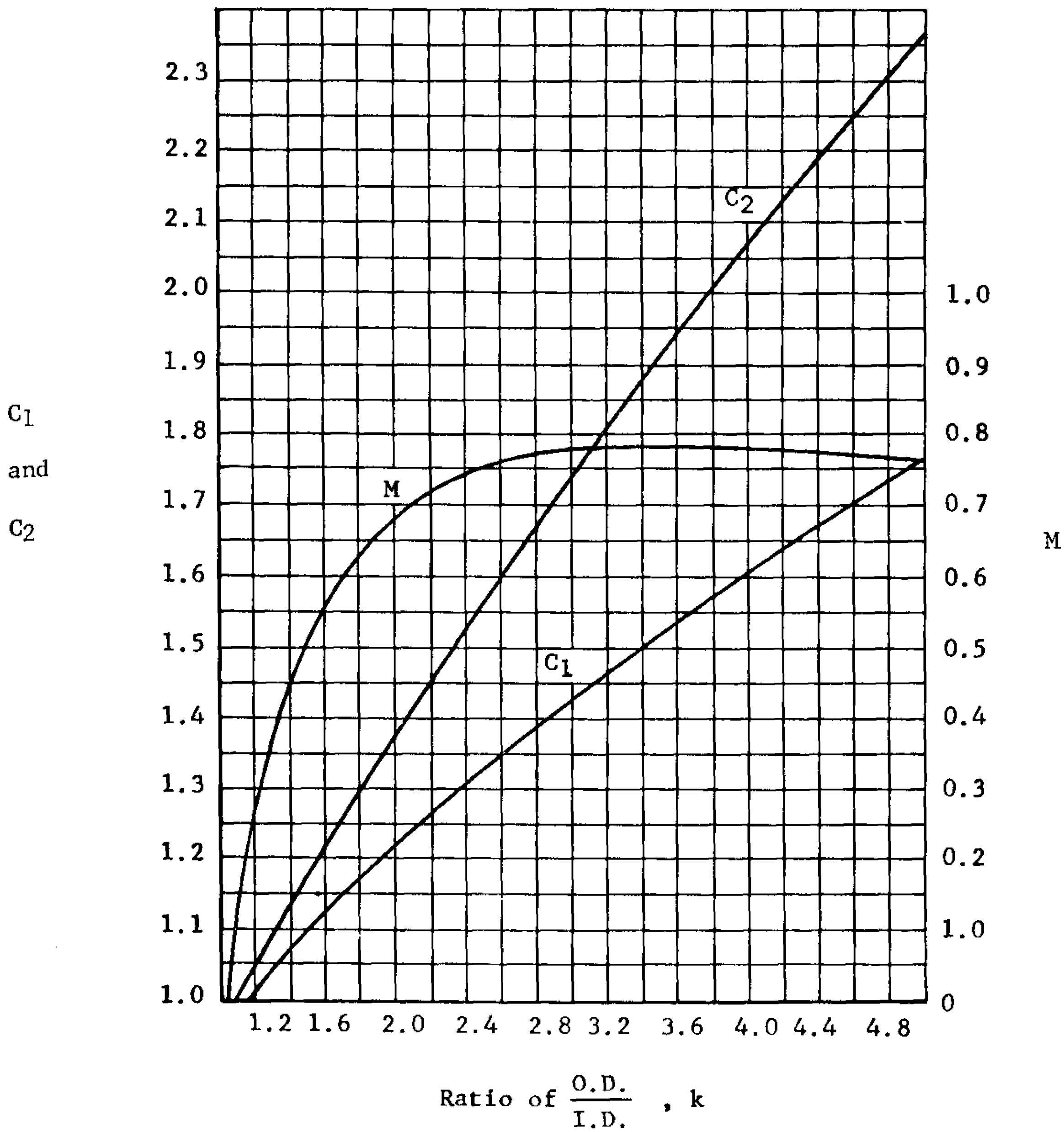


Fig.B 3.3.0-4 Belleville Spring Constants: M, C_1 and C_2

B 3.3.0 Belleville Springs or Washers (Cont'd)

Example Problem

Given:

O.D. = 2"
I.D. = 1.25"
Load to deflect .02" = 675 lb.

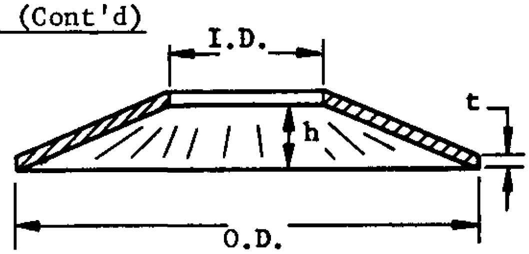


Fig. B 3.3.0-5

Required:

Determine required thickness, t and dimension h.

Solution:

$$k = \frac{O.D.}{I.D.} = \frac{2.00}{1.25} = 1.6$$

The constants M, C_1 and C_2 may be taken from the curves in Fig. B 3.3.0-4 or may be calculated as follows:

$$M = \frac{6}{\pi \log_e k} \frac{(k-1)^2}{k^2} = \frac{6}{3.14(0.47)} \frac{(1.6 - 1.0)^2}{(1.6)^2} = 0.57$$

$$C_1 = \frac{6}{\pi \log_e k} \left[\frac{k-1}{\log_e k} - 1 \right] = \frac{6}{3.14(0.47)} \left[\frac{1.6 - 1.0}{0.47} - 1 \right] = 1.123$$

$$C_2 = \frac{6}{\pi \log_e k} \left[\frac{k-1}{2} \right] = \frac{6}{3.14(0.47)} \left[\frac{1.6 - 1.0}{2} \right] = 1.220$$

The Deflection-Stress Formula

$$f = \frac{E\delta}{(1-\nu^2)M a^2} \left[C_1 \left(h - \frac{\delta}{2} \right) + C_2 t \right]$$

may be written in the form

$$h = \frac{fMa^2 (1-\nu^2)}{C_1 E\delta} + \frac{\delta}{2} - \frac{C_2}{C_1} t$$

B 3.3.0 Belleville Springs or Washers (Cont'd)

Assume that the washer shown in Fig. B 3.3.0-5 is steel

f_{max} = 200,000 psi

E = 30,000,000 psi

ν = .3

δ = 0.02 in.

M = 0.57

C_1 = 1.123

C_2 = 1.220

a = 1.00 (half outside dia., inches)

try t = .04 and solve for dimension h

$$h = \frac{200{,}000\ (1)^2 (.57)(1-.3^2)}{(1.123)(.02)(30)10^6} + \frac{.02}{2} - \frac{1.220}{1.123}(.04) = .120 \text{ in.}$$

This value for (h) is then substituted in the deflection-load formula to obtain the load.

$$P = \frac{E\,\delta}{(1-\nu^2)M\,a^2}\left[\left(h - \frac{\delta}{2}\right)\left(h - \delta\right) t + t^3\right]$$

$$= \frac{30(10^6)(.02)}{(1-.3^2)(.57)(1)^2}\left[(.121-.01)(.121-.02)(.04)+(.04)^3\right] = 600 \text{ lb.}$$

Since this load is too low, the calculation is repeated using a stock thickness (t) of .05 in.

Then, solving again for h, the result is

h = .110 in.

Therefore, substituting this value of h into the formula used to calculate the previous value of load (P), the new value of (P) is

P = 665 lb.

This is as close as calculation need be carried. It is not expected that this or a similarly calculated spring will be deflected beyond the amount used in the calculation.

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