SECTION C2

STABILITY OF PLATES

TABLE OF CONTENTS

TABLE OF CONTENTS (Continued)

TABLE OF CONTENTS(Concluded)

SECTION C2. STABILITY OF PLATES

2.1 BUCKLING OF FLAT PLATES.

This section contains design information for predicting the buckling of flat plates. Various geometrical shapes under several types of loading common to aerospace structures are considered. In most cases the methods presented may be used in either the elastic or plastic stress range. For plates subjected to thermal gradients which may cause buckling, reference should be made to Section D4.0.2, "Thermal Buckling of Plates."

2.1.1 UNSTIFFENED PLATES.

With few exceptions, plate critical stress equations take the following form:

$$F_{cr} = \eta \bar{\eta} \frac{k \pi^2 E}{12(1 - \nu_e^2)} \left(\frac{t}{b}\right)^2 \qquad (1)$$

where the terms are defined as follows:

F_{cr}	buckling stress which includes the effects of plasticity and cladding (psi)
η	plasticity reduction factor
$\bar{\eta}$	cladding reduction factor
k	buckling coefficient
E	Young's Modulus (elasticity) (psi)
ν_e	elastic Poisson's ratio

t plate thickness (in.)

b dimension of plate (usually short dimension, in.)

The buckling constant, k, depends only on the plate dimensions, excluding its thickness, and upon the condition of support at the edges. For the material, temperature, and stress level used, the proper values of E, ν_e, η, and $\bar{\eta}$ must be substituted into the equation above.

Buckling curves are used to find values of the buckling coefficient, k, for numerous loading conditions and various boundary conditions. By knowing only the plate aspect ratio, a/b, values of k can be read directly.

The wavelength of the buckled surface is an important factor in establishing the critical buckling stress. A plate will buckle into a "natural" wavelength corresponding to a minimum load. This principle has been applied to advantage in structures to increase the efficiency of the flat sheet. That is, if by any structural means the natural wavelength of buckling can be prevented, the plate will carry more load.

I. Plasticity Reduction Factor

A tremendous amount of theoretical and experimental work has been done relative to the value of the so-called plasticity correction factor. Possibly the first values used by design engineers were $\eta = E_t/E$ or $\eta = E_{sec}/E$. Whatever the expression for η, it must involve a measure of the stiffness of the material in the inelastic stress range; and, since the stress-strain relation in the plastic range is nonlinear, a resort must be made to the stress-strain curve to obtain a plasticity correction factor. This complication is greatly simplified by using the Ramberg and Osgood equations for the stress-strain curve which involves three simple parameters:

$$\frac{E\epsilon}{F_{0.7}} = \frac{f}{F_{0.7}} + \left(\frac{f}{F_{0.7}}\right)^n \qquad (2)$$

where $n = 1 + \log_e(17/7) / \log_e(F_{0.7}/F_{0.85})$, and the terms are defined as follows:

$F_{0.7}$ secant yield stress taken as the intersection of the curve and a slope 0.7E drawn from origin

n a parameter which describes the shape of the stress-strain curve on the yield region

$F_{0.85}$ stress at the intersection of the curve by a line of slope of 0.85E through the origin

Reference 1 gives values for $F_{0.7}$, $F_{0.85}$, and many flight vehicle materials; some of these are given in Table C2-1.

There is usally a maximum, or "cutoff" stress, above which it is considered unsafe to stress the material. The value of this cutoff stress differs with the type of loading, and may vary according to the design criteria established for each design. Suggested values of the cutoff stress are presented in Table C2-2. A check should be made to ensure that the buckling stress is equal to, or less than, the cutoff stress.

With the use of the Ramberg-Osgood parameters, plasticity reduction factors will be given for various types of loading in the paragraphs which follows.

II. Cladding Reduction Factors

Aluminum alloy sheets are available with a thin covering of practically pure aluminum and is widely used in aircraft structures. Such material is referred to as alclad or clad aluminum alloy. The mechanical strength properties of this clad material is considerably lower than the core material. Since the clad is located at the extreme fibers of the alclad sheet, it is located where the strains attain their value when buckling takes place. Figure C2-1 shows the makeup of an alclad sheet and Fig. C2-2 shows the stress-strain curves for cladding, core, and combinations. Thus, a further correction must be made for alclad sheets because of the lower strength clad covering material. Reference 1 gives simplified cladding reduction factors as summarized in Table C2-3.

2.1.1.1 Rectangular Plates.

Rectangular plates subjected to loads which cause instability constitute one of the major elements encountered in the structural design of space vehicles. Rectangular plate simulation occurs in such areas as beam webs, panels, and flanges.

I. Compressive Buckling

Figure C2-3 shows the change in buckled shape of rectangular plates as the boundary conditions are changed on the unloaded edges from free to restrained. In Fig. C2-3(a) the sides are free; thus, the plate acts as a column. In Fig. C2-3(b) one side is restrained and the other side is free; such a restrained plate is referred to as a flange. In Fig. C2-3(c) both sides are restrained; this restrained element is referred to as a plate.

Critical compressive stress for buckling of plate columns (free at two unloaded edges) can be obtained from Fig. C2-4. As can be seen from this figure, a transition occurs with changing $b/L\sqrt{c}$ values as evidenced by the varying value of ψ between the limits $(1 - \nu_e^2) < \psi < 1$. The increased load-carrying capacity of a wide plate column, $\psi = 1$, is due to antielastic bending effects in the plate at buckling. For narrow columns, $\psi = 1 - \nu_e^2$, the equation reduces to the Euler equation.

Figure C2-5 gives curves for finding the buckling coefficient, k, to use in equation (1) for various boundary, or edge, conditions and a/b ratio of the plate. The letter C on an edge means clamped or fixed against rotation. The letter F means a free edge and SS means simply supported or hinged. From these curves it can be seen that for long plates, (a/b) > 4, the effect of the loaded edge support condition is negligible.

The buckling of a rectangular plate compressed by two equal and opposite forces located at the midpoint of its long side (Fig. C2-6) is given in Reference 2. For simply supported sides, the following equation is true:

$$P_{cr} = \frac{\pi^2 D}{2b}\left[\frac{1}{\frac{\pi}{8}\left(\tanh\frac{\pi\beta}{2} - \frac{\frac{\pi\beta}{2}}{\cosh^2\frac{\pi\beta}{2}}\right)}\right] \qquad (3)$$

For long plates, $(a/b) > 2$, this reduces to

$$P_{cr} = \frac{4\pi D}{b} \quad . \qquad (4)$$

If the long sides of the plate are clamped, the solution reduces to

$$P_{cr} = \frac{8\pi D}{b} \quad . \qquad (5)$$

Figure C2-7 shows curves for k_c for various degrees of restraint (μ) along the sides of the sheet panel, where μ is the ratio of rotational rigidity of the plate. Figure C2-8 shows curves for k_c for a flange that has one edge free and the other with various degrees of edge restraint.

Figure C2-9 gives the k_c factor for a long sheet panel with two extremes of edge stiffener, namely a zee-stiffener which is a torsionally weak stiffener and a hat section which is a closed section and, therefore, a relatively strong stiffener torsionally.

To account for buckling in the inelastic range, one must obtain the plasticity reduction factor. By using the Ramberg-Osgood parameters of Paragraph 2. 1. 1 along with Figs. C2-10 and C2-11, one can find the compression buckling stress for flat plates with various boundary conditions.

Cladding reduction factors should be obtained from Paragraph 2. 1. 1.

II. Shear Buckling

The critical elastic shear buckling stress for flat plates with various boundary conditions is given by the following equation:

$$F_{s_{cr}} = \frac{\pi^2 k_s E}{12(1-\nu_e^2)} \left(\frac{t}{b}\right)^2 \qquad (6)$$

where b is always the shorter dimension of the plate, as all edges carry shear. The shear buckling coefficient, k_s, is plotted as a function of the plate aspect ratio a/b in Fig. C2-12 for simply supported edges and clamped edges.

It is interesting to note that a long rectangular plate subjected to pure shear produces internal compressive stresses on planes at 45 degrees with the plate edges. Thus, these compressive stresses cause the long panel to buckle in patterns at an angle to the plate edges as illustrated in Fig. C2-13; the buckle patterns have a half-wave length of 1.25 b.

Shear buckling of rectangular plates with mixed boundary conditions has been investigated by Cook and Rockey [3]. The results are tabulated in Table C2-4.

If buckling occurs at a stress above the proportional limit stress, a plasticity correction factor should be included in equation (6). This factor can be taken as $\eta_s = G_s/G$ where G is the shear modulus and G_s the shear secant modulus as obtained from a shear stress-strain diagram for the material. Also, Fig. C2-14 can be used for panels with edge rotational restraint if the values of $\sigma_{0.7}$ and n are known.

III. Bending Buckling

The critical elastic bending buckling stress for flat plates is

$$F_{b_{cr}} = \frac{\pi^2 k_b E}{12(1-\nu_e^2)} \left(\frac{t}{b}\right)^2 \quad . \tag{7}$$

When a plate buckles in bending, it involves relatively short wavelength buckles equal to (2/3)b for long plates with simply supported edges (Fig. C2-15). Thus, the smaller buckle patterns cause the buckling coefficient k_b to be larger than k_c or k_s.

Figure C2-16 gives the critical stress coefficients for a plate in bending in the plane of the plate with all edges simply supported. Figure C2-17 gives the coefficients for the case when the plate tension side is simply supported and the compression side is fixed. Figure C2-18 gives the coefficients as a function of a/b for various degrees of edge rotational restraint.

The plasticity reduction factor can be obtained from Fig. C2-10 using simply supported edges.

IV. Buckling Under Combined Loads

Practical design of plates usually involves a combined load system. The buckling strength of plates under combined loads will be determined by use of interaction equations.

A. Combined Bending and Longitudinal Compression

The interaction equation that is accepted for combined bending and longitudinal compression is

$$R_b^{1.75} + R_c = 1.0 \tag{8}$$

This equation is plotted in Fig. C2-19. Also shown are curves for various margin-of-safety (M. S.) values.

B. Combined Bending and Shear

The interaction equation for combined bending and shear is

$$R_b^2 + R_s^2 = 1.0 \quad , \tag{9}$$

and the expression for margin of safety is

$$M.S. = \frac{1}{\sqrt{R_b^2 + R_s^2}} - 1 \tag{10}$$

Figure C2-20 is a plot of equation (9). Curves showing various M. S. values are also shown. R_s is the stress ratio due to torsional shear stress and R_s is the stress ratio for transverse or flexural shear stress.

C. Combined Shear and Longitudinal Direct Stress (Tension or Compression)

The interaction equation for this combination of loads is

$$R_L + R_s^2 = 1.0 \quad , \tag{11}$$

and the expression for margin of safety is

$$M.S. = \frac{2}{\left(R_L + \sqrt{R_L^2 + 4R_s^2}\right)} - 1 \quad . \tag{12}$$

Figure C2-21 is a plot of equation (11). If the direct stress is tension, it is included on the figure as negative compression using the compression allowable.

D. Combined Compression, Bending, and Shear

The conditions for buckling under combined compression, bending, and shear are represented by the interaction curves of Fig. C2-22. This figure tells whether or not the plate will buckle but will not give the margin of safety. Given the ratios R_c, R_s, and R_b: If the value of the R_c curve defined by the given value of R_b and R_s is greater numerically than the given value of R_c, then the panel will buckle.

The margin of safety of elastically buckled flat plates may be determined from Fig. C2-23. The dashed lines indicate a typical application where $R_c = 0.161$, $R_s = 0.23$, and $R_b = 0.38$. Point 1 is the first determined for the specific value of R_s and R_b. The dashed diagonal line from the origin 0 through point 1, intersecting the related R_c/R_s curve at point 2, yields the allowable shear and bending stresses for the desired margin of safety calculations. (Note: When R_c is less than R_s use the right half of the figure; in other cases use the left half.)

E. Combined Longitudinal Bending, Longitudinal Compression, and Transverse Compression

A theoretical investigation by Noel [4] has been performed on the buckling of simply supported flat rectangular plates under critical combinations of longitudinal bending, longitudinal compression, and lateral compression. Interaction curves for these loadings are presented in Fig. C2-24 for various plate aspect ratios. These curves can be used for the limiting case of two loading conditions by setting one stress ratio equal to zero. The results of the studies leading up to (and verified by) these curves indicate that the reduction in the allowable bending stress due to the additon of lateral compression is greatly magnified by the further addition of only a small longitudinal compressive load.

F. Combined Bending, Shear, and Transverse Compression

Interaction surfaces for combined bending, shear, and transverse compression have been established by Johnston and Buckert [5] for infinitely long plates. The two types of support considered were simple support along both long edges, and simple support along the tension (due to bending) edge with clamping along the compression (due to bending) edge. The resulting curves are shown in Fig. C2-25 and C2-26.

In the case of transverse compression and shear acting alone, Batdort and Houbolt [6] examined long plates with edges elastically restrained. It was found that an appreciable fraction of the critical stress in pure shear may be applied to the plate without any reduction in the transverse compressive stress necessary to produce buckling. Batdorf and Stein [7] examined simply supported plates of finite aspect ratio and found that the curve for infinitely long plates required modification for finite aspect ratios. This condition is shown in Fig. C2-27.

G. Combined Longitudinal Compression, Transverse Compression, and Shear

Johnson [8] has examined critical combinations of longitudinal compression, transverse compression, and shear for simply supported flat rectangular plates. The calculated data are presented graphically in Figs. C2-28 through C2-32. To make use of these curves, the following procedure must be observed:

1. Calculate the ratios $k_x/k_s = N_x/N_s$ and $k_y/k_s = N_y/N_s$.

2. On the curve corresponding to the plate a/b, lay off a straight line from the origin of k_x/k_s.

3. At the intersection of this line and the curve corresponding to the k_y/k_s ratio of step 1, read k_y and/or k_s.

4. Determine required plate thickness from

$$N_x = (f_x)_{appl.} (t) = \frac{k_x \pi^2 E}{12(1 - \nu_e^2)} \frac{t^3}{b^2}$$

or

$$t^3_{req'd.} = \frac{(N_x)_{appl.} B}{k_x} = \frac{(N_y)_{appl.} B}{k_y} = \frac{(N_s)_{appl.} B}{k_s}$$

where

$$B = \frac{12(1 - \nu_e^2) b^2}{\pi^2 E}$$

If desired, the value of k_y may be determined from k_s and k_y/k_s.

5. Determine the margin of safety. Assuming that the loads increase at the same rate and are therefore in the same proportion to each other at all load levels, the margin of safety based on load is given by

$$M.S. = \left(\frac{N_{cr}}{N_{appl.}}\right) - 1 = \left(\frac{t_d}{t_{req'd.}}\right)^3 - 1$$

where t_d is the design thickness. Margin of safety based on stress is given by

$$M.S. = \left(\frac{f_{cr}}{f_{appl.}}\right) - 1 = \left(\frac{t_d}{t_{req'd.}}\right)^2 - 1 \quad .$$

EXAMPLE:

Consider a simply supported plate with $a = 10$ in., $b = 5$ in., $t = 0.051$ in., $\nu_e = 0.30$, $E = 10^7$ lb/in.^{2}, $N_x = 100$ lb/in., $N_y = 32$ lb/in., and $N_s = 80$ lb/in. Determine the margin of safety.

Calculate the stress ratios and load ratios:

$$k_y/k_s = N_y/N_s = 32/80 = 0.4$$

$$k_x/k_s = N_x/N_s = 100/80 = 1.25$$

On Fig. C2-33, the interaction curves for $a/b = 2$, lay off a line from the origin of slope $k_x/k_s = 1.25$. The intersection of this line with the curve $k_y/k_s = 0.4$ determines the critical buckling coefficients for three loads. From Fig. C2-33, the following values are obtained:

$$k_x = 2.5 \quad , \quad k_s = 2.0 \quad , \quad \text{and} \quad k_y/k_s = 0.4 \quad ;$$

therefore, $k_y = 0.8$.

To determine the plate thickness required to sustain these loads, any one of the three buckling coefficients determined may be used. Using k_x, the following value is obtained:

$$t_{req'd.} = \left[\frac{N_x \, 12(1 - \nu_e^{\,2}) \, b^2}{k_x \pi^2 E} \right]^{1/3} = 0.048 \text{ in.}$$

Since the actual design thickness is 0.051 in., the margin of safety based on stress is

$$\text{M.S.} = \left[\frac{t_d}{t_{req'd.}} \right]^2 - 1 = \left(\frac{0.051}{0.048} \right)^2 - 1 = +0.1289.$$

The margin of safety based on load is

$$\text{M.S.} = \left(\frac{t_d}{t_{req'd.}} \right)^3 - 1 = +0.1995 \quad .$$

H. Combined Shear and Nonuniform Longitudinal Compression

Bleich [9] presents a solution for buckling of a plate subjected to combined shear and nonuniform longitudinal compression as shown in Fig. C2-33. The critical buckling coefficient is for

$$\alpha \geq 1: \; k = 3.85 \, \gamma^2 \beta \sqrt{\beta^2 + 3} \left(- 1.0 + \sqrt{1 + \frac{4}{\beta^2 \gamma^2}} \right) \qquad (13)$$

where

$$\gamma = \frac{5.34 + \frac{4}{\alpha^2}}{7.7} \quad ;$$

$$\frac{1}{2} \le \alpha \le 1: \ k = 3.85\, \gamma^2 \beta \sqrt{\beta^2 + 3}\left(-1 + \sqrt{1 + \frac{4}{\gamma^2 \beta^2}}\right) \qquad (14)$$

where

$$\gamma = \frac{4 + \dfrac{5.34}{\alpha^2}}{7.7 + 33\,(1-\alpha)^3}$$

V. Special Cases

A. Efficiently Tapered Plate

When a tapered plate has attained the state of unstable equilibrium, instability is characterized by deflections out of the plane of the plate in one region only. The other portions of the plate remain essentially free of such deflections. This condition of instability constitutes an inefficient design, since the same loading distribution presumably could be sustained by a lighter plate tapered in such a manner that instability under the specified loading will be characterized by deflections throughout the entire plate. For this reason Pines and Gerard [10] have examined an exponentially tapered simply supported plate subjected to compressive loads as shown in Fig. C2-34. The load variation along the plate was assumed to be produced by shear stresses small enough to have negligible influence upon the buckling characteristics of the plate. The resulting buckling coefficient versus the plate aspect ratio is plotted in Fig. C2-34 for various amounts of plate taper.

B. Compressed Plate with Variable Loading

The problem of determining the buckling stress of an axially compressed flat rectangular plate was investigated by Libove, Ferdman, and Reusch [11] for a simply supported plate with constant thickness and a linear axial load gradient. The curves appearing in Fig. C2-35 depict their results. (Long plates will buckle at the end where the maximum load is applied.)

C. Elastic Foundation

Seide [12] has obtained a solution for the problem of the compressive buckling of infintely long, flat, simply supported plates resting on an elastic foundation. It is shown that the effect of nonattachment of the plate and foundation reduces drastically the buckling load of the plate as compared to a plate with attached foundation.

2.1.1.2 Parallelogram Plates.

Parallelogram plates may exist in beam webs or in an oblique panel pattern. The technology of analysis with respect to such plates is not very well developed. However, several solutions are available which present buckling coefficients for some basic loading conditions and boundary conditions.

I. Compression

Wittrick [13] has examined the buckling stress of a parallelogram plate with clamped edges for the case of uniform compression in one direction. Results in the form of buckling curves are shown in Fig. C2-36. Comparison of these curves with those for rectangular plates shows that for compressive loads, parallelogram plates are move efficient than equivalent area rectangular plates of the same length. References 14 and 15 contain solutions for simply supported parallelogram plates subjected to longitudinal compression.

A stability analysis of a continuous flat sheet divided by nondeflecting supports into parallelogram-shaped areas (Fig. C2-37) under compressive loads has been performed by Anderson [16]. The results show that, over a wide range of panel aspect ratios, such panels are decidedly more stable than equivalent rectangular panels of the same area. Buckling coefficients are plotted in Fig. C2-37 for both transverse compression and longitudinal compression. An interaction curve for equal-sided skew panels is shown in Fig. C2-38.

Listed in Table C2-5 is a completion of critical plate buckling parameters obtained by Durvasula [17].

II. Shear

The buckling stress of a parallelogram with clamped edges subjected to shear loads has also been investigated by Wittrick [18]. It is worth noting that the shear loads are applied in such a manner that every infinitesimal rectangular element is in a state of pure shear. For such a condition to exist, the plate must be loaded as shown in Fig. C2-39. To signify this condition, the shear stresses are drawn along the y-axis in Fig. C2-40. As might be expected, unlike a rectangular plate it was found that a reversal of the direction of the shear load causes a change in its critical value. The lower shear stress value occurs when the shear is tending to increase the obliquity of the plate.

The smaller critical shear stress values are plotted in Fig. C2-40. Table C2-5 presents critical shear stress parameters for both directions of shear for several plate geometries.

2.1.1.3 Triangular Plates.

Several investigations have been performed on triangular plates. Cox and Klein [19] analyzed buckling for normal stress alone in isosceles triangles of any vertex angle. The results are shown in Fig. C2-41. The buckling of a right-angled isosceles triangular plate subjected to shear along the two perpendicular edges together with uniform compression in all directions has been considered by Wittrick [20-23]. Four combinations of boundary conditions were considered, and the buckle is assumed to be symmetrical about the bisector for the right angle. Figure C2-42 depicts the interaction curve in terms of shear and compressive stresses. In the limiting cases, these results agree with those of Cox and Klein. In Wittrick's study it was shown that for a plate subjected to shear only, the critical stress is changed considerably upon reversal of the shear. Because of this, the interaction curve is unsymmetrical and the critical compressive stress can be appreciably increased by the application of a suitable amount of shear.

2.1.1.4 Trapezoidal Plates.

Klein [24] has determined the elastic buckling loads of simply supported flat plates of isosceles trapezoidal planform loaded in compression along the parallel edges. Shear loads are assumed to act along the sloping edges so that any ratio of axial loads may act along the parallel edges of the given plate. A collocation method was used to obtain his results. The deflection function assumed does not satisfy the boundary condition for moment along the sloping edges. However, the results are accurate enough for practical purposes. (His results appear to be more incorrect for long plates where the sides comprise a large percent of the plate edges.) Buckling curves obtained are shown in Fig. C2-43 and C2-44.

Pope [25] has analyzed the buckling of a plate of constant thickness tapered symmetrically in planform and subjected to uniform compressive loading on the parallel ends. Two cases are considered:

1. Different uniform stresses applied normal to the ends, equilibrium being maintained by shear flows along the sides (Figs. C2-45 through C2-56).

2. Equal uniform stresses applied to the ends, with displacement of the sides prevented normal to the direction of taper (Figs. C2-57 through C2-60).

Boundary conditions are such that opposite pairs of edges are either simply supported or clamped. Pope has used a more rigorous analysis than Klein;

and for comparable plates, Pope's results (which represent an upper bound) are more correct and will give buckling values lower than Klein's. However, the range of applicability of Pope's curves is limited to taper angles, θ, of less than 15 degrees.

2.1.1.5 Circular Plates.

The buckling values of circular plates subjected to radial compressive loads (Fig. C2-61) have been investigated [2].

It has been shown that the critical buckling stress for a circular plate with clamped edges as shown in Fig. C2-61 is

$$f_{r_{cr}} = \frac{14.68 D}{a^2 t} \tag{15}$$

Similarly, for the case of a plate with a simply supported edge, the critical stress is

$$f_{r_{cr}} = \frac{4.20 D}{a^2 t} \tag{16}$$

The case of a circular plate subjected to unidirectional compression with clamped edges has been investigated [26] and found to be

$$N_o = \frac{32 D}{a^2} \tag{17}$$

Circular plates with a cutout center hole of radius, b, subjected to radial compressive forces have also been investigated. The critical buckling stress for these plates is

$$f_{r_{cr}} = \frac{k D}{a^2 t} \qquad (18)$$

where the values of k are given in Fig. C2-62.

2.1.2 STIFFENED PLATES.

Critical values of load for plate buckling are dependent upon the flexural rigidity of the plate. The stability of the plate can be increased by increasing its thickness, but such a design will not be economical with respect to the weight of material used. A more economical solution is obtained by keeping the thickness of the plate as small as possible and increasing the stability by introducing reinforcing ribs. For rectangular plates with longitudinal stiffeners, the stiffeners not only carry a portion of the compressive load but subdivide the plate into smaller panels, thus considerably increasing the critical stress at which the plate will buckle.

Stability analysis of flat, stiffened plates should account for both general and local modes of instability. The general mode of instability is characterized by deflection of the stiffeners while, for local instability, buckling occurs with nodes along (or nearly along) the stiffener-skin juncture. Some coupling between these two modes exists, but this effect is usually small, and, therefore, neglected. The local instability of conventionally stiffened plates and integrally stiffened plates is presented in Section C4, "Local Buckling of Stiffened Plates."

This section is concerned with the critical buckling load of the plate. It should be emphasized, however, that the problem of finding the ultimate load is distinctly different from that of finding the buckling load, and the two must not be confused. Ultimate loads of sheet-stringer combinations should be calculated using Section C1.

2.1.2.1 Conventionally Stiffened Plates in Compression.

Buckling resulting from general instability of a conventionally stiffened plate may be determined from the general equation

$$F_{cr} = \frac{k \pi^2 E_c}{12(1-\nu^2)} \left(\frac{t}{b}\right)^2 \quad . \tag{19}$$

In this case, k is a function of several of the parameters of the stiffened plate and t is the thickness of the skin. Design tables and charts will be presented for the evaluation of k for both the case where the stiffeners are parallel to the load and the case where the stiffeners are perpendicular to the load.

I. Stiffeners Parallel to Load

A. Simply Supported Plate with One Stiffener or Centerline

Consider a rectangular plate of length a, width b, and thickness t, which is reinforced by a longitudinal stiffener on the centerline (Fig. C2-63). The area of the cross section of the stiffener is A, and its moment of inertia is I, taken with respect to the axis coinciding with the outer surface of the flange. The torsional rigidity of the stiffener is regarded as small and will be neglected. Also, the following notation is used:

$$\gamma = \frac{EI}{Db} = \frac{12(1 - \nu^2)\, I}{b\, t^3} \qquad (20)$$

$$\delta = \frac{A}{bt} \qquad (21)$$

The coefficient γ is the ratio of the flexural rigidity of the stiffener to that of the plate of width b, and δ is the ratio of the cross-sectional area of the stiffener to the area bt of the plate.

If the stiffener remains straight the buckling mode is antisymmetric as shown in Fig. C2-63(c). This antisymmetric displacement form will occur when the rigidity ratio γ is larger than a certain value γ_o. For values of γ below γ_o, the symmetric displacement form in which the stiffener deflects with the plate will occur. At the ratio γ_o both configurations are equally possible.

Bleich [9] has derived the following formula for γ_o:

$$\gamma_o = 11.4\,\alpha + (1.25 + 16\,\delta)\,\alpha^2 - 5.4\sqrt{\alpha} \qquad (22)$$

where $\alpha = a/b$ and $0 \le \delta \le 0.20$. Using this, the required moment of inertia, I_o, to keep the stiffener straight is

$$I_o = 0.092\, b\, t^3\, \gamma_o \quad . \qquad (23)$$

Timoshenko [2] gives values of k to be used in equation (19) for various parameters, α, δ, and γ. These results are given in Table C2-6. The values of k above the horizontal lines in Table C2-6 indicate those proportions of the stiffener and plate for which the stiffener remains straight when the plate buckles.

B. Simply Supported Plates Having One Stiffener Eccentrically Located

Bleich [9] obtains solutions for a rectangular plate stiffened with one eccentrically located stiffener as shown in Fig. C2-64. For the particular case of $b_1/b = 1/3$, he determines a value for the moment of inertia of the stiffener required to remain undeflected during buckling. It is

$$I = 1.85\, b t^3 + 0.4\, A t^2 \qquad (\alpha \leq 1) \quad . \tag{24}$$

Also, with this (or greater) value of I, the critical buckling coefficient, k, is equal to 10.42.

C. Simply Supported Plates Having Two Equidistant Stiffeners

For the case of two stiffeners subdividing the plate into three equal panels, Timoshenko has obtained values for the coefficient k; these are given in Table C2-7 for various values of the parameters, α, δ, and γ. Bleich has obtained formulas for values of stiffener rigidity necessary for the stiffener to remain undeflected during buckling. They are

$$\gamma_o = 96 + 610\,\delta + 975\,\delta^2 \tag{25}$$

for $0 < \delta < 0.20$ and

$$I_o = 0.092\, bt^3\, \gamma_o \quad , \tag{26}$$

with the critical stress for the plate given by

$$F_{cr} = 32.5\, E \left(\frac{t}{b}\right)^2 \quad . \tag{27}$$

D. Plates Having More Than Two Stiffeners

When the number of stiffeners is equal to or greater than three, the stiffened plate can be treated as an orthotropic plate. This results in the following equation for the compression buckling coefficient:

$$k = \frac{2\left\{\left[\left[1 + \frac{N-1}{N}\left(\frac{E I_s}{bD}\right)\right]\left(1 + \frac{\frac{A\bar{z}^2}{I_s}}{1 + \frac{0.88A}{bt}}\right)\right]^{1/2} + 1\right\}}{N^2\left[\frac{N-1}{N}\left(\frac{A}{bt}\right) + 1\right]} \tag{28}$$

where the terms are defined as follows:

N number of bays

A area of stiffener cross section

I_s bending moment of inertia of stiffener cross section taken about the stiffener centroidal axis

$\bar{z}$ distance from midsurface of skin to stiffener centroidal axis

D flexural rigidity of skin per inch of width, $E\, t^3/12(1 - \nu^2)$

b spacing of stiffeners

II. Stiffeners Transverse to Load

Timoshenko [2] has studied plates with stiffeners transverse to the applied load. He has obtained several limiting values of γ at which the stiffener remains straight during buckling of the plate. These values are given in Table C2-8 for various values of α for one, two, and three transverse stiffeners.

For the case of a large number of equal and equidistant stiffeners, the plate is considered to have two different flexural rigidities in the two perpendicular directions. The critical stress is given as

$$F_{cr} = \frac{2\pi^2}{b^2 t}\left(\sqrt{D_1 D_2} + D_3\right) \qquad (29)$$

where

$D_1 = (EI)_x/(1 - \nu_x \nu_y)$, flexural rigidity in longitudinal direction;

$D_2 = (EI)_y/(1 - \nu_x \nu_y)$, flexural rigidity in transverse direction;

$D_3 = 1/2(\nu_x D_2 + \nu_y D_1) + 2(GI)_{xy}$; and

$2(GI)_{xy}$ is the average torsional rigidity.

2.1.2.2 Conventionally Stiffened Plates in Shear.

The simple cases of simply supported rectangular plates with one and two stiffeners have been investigated by Timoshenko. Tables C2-9 and C2-10 give the limiting values of the ratio γ in the case of one stiffener and two stiffeners, respectively.

Additional analysis of stiffened plates in shear is given in Section C4. 4. 0 and in Section B4. 8. 1.

2. 1. 2. 3 Conventionally Stiffened Plates in Bending.

The case of a rectangular plate reinforced with a longitudinal stiffener under bending load is common in the design of webs for shear beams. For this case, reference should be made to Section B4. 8. 1. 1.

2. 1. 2. 4 Plates Stiffened With Corrugations in Compression.

A method is presented below for the analysis of corrugated plates subjected to a compressive load applied parallel to the corrugations. Both general and local instability modes of failure are treated. General instability results in complete failure of a corrugated plate, since the corrugations are unable to develop post-buckling strength. In the case of local instability, however, the corrugations can usually develop some post-buckling strength. However, it is recommended that the lower compression buckling stress calculated for these two modes of failure be considered ultimate. It is assumed that the corrugated edges are supported in such a manner that the load is uniformly applied along these edges. All edges are assumed to be simply supported.

The compression buckling stress for the general instability mode of failure may be found by orthotropic plate analysis to be

$$F_{cr} = \eta \frac{k_o \pi^2 E_c}{12(1 - \nu^2)} \left(\frac{t}{a}\right)^2 \qquad (30)$$

where $k_o = (1 - \nu^2)\left\{ \frac{12I}{t^3 L} \left(\frac{a}{b}\right)^2 + \left(\frac{d}{L}\right)^2 \left[2\nu + 2\left(\frac{L}{d}\right)^2 + \left(\frac{b}{a}\right)^2\right]\right\}$ and

other terms are defined as follows:

E_c	modulus of elasticity in compression
a	length of the loaded edge of the plate
b	plate dimension in the direction parallel to the load
d	centerline to centerline spacing of corrugations
L	developed length per width d
I	moment of inertia of width d about neutral axis

When the plate aspect ratio a/b is greater than approximately 1/3, the computations above may be simplified since the corrugated plate behaves approximately as a wide column. For these cases, the following equation applies:

$$F_c = \eta \frac{\pi^2 E_c I}{L t b^2} \qquad (31)$$

The compression buckling stress for the local instability mode of failure may be found from the following equation when the corrugation is composed of flat elements:

$$F_{c_{cr}} = \eta \frac{k_o \pi^2 E_c}{12(1 - \nu^2)} \left(\frac{t}{b}\right)^2 \qquad (32)$$

where k_c represents simply supported edge conditions and is taken from Fig. C2-5, t is the thickness of the plate, and b is the width of the widest flat plate element of the corrugation form. The latter dimension may best be described by presenting some typical examples such as those shown in Fig. C2-65.

In the case of Fig. C2-65(d), the compression buckling stress for local instability should be based on the buckling of an axially load cylinder of radius R (see Section C3. 1).

2. 1. 2. 5 Plates Stiffened With Corrugations in Shear.

Plates stiffened with corrugations may provide a structural weight advantage for light shear loading conditions. Both local and general instability modes of failure are treated in the following methods of analysis. It is assumed in these methods that support for the corrugated edges of the plate is such that the unbuckled form of the corrugation cannot be distorted. This condition means that in the unbuckled state an externally applied shearing force will produce only shearing stresses in the corrugated plate (i. e. , no bending or torsion). In practice, this condition may be met by welding or brazing, or by rigorous mechanical joining of the corrugated plate to, for instance, a spar cap on the inner surface of a wing skin.

General instability results in the complete failure of the corrugated plate because the corrugations are unable to redistribute stresses in this mode for the development of post-buckling stress. In contrast, local instablility of the corrugations does not necessily mean failure, since some post-buckling strength can be developed for that case. It is recommended, however, that the lower shear buckling stress calculated here for these two modes of failure be considered ultimate.

The shear buckling stress for the general instability mode of failure is from Reference 1:

$$F_{s_{cr}} = \eta \frac{4k}{b^2 t} \sqrt[4]{D_1 (D_2)^3} \quad \text{when } H > 1 \qquad (33)$$

$$F_{s_{cr}} = \eta \frac{4k}{b^2 t} \sqrt{D_2 D_3} \quad \text{when } H < 1 \qquad (34)$$

where D_1 and D_2 are the flexural stiffnesses of the plate in the x and y directions, respectively; D_3 is a function of the torsional rigidity of the plate, and H is equal to $\sqrt{D_1 D_2}/D_3$. The values for k are taken from Fig. C2-66 or C2-67.

For general instability analysis, the optimum orientation of the corrugations for a reversible shear flow is parallel to the short side of the plate. For this orientation, the plate flexural stiffnesses may be expressed as follows:

$$D_1 = \frac{E_c t^3 d}{12 L} \qquad (35)$$

$$D_2 = \frac{E_c I}{d} \qquad (36)$$

$$D_3 = \nu D_1 + \frac{E_c t^3 L}{12d} \qquad (37)$$

The shear buckling stress for the local instability mode of failure may be found from the following equation when the corrugation form is composed of flat elements:

$$F_{s_{cr}} = \eta \frac{k_s \pi^2 E_c}{12(1 - \nu^2)} \left(\frac{t}{b}\right)^2 \quad . \qquad (38)$$

Here k_s represents simply supported edge conditions and is taken from Fig. C2-5. The latter dimension may best be described by referring to some typical examples such as those presented in Fig. C2-65. In the case of Fig. C2-65(d), the shear buckling stress for local instability should be based on the torsional buckling of a cylinder of radius R (see Section C3. 1).

2. 1. 2. 6 Sandwich Plates.

Procedures for the design and analysis of sandwich plates can be found in Reference 27 which contains the latest information in structural sandwich technology. It contains many formulas and charts necessary to select and check designs and its use is quite widespread in the aerospace industry.

2. 1. 2. 7 Plates of Composite Material.

The buckling of plates constructed of composite materials is presented in Section F and in Reference 28.