SECTION B9

PLATES

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B9 PLATES

B9.1 INTRODUCTION

Plate analysis is important in aerospace applications for both lateral applied loads and also for sheet buckling problems. The plate can be considered as a two-dimensional counterpart of the beam except that the plate bends in all planes normal to the plate, whereas the beam bends in one plane only.

Because of the varied behavior of plates, they have been classified into four types, as follows:

Thick Plates — Thick plate theory considers the stress analysis of plates as a three-dimensional elasticity problem. The analysis becomes, consequently, quite involved and the problem is completely solved only for a few particular cases. In thick plates, shearing stresses become important, similar to short, deep beams.

Medium-Thick Plates — In medium-thick plates, the lateral load is supported entirely by bending stresses. Also, the deflections, w, of the plate are small compared to its thickness, t, $(w < t/3)$. Theory is developed by making the following assumptions:

1. There is no in-plane deformation in the middle plane of the plate.
2. Points of the plate lying initially on a normal-to-the-middle plane of the plate remain on the normal-to-the-middle surface of the plate after bending.
3. The normal stresses in the direction transverse to the plate can be disregarded.

Thin Plates — The thin plate supports the applied load by both bending and direct tension accompanying the stretching of the middle plane. The deflections of the plate are not small compared to the thickness ($1/3t < w < 10t$) and bending of the plate is accompanied by strain in the middle surface. These supplementary tensile stresses act in opposition to the given lateral load and the given load is now transmitted partly by the flexural rigidity and partly by a membrane action of the plate. Thus, nonlinear equations can be obtained and the solution of the problem becomes much more complicated. In the case of large deflections, one must distinguish between immovable edges and edges free to move in the plane of the plate, which may have a considerable bearing upon the magnitude of deflections and stresses in the plate.

Membranes — For membranes, the resistance to lateral load depends exclusively on the stretching of the middle plane and, hence, bending action is not present. Very large deflections would occur in a membrane ($w > 10t$).

In the literature on plates, the greatest amount of information is available on medium-thick plates. Many solutions have been obtained for plates of various shapes with different loading and boundary conditions [1, 2]. However, in the aerospace industry, thin plates are the type most frequently encountered. Some approximate methods of analysis are available for thin plates for common shapes and loads.

This section includes some of the solutions for both medium-thick plates and thin plates. Plates subjected to thermal loadings are covered in Section D3.0.7. Plates constructed from composite materials are covered in Section F.

B9.2 PLATE THEORY

This section contains the theoretical solutions for medium-thick plates (small deflection), membranes, and thin plates (large deflection). Solutions for thick plates will not be given here as this type plate is seldom used in the industry.

B9.2.1 Small Deflection Theory

Technical literature on the small deflection analysis of plates contains many excellent derivations of the plate bending equations (References 1 and 2, for instance). Therefore, only key equations will be presented here.

Figure B9-1 shows the differential element of an initially flat plate acted upon by bending moments (per unit length) M_x and M_y about axes parallel to the y and x directions, respectively. Sets of twisting couples M_{xy} (= $-M_{yx}$) also act on the element.

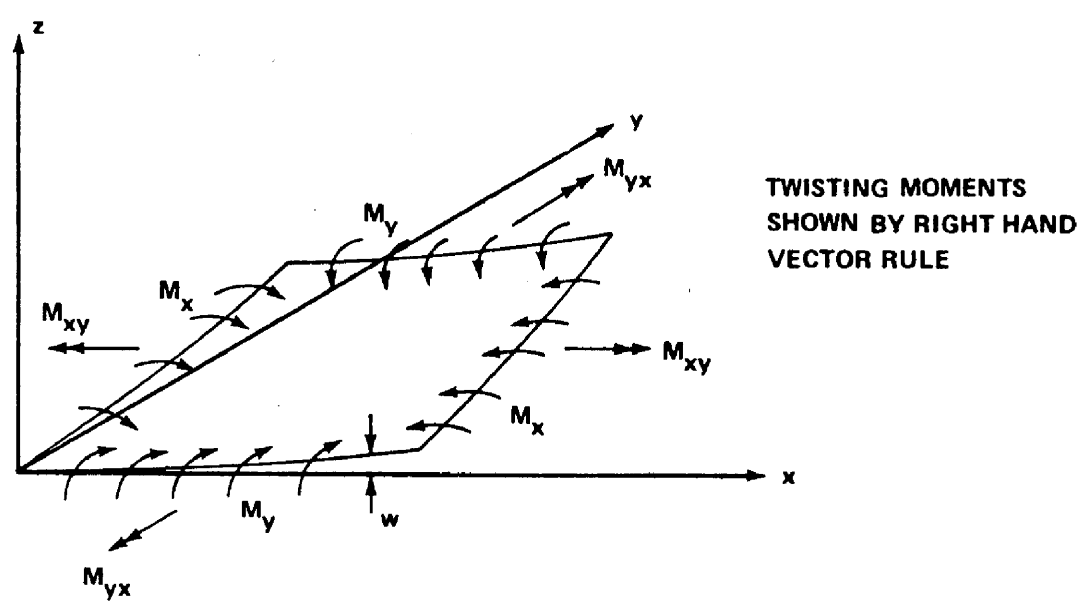


FIGURE B9-1. DIFFERENTIAL PLATE ELEMENT

As in the case of a beam, the curvature in the x, z plane, $\frac{\partial^2 w}{\partial x^2}$, is proportional to the moment M_x applied. The constant of proportionality is $\frac{1}{EI}$, the reciprocal of the bending stiffness. For a unit width of beam, $I = \frac{t^3}{12}$. In the case of a plate, due to the Poisson effect, the moment M_y also produces a (negative) curvature in the x, z plane. Thus, with both moments acting, one has

$$\frac{\partial^2 w}{\partial x^2} = \frac{12}{Et^3} \left(M_x - \mu M_y\right) \quad ,$$

where μ is Poisson's ratio. Likewise, the curvature in the y, z plane is

$$\frac{\partial^2 w}{\partial y^2} = \frac{12}{Et^3} \left(M_y - \mu M_x\right) \quad .$$

Rearranging these two equations in terms of curvature yields

$$M_x = D\left(\frac{\partial^2 w}{\partial x^2} + \mu \frac{\partial^2 w}{2y^2}\right) \tag{1}$$

$$M_y = D\left(\frac{\partial^2 w}{\partial y^2} + \mu \frac{\partial^2 w}{\partial x^2}\right) \tag{2}$$

where

$$D = \frac{Et^3}{12(1 - \mu^2)} \quad .$$

The twist of the element, $\partial^2 w/\partial x \partial y$ $(=\partial^2 w/\partial y \partial x)$ is the change in x-direction slope per unit distance in the y-direction (and vice versa). It is proportional to the twisting couple M_{xy}. A careful analysis (see References 1 and 2) gives the relation as

$$M_{xy} = D(1 - \mu)\frac{\partial^2 w}{\partial x \partial y} \quad . \tag{3}$$

Equations (1), (2), and (3) relate the applied bending and twisting couples to the distortion of the plate in much the same way as does $M = EId^2y/dx^2$ for a beam.

Figure B9-2 shows the same plate elements as the one in Fig. B9-1, but with the addition of internal shear forces Q_x and Q_y (corresponding to the "v" of beam theory) and a distributed transverse pressure load q(psi). With the presence of these shears, the bending and twisting moments now vary along the plate as indicated in Fig. B9-2a.

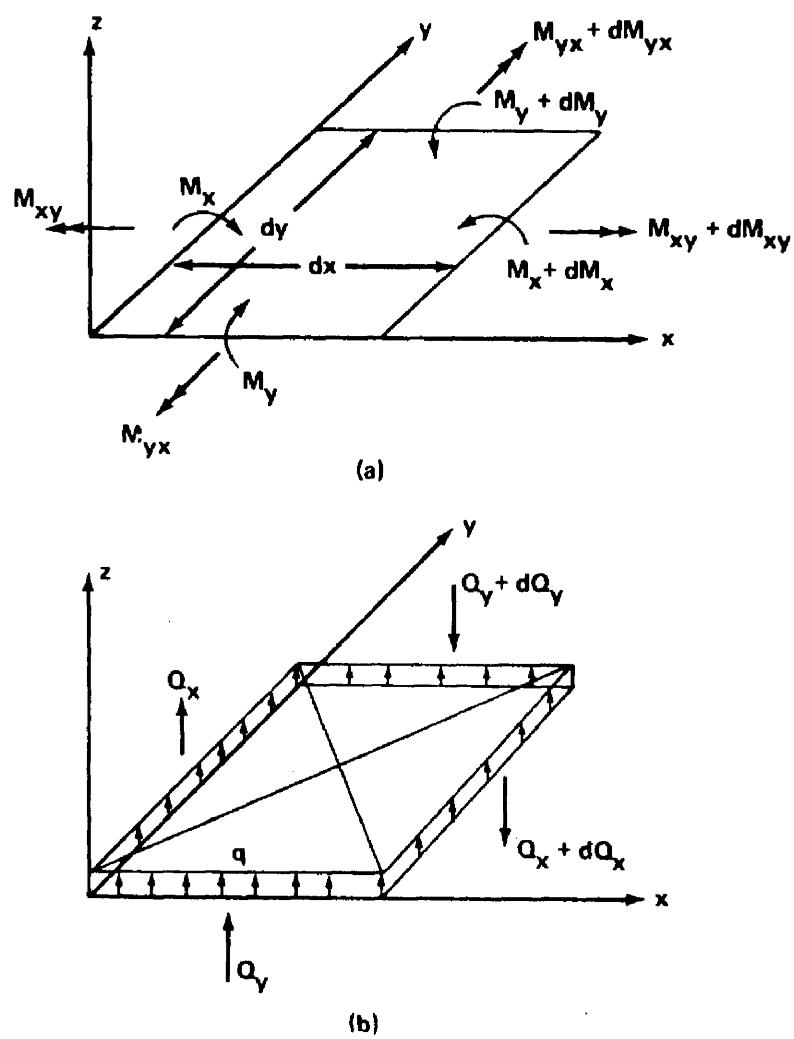


FIGURE B9-2. DIFFERENTIAL PLATE ELEMENT WITH LATERAL LOAD

By summing moments of the two loading sets of Figs. B9-2a and B9-2b about the y axis, one obtains

$$M_x dy + (M_{yx} + dM_{yx})dx + (Q_x + dQ_x)dxdy = (M_x + dM_x)dy + M_{yx}dx \quad .$$

Dividing by dxdy and discarding the term of higher order yields

$$Q_x = \frac{\partial M_x}{\partial x} - \frac{\partial M_{yx}}{\partial y} \quad , \tag{4}$$

or,

$$Q_x = \frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} \quad . \tag{4a}$$

In a similar manner, a moment summation about the x-axis yields

$$Q_y = \frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} \quad . \tag{5}$$

[Equations (4) and (5) correspond to $V = dM/dx$ in beam theory.]

One final equation is obtained by summing forces in the z-direction on the element:

$$q = \frac{\partial Q_x}{\partial y} + \frac{\partial Q_y}{\partial y} \quad . \tag{6}$$

Equations (4), (5), and (6) provide three additional equations in the three additional quantities Q_x, Q_y, and q. The plate problem is, thus, completely defined. A summary of the quantities and equations obtained above are presented in Table B9-1. For comparison, the corresponding items from the engineering theory of beams are also listed.

Table B9-1. Tabulation of Plate Equations

Class	Item	Plate Theory	Beam Theory
Geometry	Coordinates	$x \quad y$	x
	Deflections	w	y
	Distortions	$\frac{\partial^2 w}{\partial x^2}, \frac{\partial^2 w}{\partial y^2}, \frac{\partial^2 w}{\partial x \partial y}$	$\frac{d^2 y}{dx^2}$
Structural Characteristic	Bending Stiffness	$D = \frac{Et^3}{12(1 - \mu^2)}$	EI
Loadings	Couples	M_x, M_y, M_{xy}	M
	Shears	Q_x, Q_y	V
	Lateral	q	q or w
Hooke's Law	Moment	$M_x = D\left(\frac{\partial^2 w}{\partial x^2} + \mu \frac{\partial^2 w}{\partial y^2}\right)$	
	Distortion	$M_y = D\left(\frac{\partial^2 w}{\partial y^2} + \mu \frac{\partial^2 w}{\partial x^2}\right)$	$M = EI \frac{d^2 y}{dx^2}$
	Relation	$M_{xy} = D(1 - \mu) \frac{\partial^2 w}{\partial x \partial y}$	
Equilibrium	Moments	$Q_x = \frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y}$ $Q_y = \frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x}$	$V = \frac{dM}{dx}$
	Forces	$q = \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y}$	$q = \frac{dV}{dx}$

Finally, one very important equation is obtained by eliminating all internal forces (M_x, M_y, M_{xy}, Q_x, Q_y) between the six equations above. The result is a relation between the lateral loading q and the deflections w (for a beam, $q/EI = d^4y/dx^4$):

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D} \quad . \tag{7}$$

The plate bending problem is thus reduced to an integration of equation (7). For a given lateral loading $q(x, y)$, a deflection function $w(x, y)$ is sought which satisfies both equation (7) and the specified boundary conditions. Once found, $w(x, y)$ can be used in equations (1) through (5) to determine the internal forces and stresses. Often, various approximate methods are used to solve equation (7). One of the most powerful is the finite difference technique, presented in Reference 1.

It must be emphasized that in deriving the plate-bending equations it was assumed that no stresses acted in the middle (neutral) plane of the plate (no membrane stresses). Thus, in summing forces to derive equation (6), no membrane stresses were present to help support the lateral load. In the solutions to the great majority of all plate-bending problems, the deflection surface found is a nondevelopable surface, i.e., a surface which cannot be formed from a flat sheet without some stretching of the sheet's middle surface. But, if appreciable middle surface strains must occur, then large middle surface stresses will result, invalidating the assumption from which equation (6) was derived.

Thus, practically all loaded plates deform into surfaces which induce some middle surface stresses. It is the necessity for holding down the magnitude of these very powerful middle surface stretching forces that results in the more severe rule-of-thumb restriction that plate bending formulae apply accurately only to problems in which deflections are a few tenths of the plate's thickness.

B9.2.1.1 Orthotropic Plates

In the previous discussion it was assumed that the elastic properties of the material of the plate were the same in all directions. It will now be assumed that the material of the plate has three planes of symmetry with respect to the elastic properties. Such plates are generally called orthotropic plates. The bending of plates with more general elastic properties (anisotropic plates) is considered in Section F.

For orthotropic plates the relationship between stress and strain components for the case of plane stress in the x, y plane is presented by the following equations:

$$\sigma_x = E'_x \epsilon_x + E'' \epsilon_y$$

$$\sigma_y = E'_y \epsilon_y + E'' \epsilon_x$$

$$\tau_{xy} = G \gamma_{xy} \qquad (8)$$

Following procedures outlined in Reference 1, the expression for bending and twisting moments are

$$M_x = D_x \frac{\partial^2 w}{\partial x^2} + D_1 \frac{\partial^2 w}{\partial y^2} \quad (9)$$

$$M_y = D_y \frac{\partial^2 w}{\partial y^2} + D_1 \frac{\partial^2 w}{\partial x^2} \quad (10)$$

$$M_{xy} = 2D_{xy} \frac{\partial^2 w}{\partial x \partial y} \quad (11)$$

in which

$$D_x = \frac{E'_x t^3}{12} , \quad D_y = \frac{E'_y t^3}{12} , \quad D_1 = \frac{E'' t^3}{12} , \quad D_{xy} = \frac{G t^3}{12} \quad .$$

The relationship between the lateral loading q and the deflections w becomes:

$$D_x \frac{\partial^4 w}{\partial x^4} + 2\left(D_1 + 2D_{xy}\right) \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = q \quad . \quad (12)$$

Equation (12) can be used in the investigation of plate bending for many various types of orthotropic construction which have different flexural rigidities in two mutually perpendicular directions. Specific solutions will be given in Subsection B9.5, Orthotropic Plates.

B9.2.2 Membrane Theory

Before large deflection theory of plates is discussed, one should consider the limiting case of the flat membrane which cannot support any of the lateral load by bending stresses and, hence, has to deflect and stretch to develop both the necessary curvatures and membrane stresses.

The two-dimensional membrane problem is a nonlinear one whose solution has proven to be very difficult [3]. However, we can study a simplified version whose solution retains the desired general features. The one-dimensional analysis of a narrow strip cut from an originally flat membrane whose length in the y-direction is very large (Fig. B9-3).

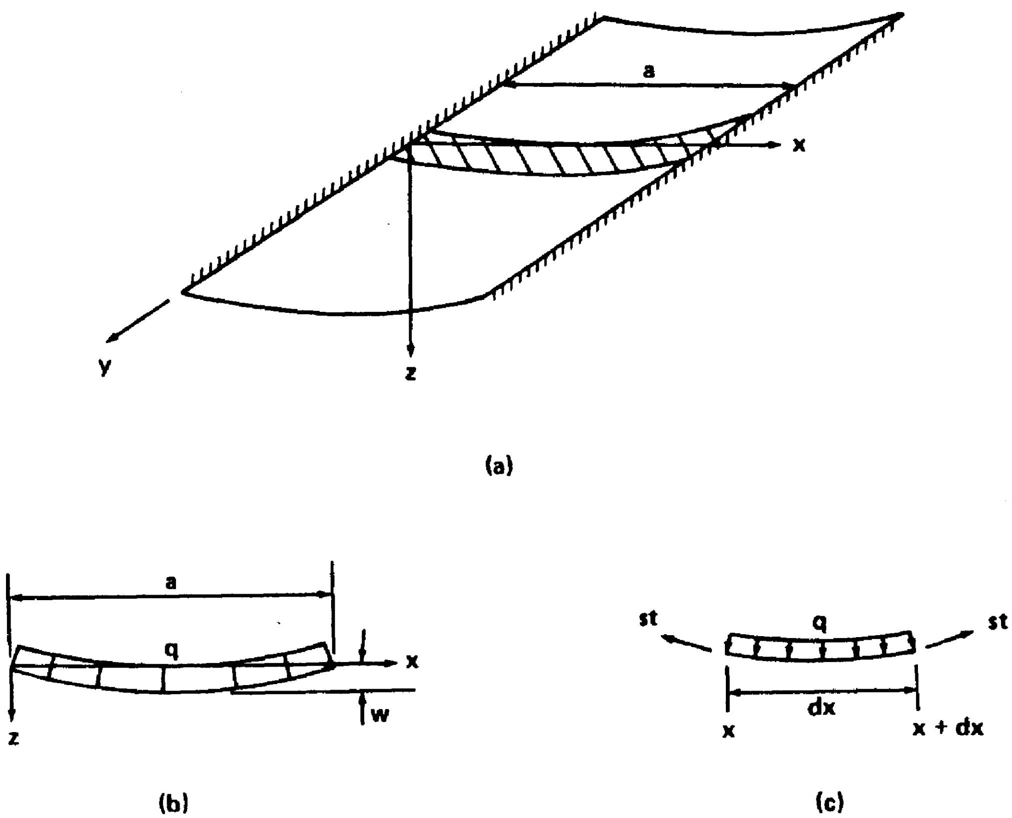


FIGURE B9-3. ONE-DIMENSIONAL MEMBRANE

Figure B9-3 shows the desired one-dimensional problem which now resembles a loaded cable. The differential equation of equilibrium is obtained by summing vertical forces on the element of Fig. B9-3c.

$$st\left(\frac{dw}{dx}\bigg|_{x+dx} - \frac{dw}{dx}\bigg|_{x}\right) + qdx = 0$$

or

$$\frac{d^2w}{dx^2} = -\frac{q}{st} \qquad , \tag{13}$$

where s is the membrane stress in psi. Equation (13) is the differential equation of a parabola. Its solution is

$$w = \frac{qx}{2st}(a-x) \quad . \tag{14}$$

The unknown stress in equation (14) can be found by computing the change in length of the strip as it deflects. From Reference 3, this stretch δ is

$$\delta = \frac{1}{2}\int_0^a \left(\frac{dw}{dx}\right)^2 dx \quad .$$

Substituting through the use of equation (14) and integrating yields

$$\delta = \frac{q^2a^3}{24s^2t^2} \quad ,$$

and consideration of the stress-strain relationship yields

$$s = \frac{\delta}{a} E \quad .$$

By equating and solving for s one finds

$$s = 0.347 \left[E\left(\frac{qa}{t}\right)^2\right]^{\frac{1}{3}} \quad . \tag{15}$$

If equation (15) is substituted into equation (14), the maximum deflection at $x = a/2$ is

$$w_{max} = 0.360\, a \left(\frac{qa}{Et}\right)^{\frac{1}{3}} \quad . \tag{16}$$

Solutions of the complete two-dimensional nonlinear membrane problem have been obtained in Reference 4, the results being expressed in forms identical to those obtained above for the one-dimensional problem.

$$w_{max} = n_1 a \left(\frac{qa}{Et}\right)^{\frac{1}{3}} \quad (17)$$

$$s_{max} = n_2 \left[E\left(\frac{qa}{t}\right)^2\right]^{\frac{1}{3}} \quad . \quad (18)$$

Here a is the length of the long side of the rectangular membrane, and n_1 and n_2 are given in Table B9-2 as functions of the panel aspect ratio a/b.

Table B9-2. Membrane Stress and Deflection Coefficients

a/b	1.0	1.5	2.0	2.5	3.0	4.0	5.0
n_1	0.318	0.228	0.16	0.125	0.10	0.068	0.052
n_2	0.356	0.37	0.336	0.304	0.272	0.23	0.205

The maximum membrane stress (s_{max}) occurs at the middle of the long side of the panel.

Experimental results reported in Reference 4 show good agreement with the theory for square panels in the elastic range.

B9.2.3 Large Deflection Theory

The theory has been outlined for the analysis of the two extreme cases of sheet panels under lateral loads. At one extreme, sheets whose bending stiffness is great relative to the loads applied (and which therefore deflect only slightly) may be analyzed satisfactorily by the plate bending solutions. At the other extreme, very thin sheets, under lateral loads great enough to cause large deflections, may be treated as membranes whose bending stiffness is ignored.

As it happens, the most efficient plate designs generally fall between these two extremes. On the one hand, if the designer is to take advantage of the presence of the interior stiffening (rings, bulkheads, stringers, etc.), which is usually present for other reasons anyway, then it is not necessary to make the skin so heavy that it behaves like a "pure" plate. On the other hand, if the skin is made so thin that it requires supporting of all pressure loads by stretching and developing membrane stresses, then permanent deformation results, producing "quilting" or "washboarding."

The exact analysis of the two-dimensional plate which undergoes large deflections and thereby supports the lateral loading partly by its bending resistance and partly by membrane action is very involved. As shown in Reference 1, the investigation of large deflections of plates reduces to the solution of two non-linear differential equations. The solution of these equations in the general case is unknown, but some approximate solutions of the problem are known and are discussed in Reference 1.

An approximate solution of the large deflection plate problem can be obtained by adding the small deflection membrane solutions in the following way:

The expression relating deflection and uniform lateral load for small deflection of a plate can be found to be

$$w_{max} = \alpha \frac{q' a^4}{Et^3} , \qquad (19)$$

where α is a coefficient dependent upon the geometry and boundary conditions of the plate.

The similar expression for membrane plates is equation (17)

$$w_{max} = n_1 a \left(\frac{q''a}{Et}\right)^{\frac{1}{3}} \quad . \tag{20}$$

Solving equations (19) and (20) for q' and q'' and adding the results yields

$$q = q' + q''$$

$$q = \frac{1}{\alpha} \frac{Et^3}{a^4\left(\frac{b}{a}\right)^4} w_{max} + \frac{1}{n_1^3} \frac{Et}{a^4} w_{max}^3 \quad . \tag{21}$$

Obviously, equation (21) is based upon summing the individual stiffnesses of the two extreme behavior mechanisms by which a flat sheet can support a lateral load. No interaction between stress systems is assumed and, since the system is nonlinear, the result can be an approximation only.

Equation (21) is best rewritten as

$$\frac{qa^4}{Et^4} = \frac{1}{\alpha}\left(\frac{w_{max}}{t}\right)\left(\frac{a}{b}\right)^4 + \frac{1}{n_1^3}\left(\frac{w_{max}}{t}\right)^3 \quad . \tag{22}$$

Figure B9-4 shows equation (22) plotted for a square plate using values of $\alpha = 0.0443$, and $n_1 = 0.318$. Also plotted are the results of an exact analysis [1]. As may be seen, equation (22) is somewhat conservative inasmuch as it gives a deflection which is too large for a given pressure.

The approximate large-deflection method outlined above has serious shortcomings insofar as the prediction of stresses is concerned. For simply supported edges, the maximum combined stresses are known to occur at the panel midpoint. Figure B9-5 shows plots of these stresses for a square panel as predicted by the approximate method (substituting q' and q'' into appropriate stress equations).

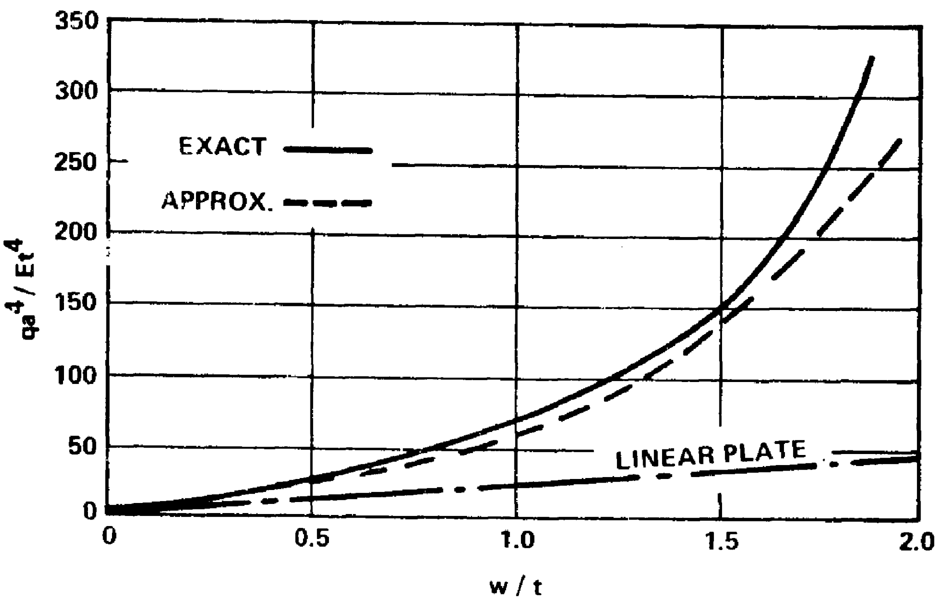


FIGURE B9-4. DEFLECTIONS AT THE MIDPOINT OF A SIMPLY SUPPORTED SQUARE PANEL BY TWO LARGE-DEFLECTION THEORIES

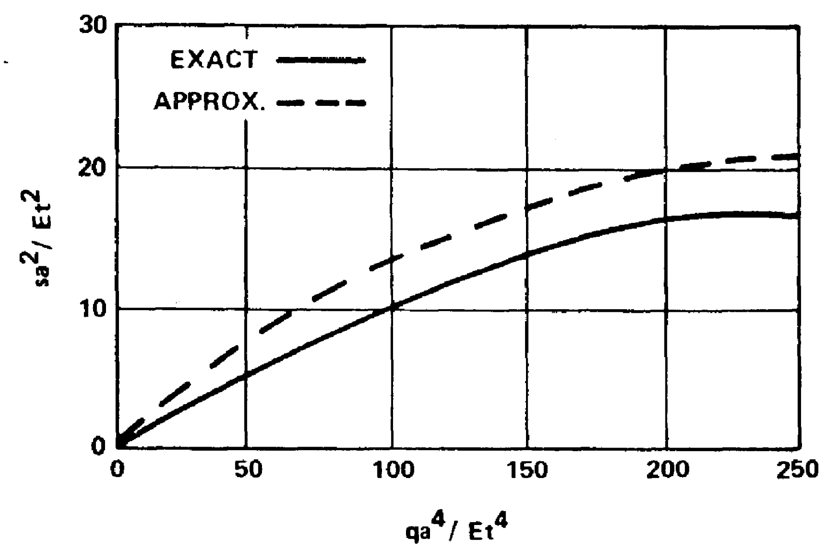


FIGURE B9-5. LARGE DEFLECTION THEORIES' MIDPANEL STRESSES; SIMPLY SUPPORTED PANEL

B9.3 MEDIUM-THICK PLATES (SMALL DEFLECTION THEORY)

This section includes solutions for stress and deflections for plates of various shapes for different loading and boundary conditions. All solutions in this section are based on small deflection theory as described in Paragraph B9.2.1.

B9.3.1 Circular Plates

For a circular plate it is naturally convenient to express the governing differential equations in polar coordinate form. The deflection surface of a laterally loaded plate in polar coordinate form is

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right)\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right) = \frac{q}{D} \quad . \qquad (23)$$

If the load is symmetrically distributed with respect to the center of the plate, w is independent of θ and the equation becomes

$$\frac{1}{r}\frac{d}{dr}\left\{r\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right]\right\} = \frac{q}{D} \quad . \qquad (24)$$

The bending and twisting moments are

$$M_r = D\left[\frac{\partial^2 w}{\partial r^2} + \mu\left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right)\right] \qquad (25)$$

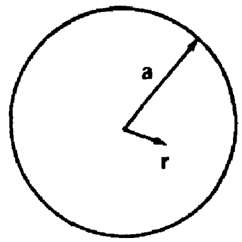
$$M_t = D\left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} + \mu\frac{\partial^2 w}{\partial r^2}\right) \qquad (26)$$

$$M_{rt} = (1-\mu)D\left(\frac{1}{r}\frac{\partial^2 w}{\partial r \partial \theta} - \frac{1}{r^2}\frac{\partial w}{\partial \theta}\right) \qquad (27)$$

B9.3.1.1 Solid, Uniform-Thickness Plates

Solutions for solid circular plates have been tabulated for many loadings and boundary conditions. The results are presented in Table B9-3.

Table B9-3. Solutions for Circular Solid Plates



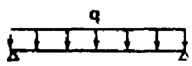
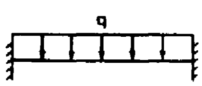
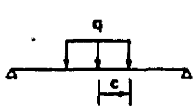
Case	Formulas For Deflection And Moments
Supported Edges, Uniform Load  	$w = \frac{qa^2}{16D(1+\mu)}(a^2-r^2)$ $\quad w_{max} = \frac{(5+\mu)}{64(1+\mu)}\frac{qa^4}{D}$ $M_r = \frac{q}{16}(3+\mu)(a^2-r^2)$ $\quad (M_r)_{max} = (M_t)_{max} = \frac{3+\mu}{16}qa^2$ $M_t = \frac{q}{16}[a^2(3+\mu) - r^2(1+3\mu)]$ At Edge $\theta = \frac{Pa}{8\pi(1+\mu)}$
Clamped Edges, Uniform Load  	$w = \frac{q}{64D}(a^2-r^2)$ $\quad w_{max} = \frac{qa^4}{64D}$ $M_r = \frac{q}{16}[a^2(1+\mu) - r^2(3+\mu)]$ $(M_r)_{max}$ at $r=a$ $= \frac{-qa^2}{8}$ $M_t = \frac{q}{16}[a^2(1+\mu) - r^2(1+3\mu)]$ $(M_r)_{r=0} = \frac{qa^2}{16}(1+\mu)$
Supported Edges, Uniform Load Over Concentric Circular Area of Radius, c  $P = \pi c^2 q$	$w = \frac{P}{16\pi D}\left\{\frac{3+\mu}{1+\mu}(a^2-r^2) + 2r^2\log\frac{r}{a} + c^2\left[\log\frac{r}{a} - \frac{1-\mu}{2(1+\mu)}\frac{a^2-r^2}{a^2}\right]\right\}$ $w_{r=0} = \frac{P}{16\pi D}\left[\frac{3+\mu}{1+\mu}a^2 + c^2\log\frac{c}{a} - \frac{7+3\mu}{4(1+\mu)}c^2\right]$ At Center $M_{max} = \frac{P}{4\pi}\left[(1+\mu)\log\frac{a}{c} + 1 - \frac{(1-\mu)c^2}{4a^2}\right]$ At Edge $\theta = \frac{Pa}{4\pi(1+\mu)}$

Table B9-3. (Continued)

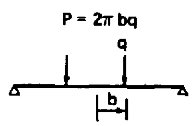
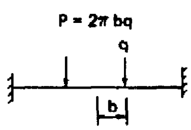
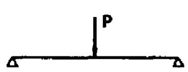
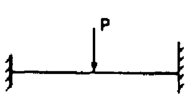
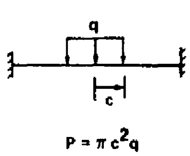
Case	Formulas For Deflection And Moments
Simply Supported, Uniform Load On Concentric Circular Ring Of Radius, b 	$(w)_{r=b} = \frac{P}{8\pi D}\left[(a^2-b^2)\left(1 + \frac{1}{2}\frac{1-\mu}{1+\mu}\frac{a^2-b^2}{a^2}\right) + 2b^2\log\frac{b}{a}\right]$ $\max(w)_{r=0} = \frac{P}{8\pi D}\left[b^2\log\frac{b}{a} + (a^2-b^2)\frac{(3+\mu)}{2(1+\mu)}\right]$ $M_{r=b} = \frac{(1+\mu)P(a^2-b^2)}{8\pi a^2} - \frac{(1+\mu)\,P\log\frac{b}{a}}{4\pi}$
Fixed Edges, Uniform Load On Concentric Circular Ring Of Radius, b 	$(w)_{r=b} = \frac{P}{8\pi D}\left(\frac{a^4-b^4}{2a^2} + 2b^2\log\frac{b}{a}\right)$ $\max(w)_{r=0} = \frac{P}{8\pi D}\left[b^2\log\frac{b}{a} + \frac{(a^2-b^2)}{2}\right]$ $M_{r=a} = \frac{P}{4\pi}\frac{a^2-b^2}{a^2}$
Simply Supported, Concentrated Load At Center 	$w = \frac{P}{16\pi D}\left[\frac{3+\mu}{1+\mu}(a^2-r^2) + 2r^2\log\frac{r}{a}\right]$ $w_{max} = \frac{(3+\mu)}{16\pi(1+\mu)}\frac{Pa^2}{D}$ $M_r = \frac{P}{4\pi}(1+\mu)\log\frac{a}{r}$ $M_t = \frac{P}{4\pi}\left[(1+\mu)\log\frac{a}{r} + 1 - \mu\right]$
Fixed Edges, Concentrated Load At Center 	$w = \frac{Pr^2}{8\pi D}\log\frac{r}{a} + \frac{P}{16\pi D}(a^2-r^2)$ $w_{max} = \frac{3}{48\pi}\frac{Pa^2}{D}$ $M_r = \frac{P}{4\pi}\left[(1+\mu)\log\frac{a}{r} - 1\right]$ $M_t = \frac{P}{4\pi}\left[(1+\mu)\log\frac{a}{r} - \mu\right]$
Clamped Edges, Uniform Load Over Concentric Circular Area Of Radius, c 	$w_{max}(r=0) = \frac{P}{64\pi D}\left(4a^2 - 4c^2\log\frac{a}{c} - 3c^2\right)$ At r=a $M_r = \frac{P}{4\pi}\left(1 - \frac{c^2}{2a^2}\right) \qquad M_t = \mu M_r$ At r=0 $M_r = M_t = \frac{P(1+\mu)}{4\pi}\left(\log\frac{a}{c} + \frac{c^2}{4a^2}\right)$

Table B9-3. (Continued)

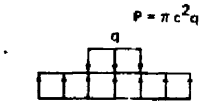
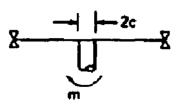
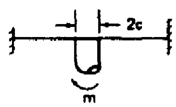
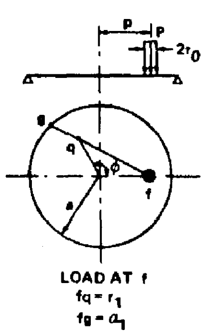
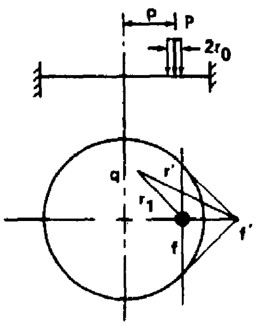
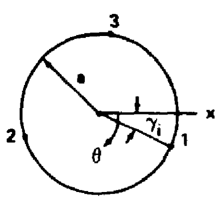
Case	Formulas For Deflection And Moments
Supported By Uniform Pressure Over Entire Lower Surface, Uniform Load Over Concentric Circular Area Of Radius, c  	At r=0 $w = \frac{P}{64\pi D}\left[4c^2 \log\frac{a}{c} + 2c^2\left(\frac{3+\mu}{1+\mu}\right) + \frac{c^4}{a^2} - a^2\left(\frac{7+3\mu}{1+\mu}\right) + \frac{(a^2-c^2)c^2}{a^2}\right]$ $M_r = M_t = \frac{P}{4\pi}\left[(1+\mu)\log\frac{a}{c} + \frac{1}{4}(1-\mu)\left(1-\frac{c^2}{a^2}\right)\right]$ If $c \to 0$ $w = \frac{Pa^2}{64\pi D}\frac{(7+3\mu)\mu}{(1+\mu)}$
No Support, Uniform Edge Moment  	$w = \frac{M(a^2-r^2)}{2D(1+\mu)}$ $\qquad w_{r=0} = \frac{Ma^2}{2D(1+\mu)}$ Edge Rotation $\theta = \frac{Ma}{D(1+\mu)}$
Edges Supported, Central Couple (Trunnion Loading)  	At r=c $M = \frac{9m}{2\pi c}\left[1 + (1+\mu)\log\frac{2(a-c)}{Ka}\right]$ where $K = \frac{0.49\,a^2}{(c+0.7a)^2}$
Edges Clamped, Central Couple (Trunnion Loading)  	At r=c $M = \frac{9m}{2\pi c}\left[1 + (1+\mu)\log\frac{2(0.45\,a-c)}{0.45\,ka}\right]$ where $k = \frac{0.1\,a^2}{(c+0.28\,a)^2}$
Edges Supported, Uniform Load Over Small Eccentric Circular Area Of Radius, r_o  	At Point of Load: $M_r = M_t = \frac{P}{4\pi}\left\{1 + (1+\mu)\log\frac{a-p}{r_o} - (1-\mu)\left[\frac{r_o^2}{4(a-p)^2}\right]\right\}$ At Point q: $w = K_0(r^3-b_0ar^2+c_0a^3) + K_1(r^4-b_1ar^3+c_1a^3r)\cos\phi + K_2(r^4-b_2ar^3+c_2a^2r^2)\cos\phi$ where $K_0 = \frac{2(1+\mu)P(p^3-b_0ap^2+c_0a^3)}{9(5+\mu)K\pi a^4}$, $K = \frac{Et^3}{12(1-\mu^2)}$ $K_1 = \frac{2(3+\mu)P(p^4-b_1ap^3+c_1a^3p)}{3(9+\mu)K\pi a^6}$, $b_0 = \frac{3(2+\mu)}{2(1+\mu)}$ $K_2 = \frac{(4+\mu)^2P(p^4-b_2ap^3+c_2a^2p^2)}{(9+\mu)(5+\mu)K\pi a^6}$, $b_1 = \frac{3(4+\mu)}{2(3+\mu)}$ $b_2 = \frac{2(5+\mu)}{4+\mu}$, $c_0 = \frac{4+\mu}{2(1+\mu)}$, $c_1 = \frac{6+\mu}{2(3+\mu)}$, $c_2 = \frac{6+\mu}{4+\mu}$

Table B9-3. (Concluded)

Case	Formulas For Deflection and Moments
Edges Fixed, Uniform Load Over Small Eccentric Circular Area of Radius, r_0  	At Point of Load: $M_r = \frac{P}{4\pi}\left[(1+\mu)\log\frac{a-p}{r_0} + (1+\mu)\frac{r_0^2}{4(a-p)^2}\right]$ = max M when $r_0<0.6(a-p)$ $w = \frac{3P(1-\mu^2)(a^2-p^2)^2}{4\pi Et^3a^2}$ At Point q: $w = \frac{3P(1-\mu^2)}{2\pi Et^3}\left[\frac{1}{2}\left(\frac{p^2r'^2}{a^2} - r_1^2\right) - r_1^2\log\frac{pr'}{ar_1}\right]$ At Edge: $M_r = \frac{P}{4\pi}\left[1 - \frac{r_0^2}{2(a-p)^2}\right]$ = max M when $r_0 > 0.6(a-p)$
Supported At Several Points Along The Boundary  	Supported At Two Points: ($\gamma_1 = 0,\ \gamma_2 = \pi$) Load P at Center: $w_{r=0} = 0.116\frac{Pa^2}{D}$ $w_{r=a,\ \theta=\pi/2} = 0.118\frac{Pa^2}{D}$ Uniformly Loaded Plate: $w_{r=0} = 0.269\frac{qa^4}{D}$ $w_{r=a,\ \theta=\pi/2} = 0.371\frac{qa^4}{D}$ Supported At Three Points 120 Deg Apart: Load P at Center $w_{r=0} = 0.0670\frac{Pa^2}{D}$ Uniformly Loaded $w_{r=0} = 0.1137\frac{qa^4}{D}$
Edge Supported, Linearly Distributed Load Symmetrical About Diameter  	$\max M_r = \frac{qa^2(5+\mu)}{72\sqrt{3}}$ at $r = 0.577\,a$ $\max M_t = \frac{qa^2(5+\mu)(1+3\mu)}{72(3+\mu)}$ at $r = 0.675\,a$ max edge reaction per linear inch $= \frac{1}{4}qa$ $\max w = 0.042\frac{qa^4}{Et^3}$ at $r = 0.503\,a$ ($\mu = 0.3$)

B9.3.1.2 Annular, Uniform-Thickness Plates

Solutions for annular circular plates with a central hole are tabulated in Table B9-4.

B9.3.1.3 Solid, Nonuniform-Thickness Plates

For the plates treated here, the thickness is a function of the radial distance, and the acting load is symmetrical with respect to the center of the plate.

I. Linearly Varying Thickness:

The plate of this type is shown in Fig. B9-6.

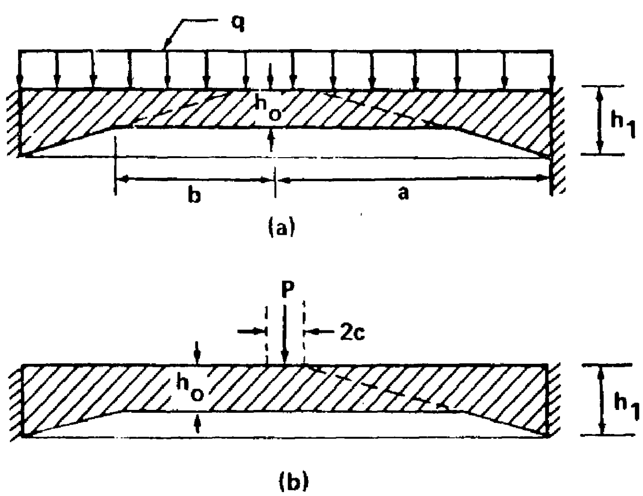
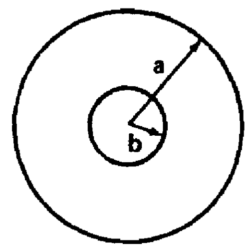


FIGURE B9-6. CIRCULAR PLATE WITH LINEARLY VARYING THICKNESS

Tables B9-5 and B9-6 give the deflection w_{max} and values of bending moments of the plate in two cases of loading. To calculate the bending moment at the center in the case of a central load P, one may assume a uniform distribution of that load over a small circular area of a radius c. The moment $M_r = M_t$ at $r = 0$ can be expressed in the form

$$M_{max} = \frac{P(1 + \mu)}{4\pi}\left(\log \frac{a}{c} + \frac{c^2}{4a^2}\right) + \gamma_1 P \quad . \tag{28}$$

Table B9-4. Solutions For Annular, Uniform-Thickness Plates



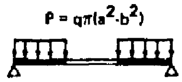
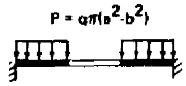
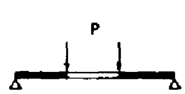
Case	Formulas For Deflection And Moments
Outer Edge Supported, Uniform Load Over Entire Actual Surface P = qπ(a²-b²) 	At Inner Edge: $\max M = M_t = \frac{q}{8(a^2-b^2)}\left[a^4(3+\mu) + b^4(1-\mu) - 4a^2b^2 - 4(1+\mu)a^2b^2 \log\frac{a}{b}\right]$ When b Is Very Small $\max M = M_t = \frac{qa^2}{8}(3+\mu)$ $\max w = \frac{q}{8D}\left[\frac{a^4(5+\mu)}{8(1+\mu)} + \frac{b^4(7+3\mu)}{8(1+\mu)} - \frac{a^2b^2(3+\mu)}{2(1+\mu)} + \frac{a^2b^2(3+\mu)}{2(1-\mu)}\log\frac{a}{b} - \frac{2a^2b^4(1+\mu)}{(a^2-b^2)(1-\mu)}\left(\log\frac{a}{b}\right)^2\right]$
Outer Edge Clamped, Uniform Load Over Entire Actual Surface P = qπ(a²-b²) 	At Outer Edge: $\max M_r = \frac{q}{8}\left[a^2 - 2b^2 + \frac{b^4(1-\mu) - 4b^4(1+\mu)\log\frac{a}{b} + a^2b^2(1+\mu)}{a^2(1-\mu) + b^2(1+\mu)}\right]$ $\max w = \frac{q}{64D}\left\{a^4 + 5b^4 - 6a^2b^2 + 8b^4\log\frac{a}{b} - \frac{[8b^6(1+\mu) - 4a^2b^2(3+\mu) - 4a^2b^2(1+\mu)]\log\frac{a}{b} + 16a^2b^4(1+\mu)\left(\log\frac{a}{b}\right)^2}{a^2(1-\mu) + b^2(1+\mu)} + \frac{4a^2b^4 - 2a^4b^2(1+\mu) + 2b^6(1-\mu)}{a^2(1-\mu) + b^2(1+\mu)}\right\}$
Outer Edge Supported, Uniform Load Along Inner Edge 	At Inner Edge: $\max M = M_t = \frac{P}{4\pi}\left[\frac{2a^2(1+\mu)}{a^2-b^2}\log\frac{a}{b} + (1-\mu)\right]$ $\max w = \frac{P}{16\pi D}\left[\frac{(a^2-b^2)(3+\mu)}{(1+\mu)} + \frac{4a^2b^2(1+\mu)}{(1-\mu)(a^2-b^2)}\left(\log\frac{a}{b}\right)^2\right]$

Table B9-4. (Continued)

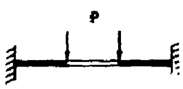
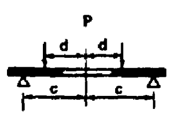
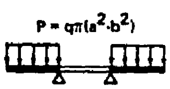
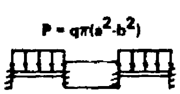
Case	Formulas For Deflection And Moments
Outer Edge Clamped, Uniform Load Along Inner Edge 	At Outer Edge: $\max M_r = \frac{P}{4\pi}\left[1 - \frac{2b^2 - 2b^2(1+\mu)\log\frac{a}{b}}{a^2(1-\mu) + b^2(1+\mu)}\right] = \max M \text{ when } \frac{a}{b} < 2.4$ At Inner Edge: $\max M_t = \frac{P\mu}{4\pi}\left[1 + \frac{a^2(1-\mu) - b^2(1+\mu) - 2(1-\mu^2)a^2\log\frac{a}{b}}{\mu a^2(1-\mu) + b^2(1+\mu)}\right]$ $= \max M \text{ when } \frac{a}{b} > 2.4$ $\max w = \frac{P}{16\pi D}\left[a^2 - b^2 + \frac{2b^2(a^2-b^2) - 8a^2b^2\log\frac{a}{b} + 4a^2b^2(1+\mu)\left(\log\frac{a}{b}\right)^2}{a^2(1-\mu) + b^2(1+\mu)}\right]$
Supported Along Concentric Circle Near Outer Edge, Uniform Load Along Concentric Circle Near Inner Edge 	At Inner Edge: $\max M = M_t = \frac{P}{4\pi}\left[\frac{2a^2(1+\mu)}{a^2-b^2}\log\frac{c}{\alpha} + (1-\mu)\frac{c^2-d^2}{a^2-b^2}\right]$
Inner Edge Supported, Uniform Load Over Entire Actual Surface 	At Inner Edge: $\max M = M_t = \frac{q}{8(a^2-b^2)}\left[4a^4(1+\mu)\log\frac{a}{b} + 4a^2b^2 + b^4(1-\mu) - a^4(1+3\mu)\right]$ At Outer Edge: $\max w = \frac{q}{64D}\left[a^4(7+3\mu) + b^4(5+\mu) - a^2b^2(12+4\mu) - \frac{4a^2b^2(3+\mu)(1+\mu)}{(1-\mu)}\log\frac{a}{b} + \frac{16a^4b^2(1+\mu)^2}{(a^2-b^2)(1-\mu)}\left(\log\frac{a}{b}\right)^2\right]$
Outer Edge Fixed And Supported, Inner Edge Fixed, Uniform Load Over Entire Actual Surface 	At Outer Edge: $\max M_r = \frac{q}{8}\left[(a^2 - 3b^2) + \frac{4b^4}{a^2-b^2}\left(\log\frac{a}{b}\right)^2\right]$ At Inner Edge: $M_r = \frac{q}{8}\left[(a^2 + b^2) - \frac{4a^2b^2}{(a^2-b^2)}\left(\log\frac{a}{b}\right)\right]$ $\max w = \frac{q}{64D}\left[a^4 + 3b^4 - 4a^2b^2 - 4a^2b^2\log\frac{a}{b} + \frac{16a^2b^2}{a^2-b^2}\left(\log\frac{a}{b}\right)^2\right]$

Table B9-4. (Continued)

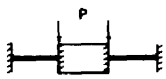
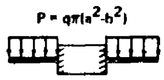
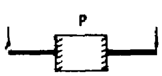
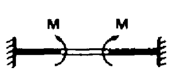
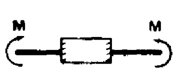
Case	Formulas For Deflection And Moments
Outer Edge Fixed And Supported, Inner Edge Fixed, Uniform Load Along Inner Edge 	At Outer Edge: $M_r = \frac{P}{4\pi}\left[1 - \frac{2b^2}{a^2 - b^2}\left(\log \frac{a}{b}\right)\right]$ At Inner Edge: $\max M_r = \frac{P}{4\pi}\left[1 - \frac{2a^2}{a^2 - b^2}\left(\log \frac{a}{b}\right)\right]$ $\max w = \frac{P}{16\pi D}\left[a^2 - b^2 - \frac{4a^2b^2}{a^2 - b^2}\left(\log \frac{a}{b}\right)^2\right]$
Inner Edge Fixed And Supported, Uniform Load Over Entire Actual Surface 	At Inner Edge: $\max M_r = \frac{q}{8}\left[\frac{4a^4(1+\mu)\log \frac{a}{b} - a^4(1+3\mu) + b^4(1-\mu) + 4a^2b^2\mu}{a^2(1+\mu) + b^2(1-\mu)}\right]$ At Outer Edge: $\max w = \frac{q}{64D}\left\{\frac{a^6(7+3\mu) + b^6(1-\mu) - a^4b^2(1+7\mu) - a^2b^4(7-5\mu)}{a^2(1+\mu) + b^2(1-\mu)} - \frac{4a^2b^2[a^2(5-\mu) + b^2(1+\mu)]\log \frac{a}{b} + 16a^4b^2(1+\mu)\left(\log \frac{a}{b}\right)^2}{a^2(1+\mu) + b^2(1-\mu)}\right\}$
Inner Edge Fixed And Supported, Uniform Load Along Outer Edge 	At Inner Edge: $\max M_r = \frac{P}{4\pi}\left[\frac{2a^2(1+\mu)\log \frac{a}{b} + a^2(1-\mu) - b^2(1-\mu)}{a^2(1+\mu) + b^2(1-\mu)}\right]$ At Outer Edge: $\max w = \frac{P}{16\pi D}\left[\frac{a^4(3+\mu) - b^4(1-\mu) - 2a^2b^2(1+\mu) - 8a^2b^2\log \frac{a}{b}}{a^2(1+\mu) + b^2(1-\mu)} - \frac{4a^2b^2(1+\mu)\left(\log \frac{a}{b}\right)^2}{a^2(1+\mu) + b^2(1-\mu)}\right]$
Outer Edge Fixed, Uniform Moment Along Inner Edge 	At Inner Edge: $\max w = \frac{M}{2D}\left[\frac{a^2b^2 - b^4 - 2a^2b^2\log \frac{a}{b}}{a^2(1-\mu) + b^2(1+\mu)}\right]$ At Outer Edge: $\max M_r = M\left[\frac{2b^2}{(1+\mu)b^2 + (1-\mu)a^2}\right]$
Inner Edge Fixed, Uniform Moment Along Outer Edge 	At Inner Edge: $\max M_r = M\left[\frac{2a^2}{(1+\mu)a^2 + (1-\mu)b^2}\right]$ At Outer Edge: $\max w = \frac{M}{2D}\left[\frac{a^4 - a^2b^2 - 2a^2b^2\log \frac{a}{b}}{a^2(1+\mu) + b^2(1-\mu)}\right]$

Table B9-4. (Continued)

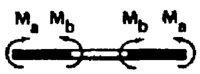
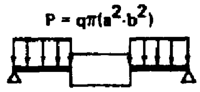
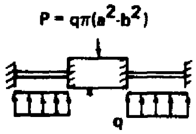
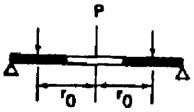
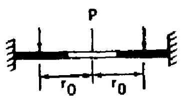
Case	Formulas For Deflection and Moments
Outer Edge Supported, Unequal Uniform Moments Along Edges  	$M_r = \frac{1}{a^2 - b^2}\left[a^2M_a - b^2M_b - \frac{a^2b^2}{r^2}(M_a - M_b)\right]$ $w = \frac{1}{D(a^2 - b^2)}\left\{\frac{a^2 - r^2}{2}\left(\frac{a^2M_a - b^2M_b}{1+\mu}\right) + \log\frac{a}{r}\left[\frac{a^2b^2(M_a - M_b)}{(1-\mu)}\right]\right\}$
Outer Edge Supported, Inner Edge Fixed, Uniform Load Over Entire Actual Surface  	At Inner Edge: $\max M_r = \frac{q}{8}\left[\frac{4a^2b^2(1+\mu)\log\frac{a}{b} - a^4(3+\mu) + a^2b^2(5+\mu)}{a^2(1+\mu) + b^2(1-\mu)} - b^2\right]$ $\max w = \frac{q}{64D}\left\{a^4 - 3b^4 + 2a^2b^2 - 8a^2b^2\log\frac{a}{b}\right.$ $- \frac{16(1+\mu)a^2b^2\log^2\frac{a}{b} + [4(7+3\mu)a^2b^4 - 4(5+3\mu)]\log\frac{a}{b}}{a^2(1+\mu) + b^2(1-\mu)}$ $\left. - \frac{4(4+\mu)a^4b^2 - 2(3+\mu)a^6 - 2(5+\mu)a^2b^4}{a^2(1+\mu) + b^2(1-\mu)}\right\}$
Both Edges Fixed, Balanced Loading (Piston)  	At Inner Edge: $\max M_r = \frac{q}{8}\left(\frac{4a^4}{a^2 - b^2}\log\frac{a}{b} - 3a^2 + b^2\right)$ $\max w = \frac{q}{64D}\left[3a^4 - 4a^2b^2 + b^4 + 4a^2b^2\log\frac{a}{b} - \frac{16a^4b^2}{a^2 - b^2}\left(\log\frac{a}{b}\right)^2\right]$
Outer Edge Supported, Inner Edge Free, Uniform Load On Concentric Circular Ring of Radius, r_0  	At Inner Edge: $\max M_t = \frac{P}{4\pi}\left[\frac{1}{2}(1-\mu) + (1+\mu)\log\frac{a}{r_0} - (1-\mu)\frac{r_0^2}{2a^2}\right] - \frac{c(a^2 + b^2)}{(a^2 - b^2)}$ $\max w = \frac{P}{8\pi D}\left[\frac{(a^2 - b^2)(3+\mu)}{2(1+\mu)} - (b^2 + r_0^2)\log\frac{a}{b} - \frac{r_0^2(a^2 - b^2)(1-\mu)}{2a^2(1+\mu)}\right]$ $- \frac{c}{2D}\left[\frac{b^2}{(1+\mu)} + \frac{2a^2b^2}{(a^2 - b^2)(1-\mu)}\log\frac{a}{b}\right]$ where $c = \frac{P}{8\pi}\left[(1-\mu) + 2(1+\mu)\log\frac{a}{r_0} - (1-\mu)\frac{r_0^2}{a^2}\right]$

Table B9-4. (Concluded)

Case: Outer Edge Fixed, Inner Edge Free, Uniform Load Of Concentric Circular Ring Of Radius, r_0



Formulas For Deflection and Moments:

At Inner Edge:

$$\max M_t = \frac{P}{8\pi}\left[(1+\mu)\left(2\log\frac{a}{r_0} + \frac{r_0^2}{a^2} - 1\right)\right] - c\left[\frac{a^2(1-\mu) - b^2(1+\mu)}{a^2(1-\mu) + b^2(1+\mu)}\right]$$

At Outer Edge:

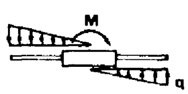
$$M_r = \frac{P}{8\pi}\left(1 - \frac{r_0^2}{a^2}\right) + c\left[\frac{2b^2}{a^2(1-\mu) + b^2(1+\mu)}\right]$$

$$\max w = \frac{P}{8\pi D}\left[\frac{(a^2 + r_0^2)(a^2 - b^2)}{2a^2} - (b^2 + r_0^2)\log\frac{a}{b}\right] - \frac{c}{2D}\left[\frac{b^4 + 2a^2b^2\log\frac{a}{b} - a^2b^2}{b^2(1+\mu) + a^2(1-\mu)}\right]$$

where

$$c = \frac{P}{8\pi}\left[(1+\mu)\left(2\log\frac{a}{r_0} + \frac{r_0^2}{a^2} - 1\right)\right]$$

Case: Central Couple Balanced By Linearly Distributed Pressure



$q = 4M/\pi a^3$

Formulas For Deflection and Moments:

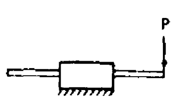
At Inner Edge:

$$\max M_r = \beta\frac{M}{6a} \quad \text{where}$$

$\frac{a}{b}$	1.25	1.50	2	3	4	5
β	0.1625	0.456	1.105	2.25	3.385	4.470

$(\mu = 0.3)$

Case: Concentrated Load Applied At Outer Edge



Formulas For Deflection and Moments:

At Inner Edge:

$$\max M_r = \beta\frac{P}{6} \quad \text{where}$$

$\frac{a}{b}$	1.25	1.50	2	3	4	5
β	3.7	4.25	5.2	6.7	7.9	8.8

for $\mu = 0.3$

Table B9-5. Deflections and Bending Moments of Clamped Circular Plates Loaded Uniformly (Fig. B9-6a) (μ = 0.25)

$\frac{b}{a}$	$w_{max} = \alpha \frac{qa^4}{Eh_0^3}$	$M_r = \beta qa^2$			$M_t = \beta_1 qa^2$		
		r=0	r=b	r=a	r=0	r=b	r=a
	α	β	β	β	β_1	β_1	β_1
0.2	0.008	0.0122	0.0040	-0.161	0.0122	0.0078	-0.040
0.4	0.042	0.0332	0.0007	-0.156	0.0332	0.0157	-0.039
0.6	0.094	0.0543	-0.0188	-0.149	0.0543	0.0149	-0.037
0.8	0.148	0.0709	-0.0591	-0.140	0.0709	0.0009	-0.035
1.0	0.176	0.0781	-0.125	-0.125	0.0781	-0.031	-0.031

Table B9-6. Deflections and Bending Moments of Clamped Circular Plates Under a Central Load (Fig. B9-6b) (μ = 0.25)

$\frac{b}{a}$	$w_{max} = \alpha \frac{Pa^2}{Eh_0^3}$	$M_r = M_t$	$M_r = \beta P$		$M_t = \beta_1 P$	
		r = 0	r = b	r = a	r = b	r = a
	α	γ_1	β	β	β_1	β_1
0.2	0.031	-0.114	-0.034	-0.129	-0.028	-0.032
0.4	0.093	-0.051	-0.040	-0.112	-0.034	-0.028
0.6	0.155	-0.021	-0.050	-0.096	-0.044	-0.024
0.8	0.203	-0.005	-0.063	-0.084	-0.057	-0.021
1.0	0.224	0	-0.080	-0.080	-0.020	-0.020

The last term is due to the nonuniformity of the thickness of the plate and the coefficient γ_1 is given in Table B9-6.

Symmetrical deformation of plates such as those shown in Fig. B9-7 have been investigated and some results are given in Tables B9-7, B9-8, and B9-9.

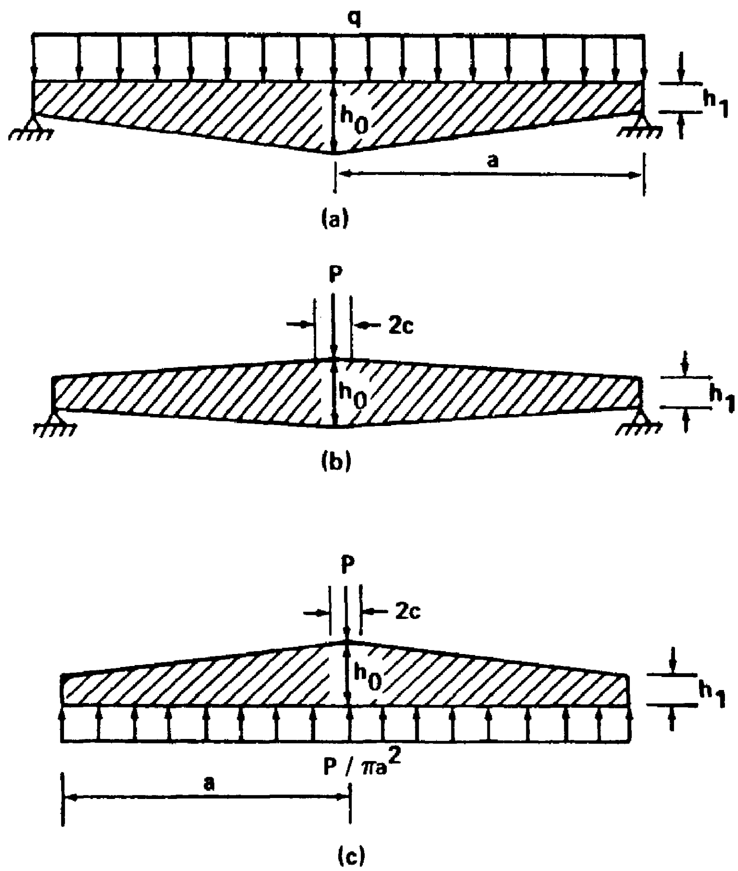


FIGURE B9-7. TAPERED CIRCULAR PLATE

For bending moments under central load P (Fig. B9-7b) the following equation is true (γ_2 is given in Table B9-8):

$$M_{max} = \frac{P}{4\pi} \; (1 + \mu) \log \frac{a}{c} + 1 - \frac{(1 - \mu) c^2}{4a^2} + \gamma_2 P \quad . \tag{29}$$

Table B9-7. Deflections and Bending Moments of Simply Supported Plates Under Uniform Load (Fig. B9-7a) (μ = 0.25)

$\frac{h_0}{h_1}$	$w_{max} = \alpha \frac{qa^4}{Eh_0^3}$	$M_r = \beta qa^2$		$M_t = \beta_1 qa^2$		
		$r = 0$	$r = \frac{a}{2}$	$r = 0$	$r = \frac{a}{2}$	$r = a$
	α	β	β	β_1	β_1	β_1
1.00	0.738	0.203	0.152	0.203	0.176	0.094
1.50	1.26	0.257	0.176	0.257	0.173	0.054
2.33	2.04	0.304	0.195	0.304	0.167	0.029

Table B9-8. Deflections and Bending Moments of Simply Supported Circular Plates Under Central Load (Fig. B9-7b) (μ = 0.25)

$\frac{h_0}{h_1}$	$w_{max} = \alpha \frac{Pa^2}{Eh_0^3}$	$M_r = M_t$	$M_r = \beta P$	$M_t = \beta_1 P$	
		$r = 0$	$r = \frac{a}{2}$	$r = \frac{a}{2}$	$r = a$
	α	γ_2	β	β_1	β_1
1.00	0.582	0	0.069	0.129	0.060
1.50	0.93	0.029	0.088	0.123	0.033
2.33	1.39	0.059	0.102	0.116	0.016

Table B9-9. Bending Moments of a Circular Plate With Central Load And Uniformly Distributed Reacting Pressure (Fig. B9-7c) (μ = 0.25)

$\frac{h_0}{h_1}$	$M_r = M_t$	$M_r = \beta P$	$M_t = \beta_1 P$	
	$r = 0$	$r = \frac{a}{2}$	$r = \frac{a}{2}$	$r = a$
	γ_2	β	β_1	β_1
1.00	-0.065	0.021	0.073	0.030
1.50	-0.053	0.032	0.068	0.016
2.33	-0.038	0.040	0.063	0.007

Of practical interest is a combination of loadings shown in Figs. B9-7a and b. For this case the γ_2 to be used in equation (29) is given in Table B9-9.

II. <u>Nonlinear Varying Thickness:</u>

In many cases the variation of the plate thickness can be represented with sufficient accuracy by the equation

$$y = e^{-\beta x^2/6} \tag{30}$$

in which β is a constant that must be chosen in each particular case so that it approximates as closely as possible the actual proportions of the plate. The variation of thickness along a diameter of a plate corresponding to various values of the constant β is shown in Fig. B9-8.

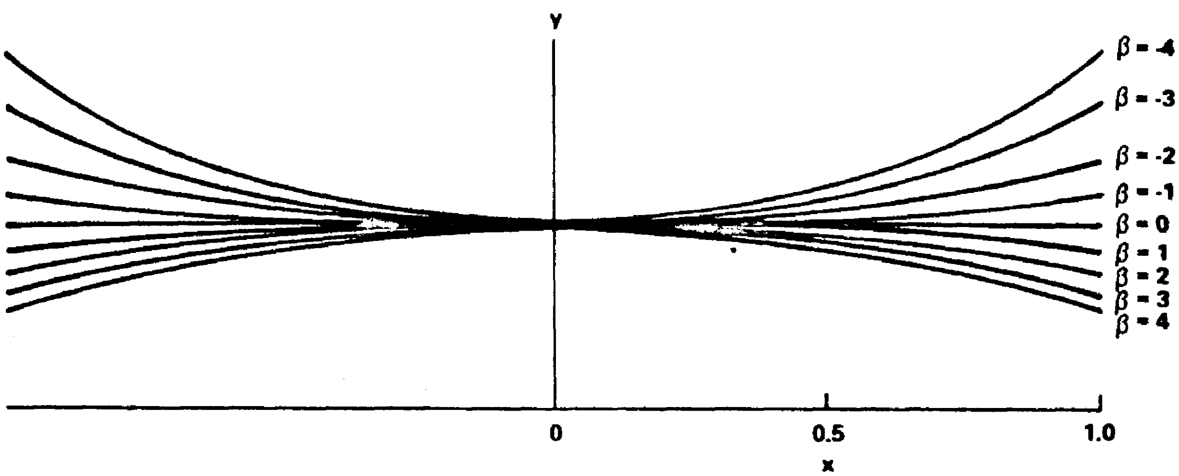


FIGURE B9-8. VARIATION OF PLATE THICKNESS FOR CIRCULAR PLATES

Solutions for this type of variation for uniformly loaded plates with both clamped edges and simply supported edges are given in Reference 1, pages 301-302.

B9.3.1.4 Annular Plates with Linearly Varying Thickness

Consider the case of a circular plate with a concentric hole and a thickness varying as shown in Fig. B9-9.

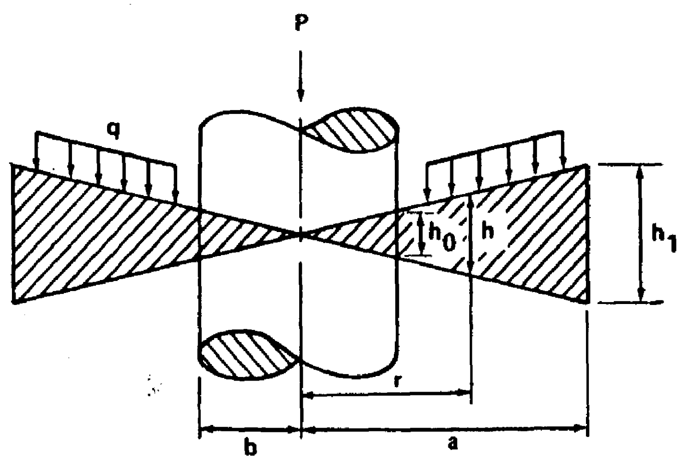


FIGURE B9-9. ANNULAR PLATE WITH LINEARLY VARYING THICKNESS

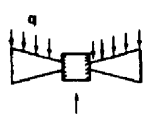
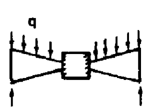
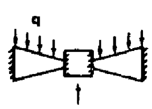
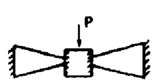
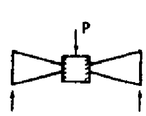
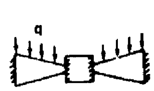
The plate carries a uniformly distributed surface load q and a line load $p = P/2\pi b$ uniformly distributed along the edge of the hole.

Table B9-10 gives values of coefficients k and k_1, to be used in the following expressions for the numerically largest stress and the largest deflection of the plate:

$$(\sigma_r)_{max} = k \frac{qa^2}{h_1^2} \quad \text{or} \quad (\sigma_r)_{max} = k \frac{P}{h_1^2}$$

$$w_{max} = k_1 \frac{qa^4}{Eh_1^3} \quad \text{or} \quad w_{max} = k_1 \frac{Pa^2}{Eh_1^3} \quad . \qquad (31)$$

Table B9-10. Values of Coefficients in Equations (31) for Various Values of the Ratio $\frac{a}{b}$ (Fig. B9-9) ($\mu = \frac{1}{3}$)

Case	Coef-ficient	$\frac{a}{b}$ = 1.25	1.5	2	3	4	5	Boundary Conditions
	k	0.249	0.638	3.96	13.64	26.0	40.6	$P = \pi q(a^2 - b^2)$, $\phi_b = 0$
	k_1	0.00372	0.0453	0.401	2.12	4.25	6.28	$M_a = 0$
	k	0.149	0.991	2.23	5.57	7.78	9.16	$P = 0$, $\phi_b = 0$
	k_1	0.00551	0.0564	0.412	1.673	2.79	3.57	$M_a = 0$
	k	0.1275	0.515	2.05	7.97	17.35	30.0	$P = \pi q(a^2 - b^2)$, $\phi_b = 0$
	k_1	0.00105	0.0115	0.0934	0.537	1.261	2.16	$\phi_a = 0$
	k	0.159	0.396	1.091	3.31	6.55	10.78	$q = 0$, $\phi_b = 0$
	k_1	0.00174	0.0112	0.0606	0.261	0.546	0.876	$\phi_a = 0$
	k	0.353	0.933	2.63	6.88	11.47	16.51	$q = 0$, $\phi_b = 0$
	k_1	0.00816	0.0583	0.345	1.358	2.39	3.27	$M_a = 0$
	k	0.0785	0.208	0.52	1.27	1.94	2.52	$P = 0$, $\phi_b = 0$
	k_1	0.00092	0.008	0.0195	0.193	0.346	0.482	$\phi_a = 0$

B9.3.1.5 Sector of a Circular Plate

The general solution developed for circular plates can also be adapted for a plate in the form of a sector (Fig. B9-10), the straight edges of which are simply supported. For a uniformly loaded plate simply supported along the straight and circular edges the expressions for the deflections and bending moments at a given point can be represented in each particular case by the following formulas:

$$w = \alpha \frac{qa^4}{D}, \quad M_r = \beta \, qa^2 , \quad M_t = \beta_1 \, qa^2 , \tag{32}$$

in which α, β, and β_1 are numerical factors. Several values of these factors for points taken on the axis of symmetry of a sector are given in Table B9-11.

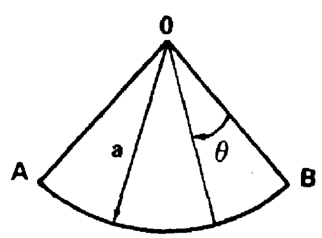


FIGURE B9-10. SECTOR OF A CIRCULAR PLATE

The coefficients for the case of a sector clamped along the circular boundary and simply supported along the straight edges are given in Table B9-12. It can be seen that in this case the maximum bending stress occurs at the midpoint of the unsupported circular edge. The following equation is used for the case when $\pi/k = \pi/2$

$$w_{max} = 0.0633 \frac{qa^4}{D} \quad .$$

The bending moment at the same point is

$$M_t = 0.1331 \, qa^2 \quad .$$

Table B9-11. Values of the Factors α, β, and B_1 for Various Angles $\frac{\pi}{k}$ of a Sector Simply Supported at the Boundary ($\mu = 0.3$)

$\frac{\pi}{k}$	$\frac{r}{a} = \frac{1}{4}$			$\frac{r}{a} = \frac{1}{2}$			$\frac{r}{a} = \frac{3}{4}$			$\frac{r}{a} = 1$		
	α	β	β_1	α	β	β_1	α	β	β_1	α	β	β_1
$\frac{\pi}{4}$	0.00006	-0.0015	0.0093	0.00033	0.0069	0.0183	0.00049	0.0161	0.0169	0	0	0.0025
$\frac{\pi}{3}$	0.00019	-0.0025	0.0177	0.00080	0.0149	0.0255	0.00092	0.0243	0.0213	0	0	0.0044
$\frac{\pi}{2}$	0.00092	0.0036	0.0319	0.00225	0.0353	0.0352	0.00203	0.0381	0.0286	0	0	0.0088
π	0.00589	0.0692	0.0357	0.00811	0.0868	0.0515	0.00560	0.0617	0.0468	0	0	0.0221

Table B9-12. Values of the Coefficients α and β for Various Angles $\frac{\pi}{k}$ of a Sector Clamped Along the Circular Boundary and Simply Supported Along the Straight Edges ($\mu = 0.3$)

$\frac{\pi}{k}$	$\frac{r}{a} = \frac{1}{4}$		$\frac{r}{a} = \frac{1}{2}$		$\frac{r}{a} = \frac{3}{4}$		$\frac{r}{a} = 1$	
	α	β	α	β	α	β	α	β
$\frac{\pi}{4}$	0.00005	-0.0008	0.00026	0.0087	0.00028	0.0107	0	-0.025
$\frac{\pi}{3}$	0.00017	-0.0006	0.00057	0.0143	0.00047	0.0123	0	-0.034
$\frac{\pi}{2}$	0.00063	0.0068	0.00132	0.0272	0.00082	0.0113	0	-0.0488
π	0.00293	0.0472	0.00337	0.0446	0.00153	0.0016	0	-0.0756

In the general case of a plate having the form of a circular sector with radial edges clamped or free, one must apply approximate methods. Another problem which allows an exact solution is that of bending of a plate clamped along two circular arcs. Data regarding the clamped semicircular plate are given in Table B9-13.

Table B9-13. Values of the Factors α, β, and β_1 for a Semicircular Plate Clamped Along the Boundary ($\mu = 0.3$)

Load Distribution	$\frac{r}{a} = 0$ β	$\frac{r}{a} = 0.483$ β_{max}	$\frac{r}{a} = 0.486$ α_{max}	$\frac{r}{a} = 0.525$ β_{1max}	$\frac{r}{a} = 1$ β
Uniform Load q	-0.0731	0.0355	0.00202	0.0194	-0.0584
Hydrostatic Load $q\frac{y}{a}$	-0.0276	—	—	—	-0.0355

I. Annular Sectored Plate:

For a semicircular annular sectored plate with outer edge supported and the other edges free, with uniform load over the entire actual surface as shown in Fig. B9-11, the equations for maximum moment and deflection are:

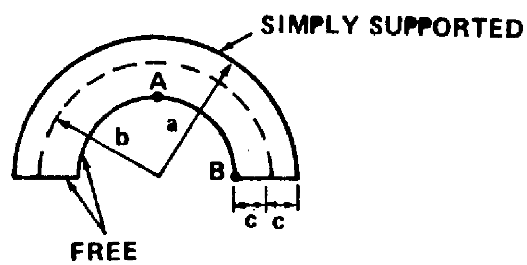


FIGURE B9-11. ANNULAR SECTORED PLATE

At A

$$M_t = qcb\left(\frac{b}{c} - \frac{1}{3}\right)\left[c_1\left(1 - \gamma_1^2\frac{c}{b}\right) + c_2\left(1 - \gamma_2^2\frac{c}{b}\right) + \frac{c}{b}\right]K \quad ,$$

At B

$$w = \frac{24qc^2b^2}{Et^3}\left(\frac{b}{c} - \frac{1}{3}\right)\left[c_1 \cosh \frac{\gamma_1\pi}{2} + c_2 \cosh \gamma_2 \frac{\pi}{2} + \frac{c}{b}\right] ,$$

where

$$c_1 = \frac{1}{\left(\frac{b}{c} - \gamma_1^2\right)(\lambda - 1) \cosh \gamma_1 \frac{\pi}{2}} , \quad c_2 = \frac{1}{\left(\frac{b}{c} - \gamma_2^2\right)\left(\frac{1}{\lambda} - 1\right) \cosh \gamma_2 \frac{\pi}{2}} ,$$

$$\gamma_1 = \frac{\gamma}{\sqrt{2}} \sqrt{1 + \sqrt{1 - \frac{4b^2}{c^2\gamma^4}}} , \quad \gamma_2 = \frac{\gamma}{\sqrt{2}} \sqrt{1 - \sqrt{1 - \frac{4b^2}{c^2\gamma^4}}} ,$$

$$\gamma = \sqrt{\frac{2b}{c} + 4\left(1 - \frac{0.625t}{2c}\right)\frac{G}{E}\left(1 + \frac{b}{c}\right)^2} ,$$

$$\lambda = \frac{\gamma_1\left(\frac{b}{c} - \gamma_1^2 + \lambda_1\right)\left(\frac{b}{c} - \gamma_2^2\right) \tanh \gamma_1 \frac{\pi}{2}}{\gamma_2\left(\frac{b}{c} - \gamma_2^2 + \lambda_1\right)\left(\frac{b}{c} - \gamma_1^2\right) \tanh \gamma_2 \frac{\pi}{2}} ,$$

$$\lambda_1 = 4\left(1 - \frac{0.625t}{2c}\right)\frac{G}{E}\left(1 + \frac{b}{c}\right)^2 .$$

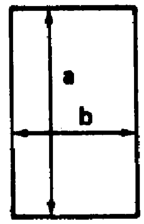
K is a function of $\frac{b-c}{b+c}$ and has the following values:

$\frac{b-c}{b+c}$ =	0.05	0.10	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
K =	2.33	2.20	1.95	1.75	1.58	1.44	1.32	1.22	1.13	1.06	1.0

B9.3.2 Rectangular Plates

Solutions for many rectangular plate problems with various loadings and boundary conditions are given in Tables B9-14 through 18. For loads and boundary conditions not covered here, solutions can be found by applying the various theoretical, approximate, or complete solutions discussed in

Table B9-14. Solutions for Rectangular Plates



b/a = α

P is load

All Edges Supported, Uniform Load Over Entire Surface

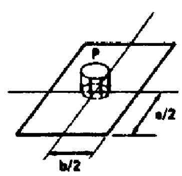
At Center:

$$M_a = (0.0375 + 0.0637\alpha^2 - 0.0533\alpha^3)qb^2$$

$$M_b = \frac{0.125qb^2}{(1 + 1.61\alpha^3)} = \max M$$

$$\max w = \frac{0.1422}{(1 + 2.21\alpha^3)} \frac{qb^4}{Et^3}$$

All Edges Supported, Uniform Load Over Small Concentric Circular Area Of Radius, r_0



At Center:

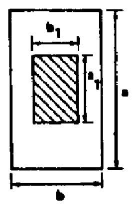
$$M_b = \frac{P}{4\pi}\left[(1 + \mu) \log \frac{b}{2r_0} + (1 + k)\right]$$

where

$$k = \frac{0.914}{1 + 1.6\alpha^3} - 0.6$$

$$\max w = \frac{0.203Pb^2}{12D(1 + 0.462\alpha^4)}$$

All Edges Supported, Uniform Load Over Central Rectangular Area Shown Shaded



At Center: $\max \sigma = \sigma_b = \beta \frac{P}{t^2}$ where β is found in the following ($\mu = 0.3$):

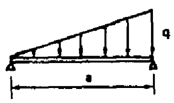
b_1/b \ a_1/b	a = b					
	0	0.2	0.4	0.6	0.8	1.0
0		1.82	1.38	1.12	0.93	0.76
0.2	1.82	1.28	1.08	0.90	0.76	0.63
0.4	1.39	1.07	0.84	0.72	0.62	0.52
0.6	1.12	0.90	0.72	0.60	0.52	0.43
0.8	0.92	0.76	0.62	0.51	0.42	0.36
1.0	0.76	0.63	0.52	0.42	0.35	0.30

b_1/b \ a_1/b	a = 1.4b					
	0	0.2	0.4	0.8	1.2	1.4
0		2.0	1.55	1.12	0.84	0.75
0.2	1.78	1.43	1.23	0.95	0.74	0.64
0.4	1.39	1.13	1.00	0.80	0.62	0.55
0.6	1.10	0.91	0.82	0.68	0.53	0.47
0.8	0.90	0.76	0.68	0.57	0.45	0.40
1.0	0.75	0.62	0.57	0.47	0.38	0.33

b_1/b \ a_1/b	a = 2b					
	0	0.4	0.8	1.2	1.6	2.0
0		1.64	1.20	0.97	0.78	0.64
0.2	1.73	1.31	1.03	0.84	0.68	0.57
0.4	1.32	1.08	0.88	0.74	0.60	0.50
0.6	1.04	0.90	0.76	0.64	0.54	0.44
0.8	0.87	0.76	0.63	0.54	0.44	0.38
1.0	0.71	0.61	0.53	0.45	0.38	0.30

Table B9-14. (Continued)

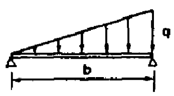
All Edges Supported, Distributed Load Varying Linearly Along Length



$\max \sigma = \beta \frac{qb^2}{t^2}$, $\max w = \delta \frac{qb^4}{Et^3}$ where β and δ are found in the following:

a/b	1	1.5	2.0	2.5	3.0	3.5	4.0
β	0.16	0.26	0.34	0.38	0.43	0.47	0.49
δ	0.022	0.043	0.060	0.070	0.078	0.086	0.091

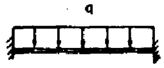
All Edges Supported, Distributed Load Varying Linearly Along Breadth



$\max \sigma = \beta \frac{qb^2}{t^2}$, $\max w = \delta \frac{qb^4}{Et^3}$ where β and δ are found as follows:

a/b	1	1.5	2.0	2.5	3.0	3.5	4.0
β	0.16	0.26	0.32	0.35	0.37	0.38	0.38
δ	0.022	0.042	0.056	0.063	0.067	0.069	0.070

All Edges Fixed, Uniform Load Over Entire Surface



At Centers of Long Edges:

$$M_b = \frac{qb^2}{12(1 + 0.623\alpha^6)} = \max M$$

At Centers of Short Edges:

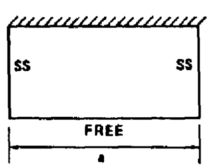
$$M_a = \frac{qb^2}{24}$$

At Center

$$M_b = \frac{qb^2}{8(3 + 4\alpha^4)} , \quad M_a = 0.009qb^2(1 + 2\alpha^2 - \alpha^4)$$

$$\max w = \frac{0.0284}{(1 + 1.056\alpha^5)} \frac{qb^4}{Et^3}$$, formulas for M_b, $\mu = 0.3$; others $\mu = 0$

One Long Edge Fixed, Other Free, Short Edges Supported, Uniform Load Over Entire Surface



At Center of Fixed Edge:

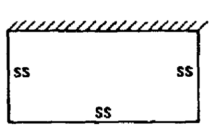
$$\max M = M_b = \frac{qb^2}{2(1 + 3.2\alpha^3)}$$

At Center of Free Edge:

$$M_a = \frac{8qa^2}{\left(1 + \frac{0.285}{\alpha^4}\right)} , \quad \max w = \frac{1.37qb^4}{Et^3(1 + 10\alpha^3)}$$

($\mu = 0.3$)

One Long Edge Clamped, Other Three Edges Supported, Uniform Load Over Entire Surface



$$\text{Max Stress } \sigma = \beta \frac{qb^2}{t^2} , \quad \max w = \frac{\alpha qb^4}{Et^3}$$

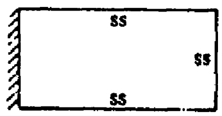
where β and α may be found from the following:

a/b	1.0	1.5	2.0	2.5	3.0	3.5	4.0
β	0.50	0.67	0.73	0.74	0.74	0.75	0.75
α	0.03	0.046	0.054	0.056	0.057	0.058	0.058

($\mu = 0.3$)

Table B9-14. (Continued)

One Short Edge Clamped, Other Three Edges Supported, Uniform Load Over Entire Surface



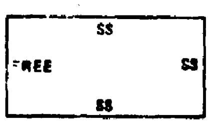
Max Stress $\sigma = \beta \frac{qb^2}{t^2}$, max $w = \frac{\alpha q b^4}{Et^3}$

where β and α may be found from the following:

$\frac{a}{b}$	1.0	1.5	2.0	2.5	3.0	3.5	4.0
β	0.50	0.67	0.73	0.74	0.75	0.75	0.75
α	0.03	0.071	0.101	0.122	0.132	0.137	0.139

(μ = 0.3)

One Short Edge Free, Other Three Edges Supported, Uniform Load Over Entire Surface



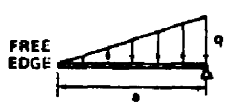
max $\sigma = \frac{\beta q b^2}{t^2}$, max $w = \frac{\alpha q b^4}{Et^3}$

where β and α are found from the following:

$\frac{a}{b}$	1.0	1.5	2.0	4.0
β	0.67	0.77	0.79	0.80
α	0.14	0.16	0.165	0.167

(μ = 0.3)

One Short Edge Free, Other Three Edges Supported, Distributed Load Varying Linearly Along Length



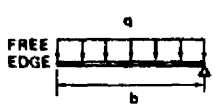
max $\sigma = \frac{\beta q b^2}{t^2}$, max $w = \frac{\alpha q b^4}{Et^3}$

where β and α are found from the following:

$\frac{a}{b}$	1.0	1.5	2.0	2.5	3.0	3.5	4.0
β	0.2	0.28	0.32	0.35	0.36	0.37	0.37
α	0.04	0.05	0.058	0.064	0.067	0.069	0.070

(μ = 0.3)

One Long Edge Free, Other Three Edges Supported, Uniform Load Over Entire Surface



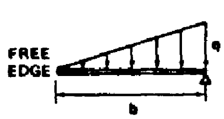
max $\sigma = \frac{\beta q b^2}{t^2}$, max $w = \frac{\alpha q b^4}{Et^3}$

where β and α are found from the following:

$\frac{a}{b}$	1.0	1.5	2.0
β	0.67	0.45	0.36
α	0.14	0.106	0.080

(μ = 0.3)

One Long Edge Free, Other Three Edges Supported, Distributed Load Varying Linearly Along Length



max $\sigma = \frac{\beta q b^2}{t^2}$, max $w = \frac{\alpha q b^4}{Et^3}$

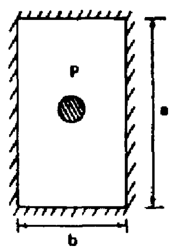
where β and α are found from the following:

$\frac{a}{b}$	1.0	1.5	2.0
β	0.2	0.16	0.11
α	0.04	0.033	0.026

(μ = 0.3)

Table B9-14. (Continued)

All Edges Fixed, Uniform Load Over Small Concentric Circular Area Of Radius, r_0



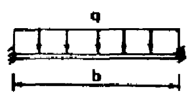
At Center:

$$M_b = \frac{P}{4\pi}\left[(1+\mu)\log\frac{b}{2r_0} + 5(1-\alpha)\right] = \max M \quad , \quad w = \beta\frac{Pb^2}{12}$$

where β has values as follows:

$\frac{a}{b}$	4	2	1
β	0.072	0.0816	0.0624

Long Edges Fixed, Short Edges Supported, Uniform Load Over Entire Surface



At Centers of Long Edges:

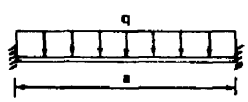
$$\max M = M_b = \frac{qb^2}{12(1+0.2\alpha^4)}$$

At Center:

$$M_b = \frac{qb^2}{24(1+0.8\alpha^4)} \quad , \quad M_a = \frac{qb^2(1+0.3\alpha^2)}{80}$$

$(\mu = 0)$

Short Edges Fixed, Long Edges Supported, Uniform Load Over Entire Surface



At Centers of Short Edges:

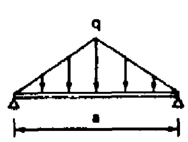
$$\max M = M_a = \frac{qb^2}{8(1+0.8\alpha^4)}$$

At Center:

$$M_b = \frac{qb^2}{8(1+0.8\alpha^2+6\alpha^4)} \quad , \quad M_a = \frac{0.015qb^2(1+3\alpha^2)}{(1+\alpha^4)}$$

$(\mu = 0)$

All Edges Supported, Distributed Load in Form of Triangular Prism



$$\max M = \beta qb^2 \quad , \quad \max w = \frac{\alpha qb^4}{D}$$

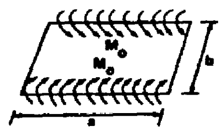
β and α found from the following:

$\frac{a}{b}$	1.0	1.5	2.0	3.0	∞
β	0.034	0.0548	0.0707	0.0922	0.1250
α	0.00263	0.00308	0.00686	0.00868	0.01302

$(\mu = 0.3)$

Table B9-14. (Continued)

All Edges Supported, Uniformly Distributed Edge Moment



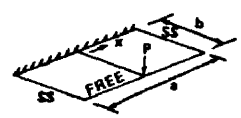
At Center:

$M_a = \beta M_0$, $M_b = \beta_1 M_0$, $w = \frac{\alpha M_0 b^2}{D}$

β, β_1, and α are found from the following:

$\frac{b}{a}$	α	β	β_1
0	0.1250	0.300	1.000
0.5	0.0964	0.387	0.770
0.75	0.0620	0.424	0.476
1.00	0.0368	0.394	0.256
1.50	0.0280	0.264	0.046
2.00	0.0174	0.153	-0.010

One Edge Fixed, Opposite Edge Free, Other Edges Supported, Concentrated Load On Center Of Free Edge



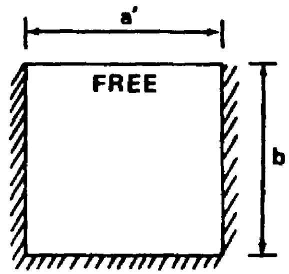
On Free Edge: $w = \frac{\alpha P b^2}{D}$ where

x	0	$\frac{b}{4}$	$\frac{b}{2}$	b	2b
α	0.168	0.150	0.121	0.068	0.016

At Center of Fixed Edge: $M = \beta P$ where

$\frac{b}{a}$	4	2	1.5	1	$\frac{2}{3}$	0.5	0.25
β	-0.000039	-0.0117	-0.0455	-0.163	-0.366	-0.436	-0.507

Table B9-15. Coefficients For Maximum Moments For Various Loads, Plate With Three Sides Fixed, One Free ($\mu = 0.2$)



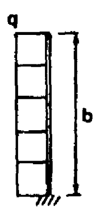
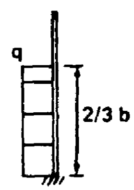
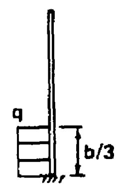
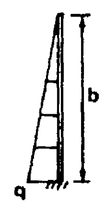
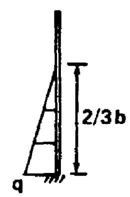
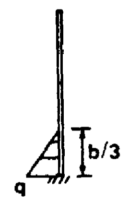
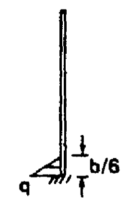
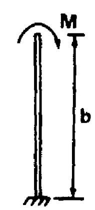
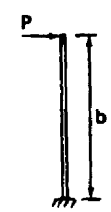
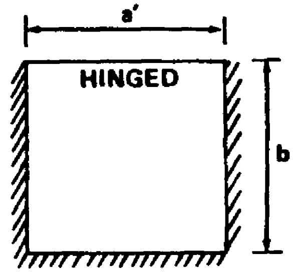
Load / $^a/_b$									
1/4	0.0052 qb^2	0.0051 qb^2	0.0044 qb^2	0.0038 qb^2	0.0032 qb^2	0.0017 qb^2	0.0004 qb^2	1.000 M	0.0471 Pb
1/2	0.0209 qb^2	0.0184 qb^2	0.0105 qb^2	0.0114 qb^2	0.0084 qb^2	0.0040 qb^2	0.0009 qb^2	1.000 M	0.1522 Pb
3/4	0.0476 qb^2	0.0330 qb^2	0.0140 qb^2	0.0208 qb^2	0.0131 qb^2	0.0051 qb^2	0.0013 qb^2	1.1461 M	0.2723 Pb
1	0.0852 qb^2	0.0433 qb^2	0.0131 qb^2	0.0277 qb^2	0.0165 qb^2	0.0050 qb^2	0.0012 qb^2	1.3643 M	0.3938 Pb
3/2	0.1788 qb^2	0.0617 qb^2	0.0140 qb^2	0.0433 qb^2	0.0190 qb^2	0.0042 qb^2	0.0010 qb^2	1.6292 M	0.6266 Pb
2	0.2613 qb^2	0.0757 qb^2	0.0136 qb^2	0.0644 qb^2	0.0208 qb^2	0.0039 qb^2	0.0008 qb^2	1.7779 M	0.8094 Pb
3	0.3304 qb^2	0.1036 qb^2	0.0146 qb^2	0.0857 qb^2	0.0270 qb^2	0.0038 qb^2	0.0006 qb^2	1.7980 M	0.9388 Pb

Table B9-16. Coefficients For Maximum Moments For Various Loads, Plate With Three Sides Fixed, One Hinged ($\mu = 0.2$)



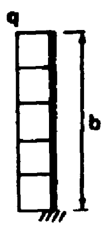
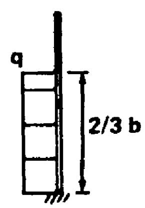
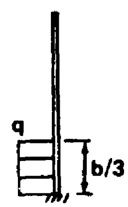
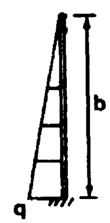
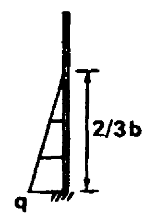
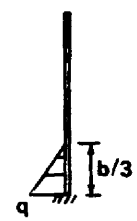
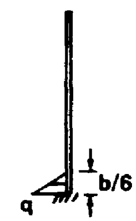
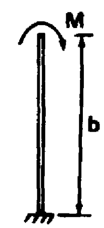
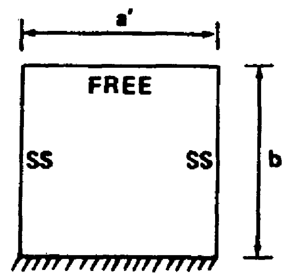
Load / a/b								
1/4	$0.0052\ qb^2$	$0.0051\ qb^2$	$0.0044\ qb^2$	$0.0038\ qb^2$	$0.0032\ qb^2$	$0.0017\ qb^2$	$0.0014\ qb^2$	1.00 M
1/2	$0.0201\ qb^2$	$0.0185\ qb^2$	$0.0105\ qb^2$	$0.0114\ qb^2$	$0.0084\ qb^2$	$0.0040\ qb^2$	$0.0025\ qb^2$	1.00 M
3/4	$0.0403\ qb^2$	$0.0329\ qb^2$	$0.0132\ qb^2$	$0.0207\ qb^2$	$0.0131\ qb^2$	$0.0051\ qb^2$	$0.0027\ qb^2$	1.00 M
1	$0.0572\ qb^2$	$0.0425\ qb^2$	$0.0131\ qb^2$	$0.0269\ qb^2$	$0.0163\ qb^2$	$0.0050\ qb^2$	$0.0032\ qb^2$	1.00 M
3/2	$0.0695\ qb^2$	$0.0472\ qb^2$	$0.0132\ qb^2$	$0.0302\ qb^2$	$0.0176\ qb^2$	$0.0041\ qb^2$	$0.0036\ qb^2$	1.00 M
2	$0.0664\ qb^2$	$0.0451\ qb^2$	$0.0120\ qb^2$	$0.0289\ qb^2$	$0.0161\ qb^2$	$0.0035\ qb^2$	$0.0038\ qb^2$	1.00 M
3	$-0.0704\ qb^2$	$-0.0477\ qb^2$	$-0.0111\ qb^2$	$-0.0297\ qb^2$	$-0.0154\ qb^2$	$-0.0029\ qb^2$	$0.0039\ qb^2$	1.00 M

Table B9-17. Coefficients For Maximum Moments For Various Loads, Plate Fixed Along One Edge, Free On Opposite Edge And Hinged On Other Two Edges ($\mu = 0.2$)



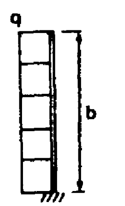
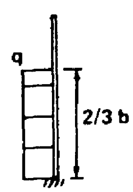
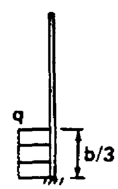
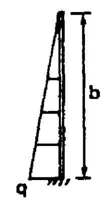
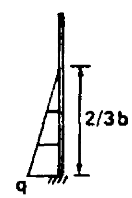
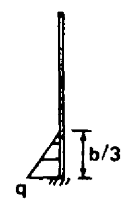
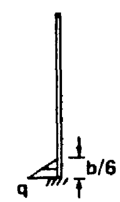
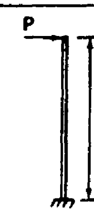
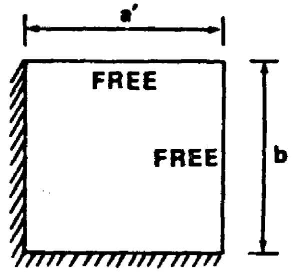
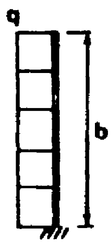
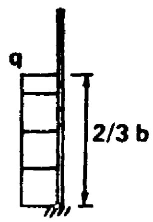
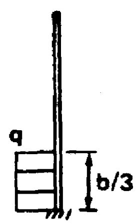
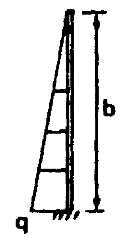
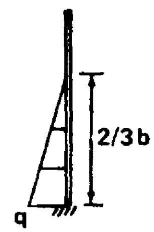
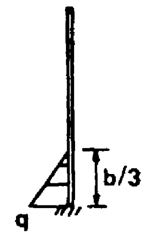
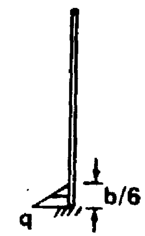
Load / a/b									
1/4	$-0.0080\ qb^2$	$0.0073\ qb^2$	$0.0066\ qb^2$	$0.0061\ qb^2$	$0.0055\ qb^2$	$0.0038\ qb^2$	$0.0020\ qb^2$	1.0 M	-0.0534 Pb
1/2	$-0.0317\ qb^2$	$0.0269\ qb^2$	$0.0177\ qb^2$	$0.0199\ qb^2$	$0.0156\ qb^2$	$0.0080\ qb^2$	$0.0030\ qb^2$	1.0 M	-0.1300 Pb
3/4	$-0.0644\ qb^2$	$0.0497\ qb^2$	$0.0250\ qb^2$	$0.0353\ qb^2$	$0.0243\ qb^2$	$0.0101\ qb^2$	$0.0032\ qb^2$	1.0 M	-0.2007 Pb
1	$0.1108\ qb^2$	$0.0757\ qb^2$	$0.0317\ qb^2$	$0.0535\ qb^2$	$0.0333\ qb^2$	$0.0122\ qb^2$	$0.0036\ qb^2$	1.0 M	-0.2590 Pb
3/2	$0.2136\ qb^2$	$0.1216\ qb^2$	$0.0406\ qb^2$	$0.0871\ qb^2$	$0.0471\ qb^2$	$0.0147\ qb^2$	$0.0040\ qb^2$	1.0 M	-0.3114 Pb
2	$0.3007\ qb^2$	$0.1552\ qb^2$	$0.0461\ qb^2$	$0.1128\ qb^2$	$0.0565\ qb^2$	$0.0161\ qb^2$	$0.0043\ qb^2$	1.0 M	0.4831 Pb
3	$0.4084\ qb^2$	$0.1929\ qb^2$	$0.0516\ qb^2$	$0.1426\ qb^2$	$0.0666\ qb^2$	$0.0175\ qb^2$	$0.0045\ qb^2$	1.0 M	0.7513 Pb

Table B9-18. Coefficients For Maximum Moments For Various Loads, Plate Fixed On Two Adjacent Sides, Free On Other Sides ($\mu = 0.2$)



Load / a/b							
	q, b	q, 2/3 b	q, b/3	q, b	q, 2/3 b	q, b/3	q, b/6
1/8	0.0083 qb^2	0.0083 qb^2	0.0057 qb^2	0.0072 qb^2	0.0066 qb^2	0.0041 qb^2	0.0027 qb^2
1/4	0.0313 qb^2	0.0239 qb^2	0.0165 qb^2	0.0221 qb^2	0.0181 qb^2	0.0087 qb^2	0.0036 qb^2
3/8	0.0664 qb^2	0.0495 qb^2	0.0238 qb^2	0.0354 qb^2	0.0257 qb^2	0.0118 qb^2	0.0038 qb^2
1/2	0.1074 qb^2	0.0775 qb^2	0.0310 qb^2	0.0546 qb^2	0.0358 qb^2	0.0140 qb^2	0.0043 qb^2
3/4	0.2076 qb^2	0.1262 qb^2	0.0402 qb^2	0.0896 qb^2	0.0507 qb^2	0.0167 qb^2	0.0048 qb^2
1	0.2949 qb^2	0.1605 qb^2	0.0456 qb^2	0.1157 qb^2	0.0603 qb^2	0.0181 qb^2	0.0050 qb^2

B9.3.3 Elliptical Plates

For plates whose boundary is the shape of an ellipse, solutions have been found for some common loadings. Table B9-19 presents the available solutions for elliptical plates. For additional information as to method of solution to the plate differential equations see Reference 1.

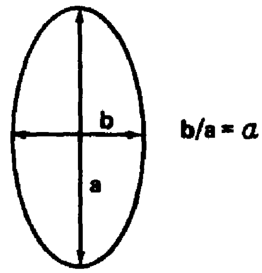
B9.3.4 Triangular Plates

Solutions for several loadings on triangular shaped plates are presented in Table B9-20.

B9.3.5 Skew Plates

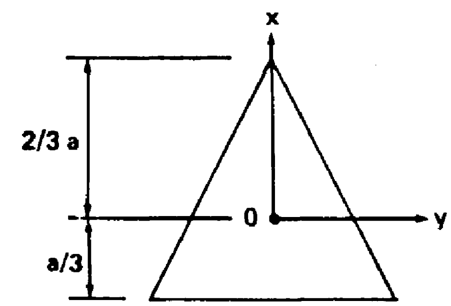
Solutions have been obtained for skew plates in References 1 and 5. The significant results from these references are presented in Table B9-21.

Table B9-19. Solutions For Elliptical, Solid Plates



Edge Supported, Uniform Load Over Entire Surface	At Center: $\text{max stress} = \sigma_b = \frac{-0.3125(2-\alpha)qb^2}{t^2}$ $\text{max } w = \frac{(0.146 - 0.1\alpha)qb^4}{Et^3} \ (\text{for } \mu = \frac{1}{3})$
Edge Supported, Uniform Load Over Small Concentric Circular Area of Radius, r_0	At Center: $\text{max } M = M_b = \frac{P}{4\pi}\left[(1+\mu)\log\frac{b}{2r_0} + 6.57\mu - 2.57\alpha\mu\right]$ $\text{max } w = \frac{Pb^2}{Et^3}(0.19 - 0.045\alpha) \quad (\mu = \frac{1}{4})$
Edge Fixed, Uniform Load Over Entire Surface	At Edge: $M_a = \frac{qb^2\alpha^2}{4(3 + 2\alpha^2 + 3\alpha^4)}, \quad M_b = \frac{qb^2}{4(3 + 2\alpha^2 + 3\alpha^4)}$ At Center: $M_a = \frac{qb^2(\alpha^2 + \mu)}{8(3 + 2\alpha^2 + 3\alpha^4)}, \quad M_b = \frac{qb^2(1 + \alpha^2\mu)}{8(3 + 2\alpha^2 + 3\alpha^4)}$ $\text{max } w = \frac{qb^4}{64D(6 + 4\alpha^2 + 6\alpha^4)}$
Edge Fixed, Uniform Load Over Small Concentric Circular Area of Radius, r_0	At Center: $M_b = \frac{P(1+\mu)}{4\pi}\left(\log\frac{b}{r_0} - 0.317\alpha - 0.376\right)$ $\text{max } w = \frac{Pb^2(0.0815 - 0.026\alpha)}{Et^3} \ (\mu = 0.25)$

Table B9-20. Solutions For Triangular Plates



Equilateral Triangle, Edges Supported, Distributed Load Over Entire Surface

$\max \sigma_x = 0.1488 \dfrac{qa^2}{t^2}$ at $y = 0$, $x = -0.062a$ $(\mu = 0.3)$

$\max \sigma_y = 0.1554 \dfrac{qa^2}{t^2}$ at $y = 0$, $x = 0.129a$ $(\mu = 0.3)$

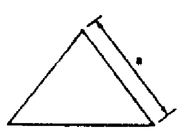
$\max w = \dfrac{qa^4}{342D}$ at point 0

Edges Supported, Load P Concentrated At 0 On Small Circular Area Of Radius, r_0

$$\max \sigma_y = \frac{3(1+\mu)P}{2\pi t^2}\left[\log \frac{0.378a}{\sqrt{1.6r_0^2 + t^2} - 0.675t} - 0.379 + \frac{(1-\mu)}{2(1+\mu)}\right]$$

$\max w = 0.06852 \dfrac{Pa^2(1-\mu^2)}{Et^3}$ at point 0

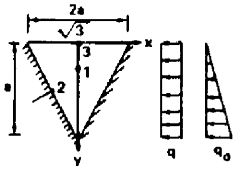
Right-Angle Isosceles Triangle, Edges Supported, Distributed Load Over Entire Surface



$\max \sigma_x = 0.131 \dfrac{qa^2}{t^2}$, $\max \sigma_y = 0.1125 \dfrac{qa^2}{t^2}$

$\max w = 0.0095 \dfrac{qa^4}{Et^3}$ $(\mu = 0.3)$

Equilateral Triangle With Two Or Three Edges Clamped, Uniform Or Hydrostatic Load

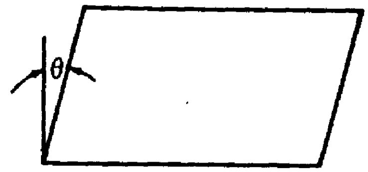


$M = \beta q a^2$ or $M = \beta_1 q_0 a^2$ where

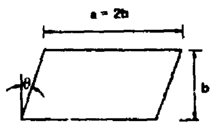
Load Distribution	Edge y = 0 Supported				Edge y = 0 Clamped			
	M_{x_1}	M_{y_1}	M_{n_2}	M_{y_3}	M_{x_1}	M_{y_1}	M_{n_2}	M_{y_3}
Uniform β	0.0126	0.0147	-0.0285	0	0.0113	0.0110	-0.0238	-0.0238
Hydrostatic β_1	0.0053	0.0035	-0.0100	0	0.0051	0.0034	-0.0091	-0.0060

$(\mu = 0.2)$

Table B9-21. Solutions for Skew Plates



All Edges Supported, Distributed Load Over Entire Surface

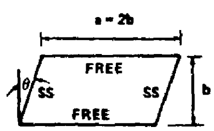


$\max \sigma = \sigma_b = \frac{\beta q b^2}{t^2}$ where β is

θ	0 deg	30 deg	45 deg	60 deg	75 deg
β	0.501	0.50	0.45	0.40	0.16

($\mu = 0.2$)

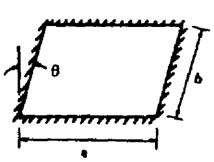
Edges b Supported, Edges a Free, Uniform Distributed Load Over Entire Surface



$\max \sigma = \sigma_b = \frac{\beta q b^2}{t^2}$ where β is

θ	0 deg	30 deg	45 deg	60 deg
β	0.762	0.615	0.437	0.250

All Edges Clamped, Uniform Distributed Load Over Entire Surface



At Center: $M = \beta q a^2$ $w = \frac{\beta_1 q a^4}{D}$

where β and β_1 are

Skew Angle θ (deg)	$\frac{a}{b} = 1$		$\frac{a}{b} = 1.25$		$\frac{a}{b} = 1.5$		$\frac{a}{b} = 2.0$	
	β	β_1	β	β_1	β	β_1	β	β_1
15	0.024	0.001123	0.019	0.00066	0.015	0.00038	0.0097	0.00014
30	0.020	0.00077	0.016	0.00045	0.0125	0.00026	0.0075	0.00009
45	0.015	0.00038	0.011	0.00022	0.014	0.00012	0.005	0.00004
60	0.0085	0.00011	0.0062	0.00006	0.0048	0.00003	0.0025	0.00001
75	0.0025	0.000009	0.0027	0.000005	0.00125	0.000002	0.00125	

Along Fixed Edge:

The coefficient β_2 for maximum bending moment along the edge at a distance fa from the acute corner is

($M = \beta_2 q a^2$ for $\frac{a}{b} = 1$)

Skew Angle (deg)	β_2	f
15	-0.0475	0.6
30	-0.0400	0.69
45	-0.0299	0.80