B7.1.2.0 DOME ANALYSIS

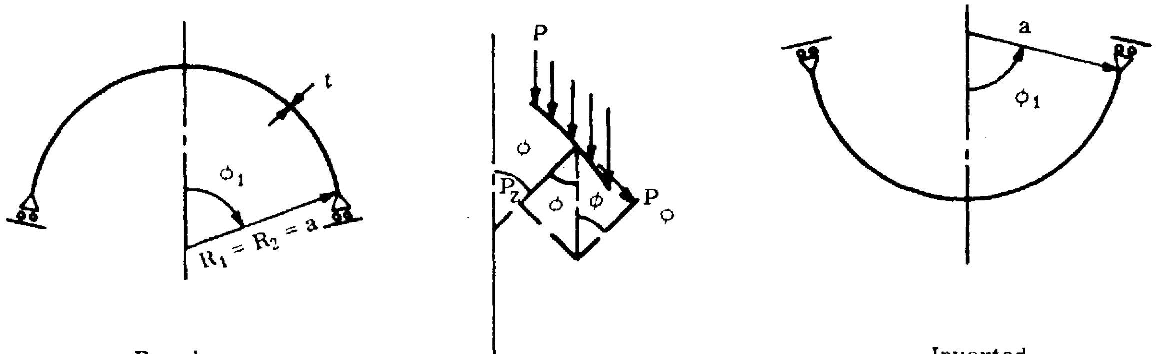
B7.1.2.1 SPHERICAL DOMES

This subsection presents the solutions for nonshallow spherical shells exposed to axisymmetric loading. Both closed and open shells will be considered. The boundaries of the shell must be free to rotate and deflect normal to the shell middle surface. No abrupt discontinuities in shell thickness shall be present.

Note that because of the geometry, $R_1 = R_2 = a$ (radius of spherical shell), and radial deflection $w = \Delta a$. Therefore, $\bar{w} \sin \phi$ and $\bar{u} = w \cos \phi$.

The following loading conditions will be considered: dead weight (Table B7.1.2.1 - 1); uniform load over base area (Table B7.1.2.1 - 2); hydrostatic pressure (Table B7.1.2.1 - 3); uniform pressure (Table B7.1.2.1 - 4); and lantern (Table B7.1.2.1 - 5). These tables begin on page 31.

Table B7.1.2.1 - 1. Dead Weight Loading
Membrane Stresses and Deformations, Closed Spherical Domes



Regular   Dead Weight Loading   Inverted

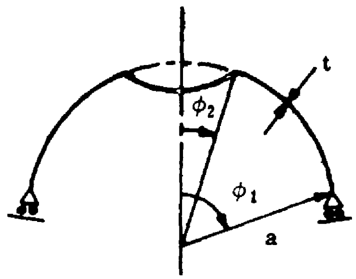
$P_\theta = 0$

$P_\phi = P\sin\phi$

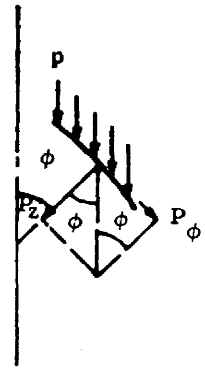
$P_z = P\cos\phi$

Regular	Inverted
$N_\phi = \dfrac{-ap}{1+\cos\phi}$	$N_\phi = \dfrac{ap}{1+\cos\phi}$
$N_\theta = -ap\left(\cos\phi - \dfrac{1}{1+\cos\phi}\right)$	$N_\theta = ap\left(\cos\phi - \dfrac{1}{1+\cos\phi}\right)$
$\sigma_\phi, \sigma_\theta = \dfrac{N_\phi}{t}, \dfrac{N_\theta}{t}$	$\sigma_\phi, \sigma_\theta = \dfrac{N_\phi}{t}, \dfrac{N_\theta}{t}$
$w = \dfrac{a^2p}{Et}\left[-\cos\phi + \dfrac{1+\mu}{\sin^2\phi}(1-\cos\phi)\right]$	$w = -\dfrac{a^2p}{Et}\left[-\cos\phi + \dfrac{1+\mu}{\sin^2\phi}(1-\cos\phi)\right]$
$\bar{w} = w\sin\phi$	$\bar{w} = w\sin\phi$
$\bar{u} = w\cos\phi$	$\bar{u} = w\cos\phi$

Table B7.1.2.1 - 1 (Concluded). Dead Weight Loading Membrane Stresses and Deformations, Open Spherical Domes



Regular

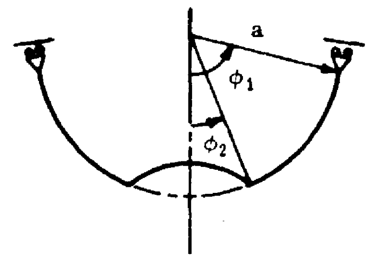


Dead Weight Loading

$P_\theta = 0$

$P_\phi = P \sin \phi$

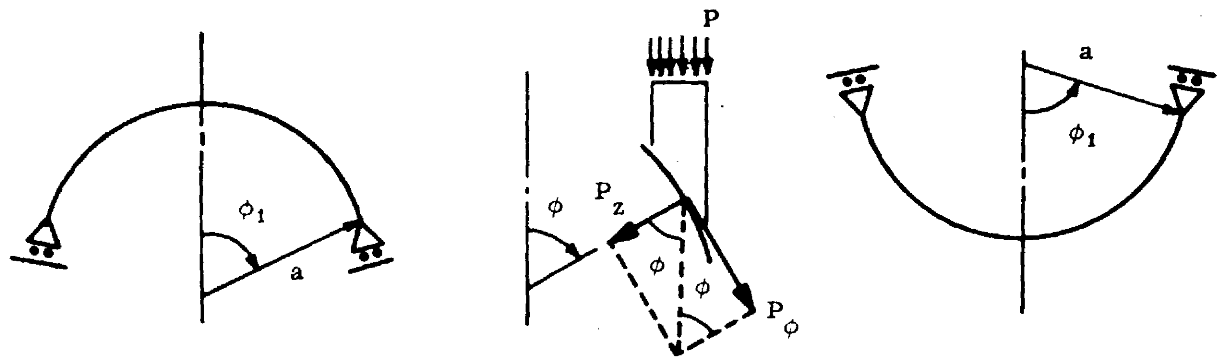
$P_z = P \cos \phi$



Inverted

Regular	Inverted
$N_\phi = -\frac{ap}{\sin^2\phi}(\cos\phi_2 - \cos\phi)$	$N_\phi = \frac{ap}{\sin^2\phi}(\cos\phi_2 - \cos\phi)$
$N_\theta = -ap\left[\cos\phi - \frac{1}{\sin^2\phi}(\cos\phi_2 - \cos\phi)\right]$	$N_\theta = ap\left[\cos\phi - \frac{1}{\sin^2\phi}(\cos\phi_2 - \cos\phi)\right]$
$\sigma_\phi, \sigma_\theta = \frac{N_\phi}{t}, \frac{N_\theta}{t}$	$\sigma_\phi, \sigma_\theta = \frac{N_\phi}{t}, \frac{N_\theta}{t}$
$w = \frac{a^2p}{Et}\left[-\cos\phi + \frac{1+\mu}{\sin^2\phi}(\cos\phi_2 - \cos\phi)\right]$	$w = -\frac{a^2p}{Et}\left[-\cos\phi + \frac{1+\mu}{\sin^2\phi}(\cos\phi_2 - \cos\phi)\right]$
$\bar{w} = w \sin\phi$	$\bar{w} = w \sin\phi$
$\bar{u} = w \cos\phi$	$\bar{u} = w \cos\phi$

Table B7.1.2.1 - 2. Uniform Loading over Base Area
Membrane Stresses and Deformations, Closed Spherical Domes



Regular — Inverted

Uniform Loading over Base Area

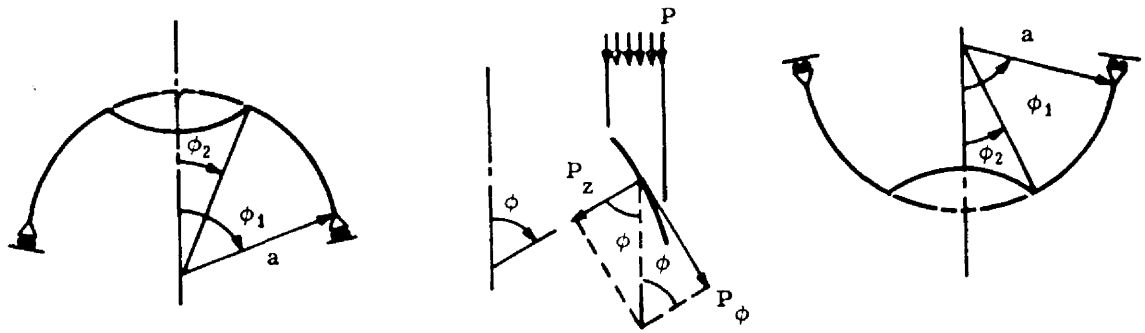
$$P_\theta = 0$$

$$P_\phi = P\cos\phi\,\sin\phi$$

$$P_z = P\cos^2\phi$$

Regular	Inverted
$N_\phi = -\frac{ap}{2}$	$N_\phi = \frac{ap}{2}$
$N_\theta = -\frac{ap}{2}\cos 2\phi$	$N_\theta = \frac{ap}{2}\cos 2\phi$
$\sigma_\phi,\ \sigma_\theta = \frac{N_\phi}{t},\ \frac{N_\theta}{t}$	$\sigma_\phi,\ \sigma_\theta = \frac{N_\phi}{t},\ \frac{N_\theta}{t}$
$w = \frac{a^2p}{Et}\left[\frac{1+\mu}{2} - \cos^2\phi\right]$	$w = -\frac{a^2p}{Et}\left[\frac{1+\mu}{2} - \cos^2\phi\right]$
$\bar{w} = w\sin\phi$	$\bar{w} = w\sin\phi$
$\bar{u} = w\cos\phi$	$\bar{u} = w\cos\phi$

Table B7.1.2.1 - 2 (Concluded). Uniform Loading over Base Area Membrane Stresses and Deflections, Open Spherical Domes



Regular | Uniform Loading over Base Area | Inverted

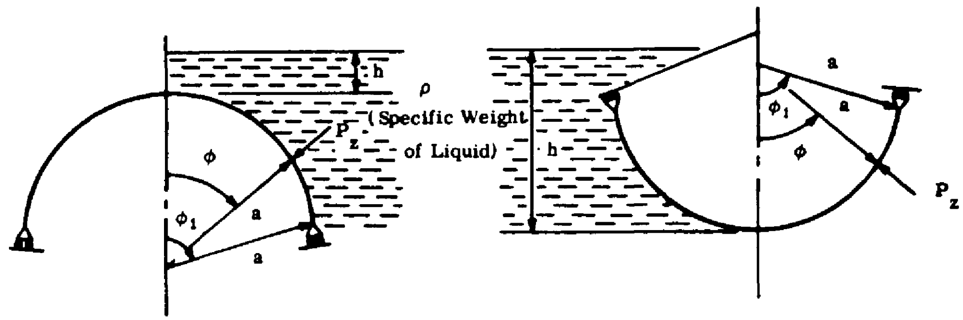
$P_\theta = 0$

$P_\phi = P\cos\phi\,\sin\phi$

$P_z = P\cos^2\phi$

Regular	Inverted
$N_\phi = -\frac{ap}{2}\left(1 - \frac{\sin^2\phi_2}{\sin^2\phi}\right)$	$N_\phi = \frac{ap}{2}\left(1 - \frac{\sin^2\phi_2}{\sin^2\phi}\right)$
$N_\theta = -\frac{ap}{2}\left(2\cos^2\phi - 1 + \frac{\sin^2\phi_2}{\sin^2\phi}\right)$	$N_\theta = \frac{ap}{2}\left(2\cos^2\phi - 1 + \frac{\sin^2\phi_2}{\sin^2\phi}\right)$
$\sigma_\phi, \sigma_\theta = \frac{N_\phi}{t}, \frac{N_\theta}{t}$	$\sigma_\phi, \sigma_\theta = \frac{N_\phi}{t}, \frac{N_\theta}{t}$
$w = \frac{a^2p}{Et}\left[-\cos^2\phi + \frac{1+\mu}{2}\left(1 - \frac{\sin^2\phi_2}{\sin^2\phi}\right)\right]$	$w = -\frac{a^2p}{Et}\left[-\cos^2\phi + \frac{1+\mu}{2}\left(1 - \frac{\sin^2\phi_2}{\sin^2\phi}\right)\right]$
$\bar{w} = w\sin\phi$	$\bar{w} = w\sin\phi$
$\bar{u} = w\cos\phi$	$\bar{u} = w\cos\phi$

Table B7.1.2.1 - 3. Hydrostatic Pressure Loading Membrane Stresses and Deformations, Closed Spherical Domes



Regular Inverted

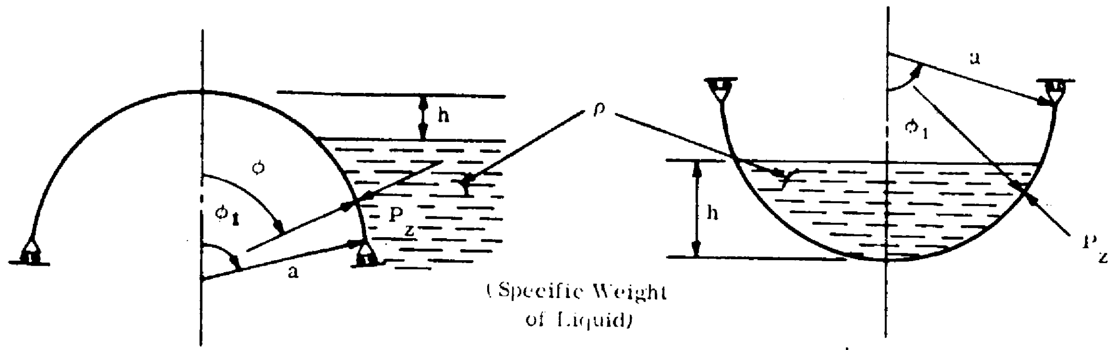
$$P_\theta = P_\phi = 0$$

$$P_z = \rho[h + a(1 - \cos\phi)]$$

$$P_z = \rho[h - a(1 - \cos\phi)]$$

Regular	Inverted
$N_\phi = \frac{\rho a^2}{6}\left(1 + \frac{3h}{a} - \frac{2\cos^2\phi}{1 + \cos\phi}\right)$	$N_\phi = \frac{\rho a^2}{6}\left(-1 + \frac{3h}{a} + \frac{2\cos^2\phi}{1 + \cos\phi}\right)$
$N_\theta = -\frac{\rho a^2}{6}\left(-1 + \frac{3h}{a} - \frac{4\cos^2\phi - 6}{1 + \cos\phi}\right)$	$N_\theta = \frac{\rho a^2}{6}\left(1 + \frac{3h}{a} - \frac{4\cos^2\phi - 6}{1 + \cos\phi}\right)$
$\sigma_\phi, \sigma_\theta = \frac{N_\phi}{t}, \frac{N_\theta}{t}$	$\sigma_\phi, \sigma_\theta = \frac{N_\phi}{t}, \frac{N_\theta}{t}$
$w = -\frac{\rho a^3}{6Et}\left[3\left(1 + \frac{h}{a}\right)(1 - \mu) - 6\cos\phi - \frac{2(1 + \mu)}{\sin^2\phi}(\cos^3\phi - 1)\right]$	$w = \frac{\rho a^3}{6Et}\left[3\left(1 + \frac{h}{a}\right)(1 - \mu) - 6\cos\phi - \frac{2(1 + \mu)}{\sin^2\phi}(\cos^3\phi - 1)\right]$
$\bar{w} = w\sin\phi$	$\bar{w} = w\sin\phi$
$\bar{u} = w\cos\phi$	$\bar{u} = w\cos\phi$

Table B7.1.2.1 - 3 (Continued). Hydrostatic Pressure Loading (Partial) Membrane Stress Resultants, Closed Spherical Domes



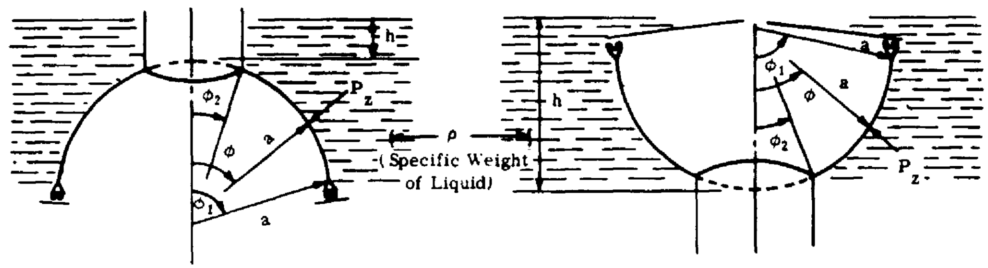
$P_z = \rho(a - a\cos\phi - h)$

$\phi' = 180 - \phi$

$P_z = -\rho(a - a\cos\phi' - h)$

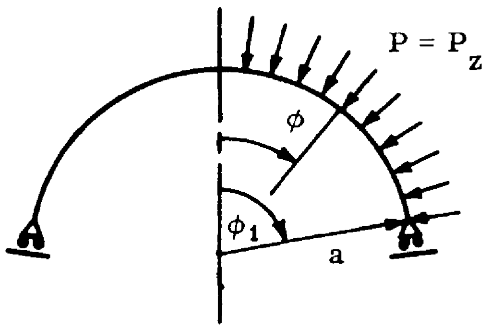
Regular	Inverted
(Above Liquid Level)	(Above Liquid Level)
$N_\phi = N_\theta = 0$	$N_\phi = \frac{\rho h^2}{6}\left(3 - \frac{h}{a}\right)\frac{1}{\sin^2\phi'}$ $N_\theta = -\frac{\rho h^2}{6}\left(3 - \frac{h}{a}\right)\frac{1}{\sin^2\phi'}$
(Below Liquid Level)	(Below Liquid Level)
$N_\phi = -\frac{\rho a^2}{6}\left\{\frac{h}{a}\left[\frac{h}{a\sin^2\phi}\left(3 - \frac{h}{a}\right) - 3\right] + 1 - \frac{2\cos^2\phi}{1 + \cos\phi}\right\}$	$N_\phi = \frac{\rho a^2}{6}\left[\frac{3h}{a} - 1 + \frac{2\cos^2\phi'}{1 + \cos\phi'}\right]$
$N_\theta = -\frac{\rho a^2}{6}\left[1 - \cos\phi - \frac{h}{a}\right] - N_\phi$	$N_\theta = \rho a^2\left[\frac{h}{a} - 1 + \cos\phi'\right] - N_\phi$
$\sigma_\phi = \frac{N_\phi}{t}, \quad \sigma_\theta = \frac{N_\theta}{t}$	$\sigma_\phi = \frac{N_\phi}{t}, \quad \sigma_\theta = \frac{N_\theta}{t}$
For Deflections See Section B7.1.1.4 - IV	For Deflections See Section B7.1.1.4 - IV

Table B7.1.2.1 - 3 (Concluded). Hydrostatic Pressure Loading Membrane Stresses and Deformations, Open Spherical Domes



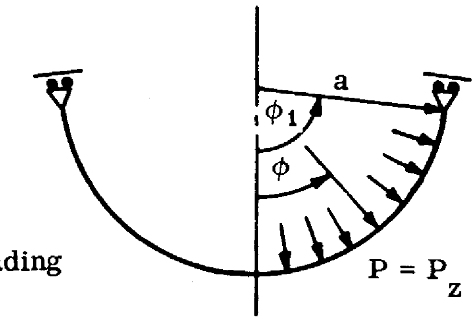
Regular	Inverted
$N_\phi = -\frac{\rho a^2}{6}\left[3\left(1+\frac{h}{a}\right)\left(1-\frac{\sin^2\phi_2}{\sin^2\phi}\right)-2\left(\frac{\cos^3\phi_2-\cos^3\phi}{\sin^2\phi}\right)\right]$	$N_\phi = \frac{\rho a^2}{6}\left[3\left(1-\frac{h}{a}\right)\left(1-\frac{\sin^2\phi_2}{\sin^2\phi}\right)-2\left(\frac{\cos^3\phi_2-\cos^3\phi}{\sin^2\phi}\right)\right]$
$N_\theta = -\frac{\rho a^2}{6}\left[3\left(1+\frac{h}{a}\right)\left(1+\frac{\sin^2\phi_2}{\sin^2\phi}\right)+\frac{2(2\cos^3\phi+\cos^3\phi_2)-6\cos\phi}{\sin^2\phi}\right]$	$N_\theta = \frac{\rho a^2}{6}\left[3\left(1-\frac{h}{a}\right)\left(1+\frac{\sin^2\phi_2}{\sin^2\phi}\right)+\frac{2(2\cos^3\phi+\cos^3\phi_2)-6\cos\phi}{\sin^2\phi}\right]$
$\sigma_\phi, \sigma_\theta = \frac{N_\phi}{t}, \frac{N_\theta}{t}$	$\sigma_\phi, \sigma_\theta = \frac{N_\phi}{t}, \frac{N_\theta}{t}$
$w = -\frac{\rho a^3}{6Et}\left\{3\left(1+\frac{h}{a}\right)\left[1-\mu+(1+\mu)\frac{\sin^2\phi_2}{\sin^2\phi}\right]-6\cos\phi + 2(1+\mu)\frac{\cos^3\phi_2-\cos^3\phi}{\sin^2\phi}\right.$	$w = \frac{\rho a^2}{6Et}\left\{3\left(1+\frac{h}{a}\right)\left[1-\mu+(1+\mu)\frac{\sin^2\phi_2}{\sin^2\phi}\right]-6\cos\phi + 2(1+\mu)\frac{\cos^3\phi_2-\cos^3\phi}{\sin^2\phi}\right\}$
$\bar{w} = w\sin\phi$	$\bar{w} = w\sin\phi$
$\bar{u} = w\cos\phi$	$\bar{u} = w\cos\phi$

Table B7.1.2.1 - 4. Uniform Pressure Loading
Membrane Stresses and Deformations, Closed Spherical Domes



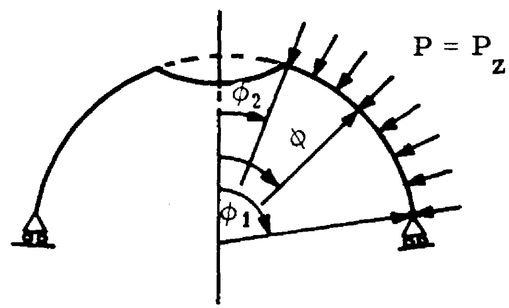
$P_\phi = P_\theta = 0$

Uniform Pressure Loading



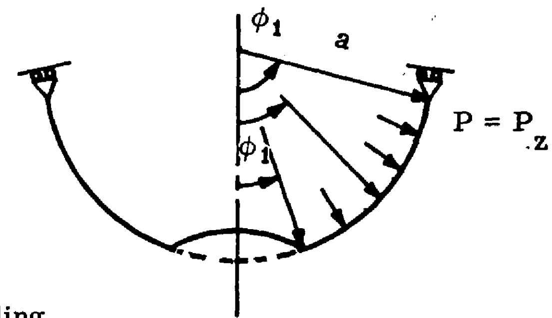
Regular	Inverted
$N_\phi = -\dfrac{ap}{2}$	$N_\phi = \dfrac{ap}{2}$
$N_\theta = -\dfrac{ap}{2}$	$N_\theta = \dfrac{ap}{2}$
$\sigma_\phi = \sigma_\theta = \dfrac{N_\phi}{t} = \dfrac{N_\theta}{t}$	$\sigma_\phi = \sigma_\theta = \dfrac{N_\phi}{t} = \dfrac{N_\theta}{t}$
$w = -\dfrac{a^2p}{2Et}(1-\mu)$	$w = \dfrac{a^2p}{2Et}(1-\mu)$
$\bar{w} = -\dfrac{a^2p}{2Et}\sin\phi\,(1-\mu)$	$\bar{w} = \dfrac{a^2p}{2Et}\sin\phi\,(1-\mu)$
$\bar{u} = -\dfrac{a^2p}{2Et}\cos\phi\,(1-\mu)$	$\bar{u} = \dfrac{a^2p}{2Et}\cos\phi\,(1-\mu)$

Table B7.1.2.1 - 4 (Concluded). Uniform Pressure Loading Membrane Stresses and Deformations, Open Spherical Shells



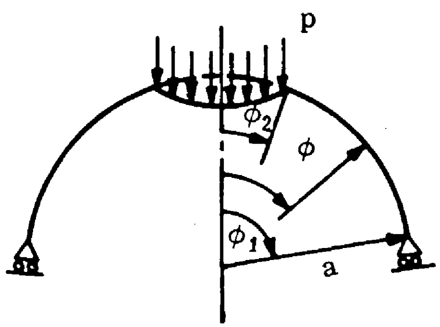
$P_\phi = P_\theta = 0$

Uniform Pressure Loading

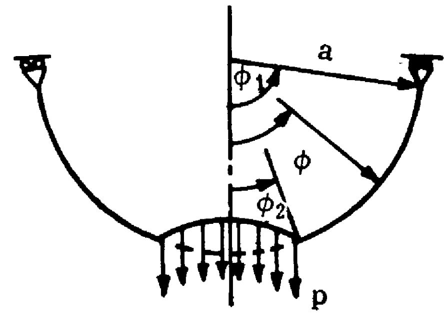


Regular	Inverted
$N_\phi = -\frac{ap}{2}\left(1 - \frac{\sin^2\phi_2}{\sin^2\phi}\right)$	$N_\phi = \frac{ap}{2}\left(1 - \frac{\sin^2\phi_2}{\sin^2\phi}\right)$
$N_\theta = -\frac{ap}{2}\left(1 + \frac{\sin^2\phi_2}{\sin^2\phi}\right)$	$N_\theta = \frac{ap}{2}\left(1 + \frac{\sin^2\phi_2}{\sin^2\phi}\right)$
$\sigma_\phi, \sigma_\theta = \frac{N_\phi}{t}, \frac{N_\theta}{t}$	$\sigma_\phi, \sigma_\theta = \frac{N_\phi}{t}, \frac{N_\theta}{t}$
$w = -\frac{a^2p}{Et}\left[1 - \frac{1+\mu}{2}\left(1 - \frac{\sin^2\phi_2}{\sin^2\phi}\right)\right]$	$w = \frac{a^2p}{Et}\left[1 - \frac{1+\mu}{2}\left(1 - \frac{\sin^2\phi_2}{\sin^2\phi}\right)\right]$
$\bar{w} = w \sin\phi$	$\bar{w} = w \sin\phi$
$\bar{u} = w \cos\phi$	$\bar{u} = w \cos\phi$

Table B7.1.2.1 - 5. Lantern Loading
Membrane Stresses and Deflections, Open Spherical Domes



$$P_z = P_\phi = P_\theta = 0$$



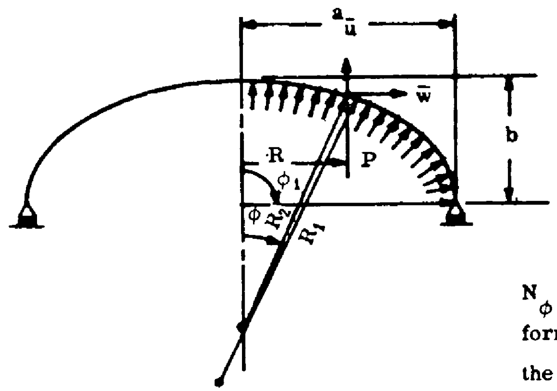
Lantern Loading

Regular	Inverted
$N_\phi = -\dfrac{p \sin\phi_2}{\sin^2\phi}$	$N_\phi = \dfrac{p \sin\phi_2}{\sin^2\phi}$
$N_\theta = \dfrac{p \sin\phi_2}{\sin^2\phi}$	$N_\theta = -\dfrac{p \sin\phi_2}{\sin^2\phi}$
$\sigma_\phi, \sigma_\theta = \dfrac{N_\phi}{t}, \dfrac{N_\theta}{t}$	$\sigma_\phi, \sigma_\theta = \dfrac{N_\phi}{t}, \dfrac{N_\theta}{t}$
$w = \dfrac{ap}{Et}\left(\dfrac{1+\mu}{\sin\phi}\right)$	$w = -\dfrac{ap(1+\mu)}{Et}\left(\dfrac{\sin\phi_2}{\sin^2\phi}\right)$
$\bar{w} = \dfrac{ap}{Et}(1+\mu)$	$\bar{w} = -\dfrac{ap}{Et}(1+\mu)\left(\dfrac{\sin\phi_2}{\sin\phi}\right)$
$\bar{u} = \dfrac{ap}{Et}(1+\mu)\cot\phi$	$\bar{u} = -\dfrac{ap}{Et}(1+\mu)\left(\dfrac{\sin\phi_2}{\sin\phi}\right)\cot\phi$

B7.1.2.2 ELLIPTICAL DOMES

This subsection presents the solutions for elliptical shells exposed to axisymmetric loadings. Only closed elliptical shells are considered. The boundaries of the elliptical shell must be free to rotate and deflect normal to the shell middle surface. Abrupt changes in the shell thickness must not be present. The following loading conditions will be considered: uniform pressure (Table B7.1.2.2 - 1); stress resultant and displacement parameters (Figs. B7.1.2.2 - 1 and -2); dead weight (Table B7.1.2.2 - 2); and uniform load over base area (Table B7.1.2.2 - 3). These tables begin on page 42.

Table B7.1.2.2 - 1. Uniform Pressure Loading Membrane Stresses and Deflections, Closed Ellipsoidal Dome



Uniform Pressure

$P_\phi = P_\theta = 0$

$P_z = P$

$$R_1 = \frac{a^2b^2}{(a^2\sin^2\phi + b^2\cos^2\phi)^{3/2}}$$

$$R_2 = \frac{a^2}{(a^2\sin^2\phi + b^2\cos^2\phi)^{1/2}}$$

N_ϕ and N_θ are plotted in nondimensional form in Figure B7.1.2.2 - 1 according to the following equations

$$N_\phi = \frac{R_2 p}{2} \qquad\qquad \frac{N_\phi}{ap} = \frac{a}{2b}\left\{1 - \left(\frac{R}{a}\right)^2\left[1 - \left(\frac{b}{a}\right)^2\right]\right\}^{-1/2}$$

$$N_\theta = \frac{R_2 p}{2}\left(2 - \frac{R_2}{R_1}\right) \qquad\qquad \frac{N_\theta}{ap} = \frac{N_\phi}{a}\left\{2 - \frac{1}{1 - \left(\frac{R}{a}\right)^2\left[1 - \left(\frac{b}{a}\right)^2\right]}\right\}$$

$$\bar{w} = \frac{pR_2^2\sin\phi}{2Et}\left[2 - \mu - \frac{R_1}{R_2}\right]$$

$$\bar{u} = \frac{a^2p}{4Et}\left\{\left[3\left(\frac{a}{b}\right)^2 - 1 - 2\mu\right]\left[\frac{\cos\phi}{1 + K^2\sin^2\phi}\right] - 2\left(\frac{a}{b}\right)^2\cos\phi + \frac{bK}{a}\left[\left(\frac{a}{b}\right)^2 - \frac{1}{2} + \mu\right] Ln\left[\frac{\frac{a}{b} - K\cos\phi}{\frac{a}{b} - K\cos\phi}\right]\right\}$$

Let $K = \sqrt{\left(\frac{a}{b}\right)^2 - 1}$

$\bar{w}$ and $\bar{u}$ are plotted in nondimensional form in Figure B7.1.2.2 - 2, for t = constant and $\mu = 0.3$

$w = \bar{w}\sin\phi - \bar{u}\cos\phi \qquad u = \bar{w}\cos\phi + \bar{u}\sin\phi$

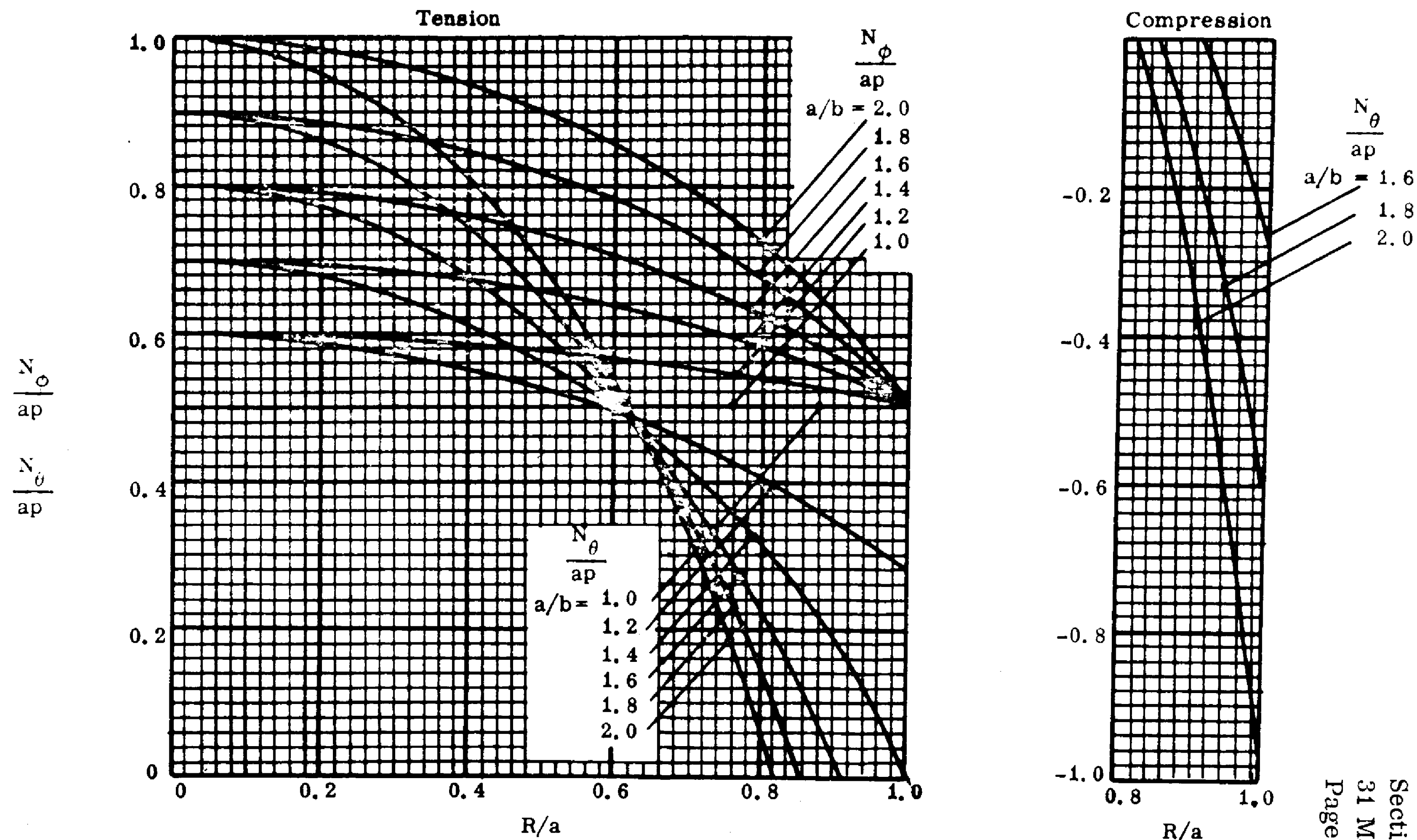


Fig. B7.1.2.2 - 1. Stress Resultant Parameters, Ellipsoidal Shell, Uniform Pressure

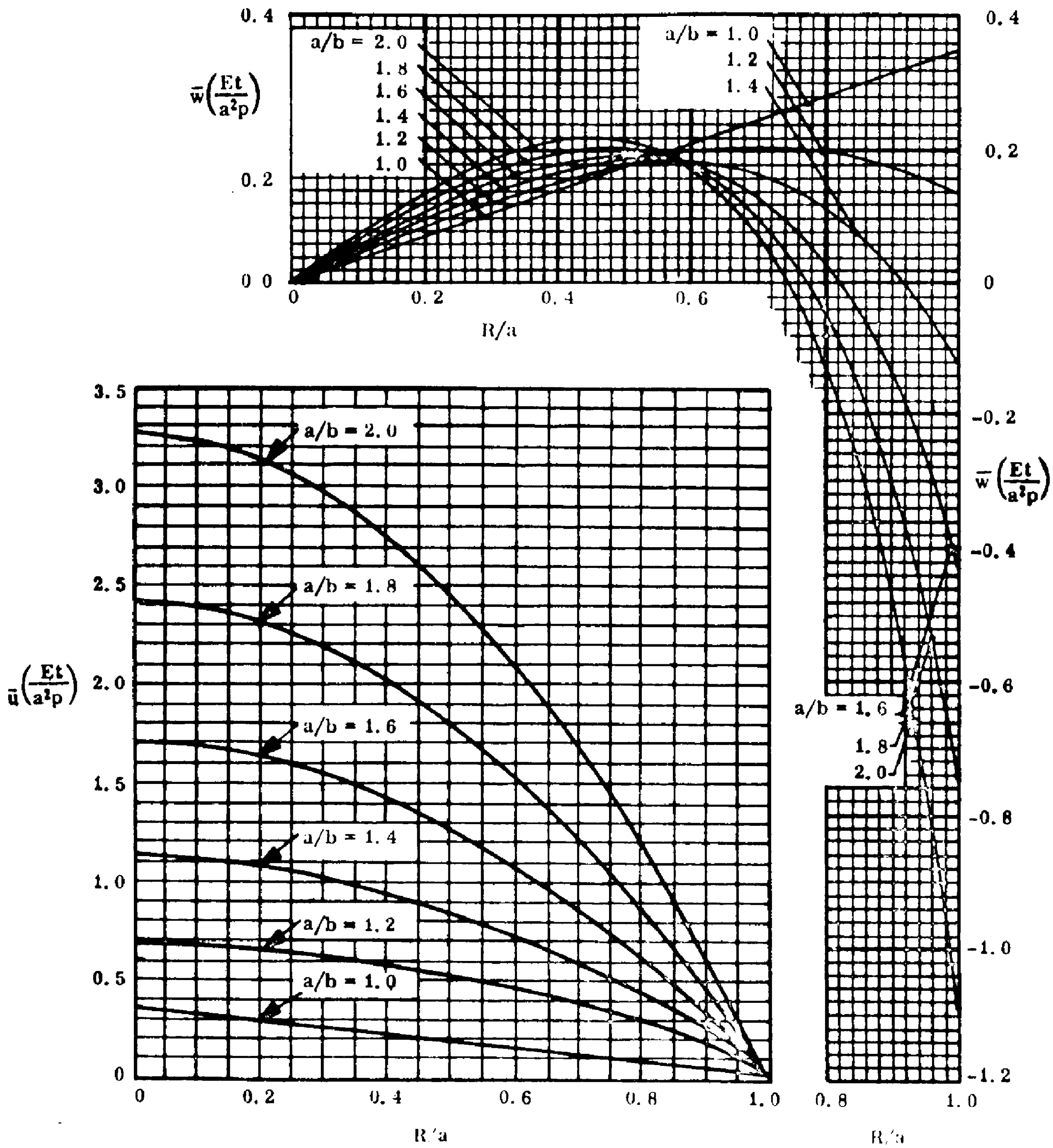
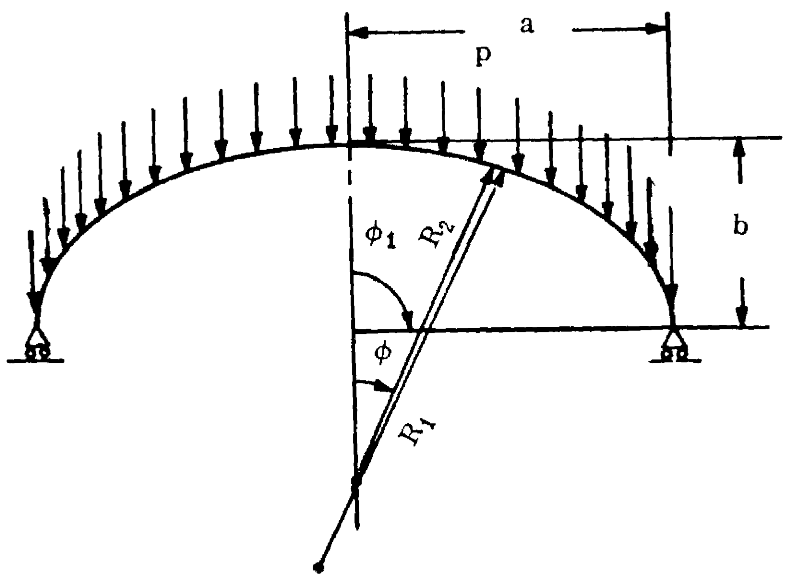


Figure B7.1.2.2 - 2. Displacement Parameters, Ellipsoidal Shells, Uniform Pressure

Table B7.1.2.2 - 2. Dead Weight Loading
Membrane Stress Resultants, Closed Ellipsoidal Dome



Dead Weight Loading

$P_\theta = 0$

$P_\phi = p\sin\phi$

$P_t = p\cos\phi$

Let $K = \sqrt{\dfrac{a^2 - b^2}{a}}$

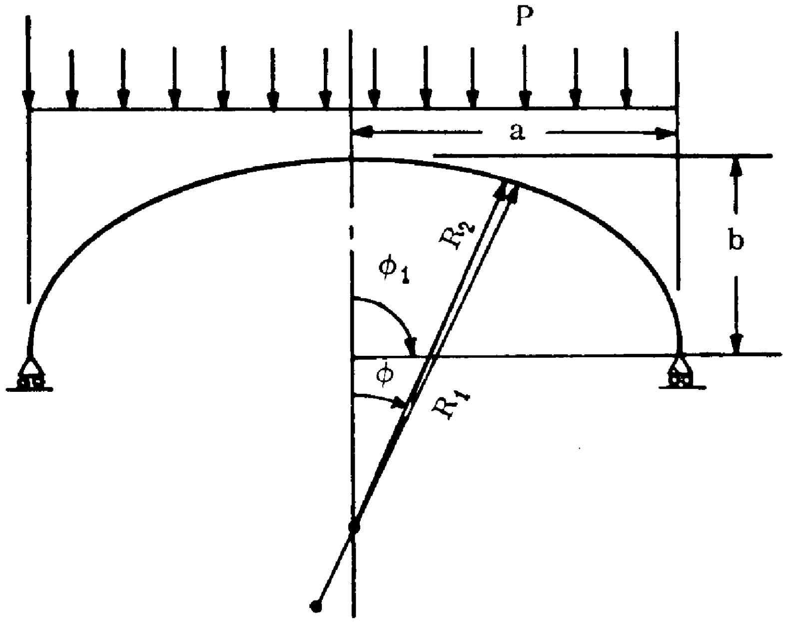
$$N_\phi = -\frac{p}{2}\left[\frac{\sqrt{a^2\tan^2\phi + b^2}}{a^2\sin\phi\tan\phi}\right]\left[a^2 - \frac{a^2b^2\sqrt{1+\tan^2\phi}}{b^2 + a^2\tan^2\phi} + \frac{b^2}{K}\,\mathrm{Ln}\frac{(1+K)\sqrt{b^2 + a^2\tan^2\phi}}{b\left(K + \sqrt{1+\tan^2\phi}\right)}\right]$$

$$N_\theta = p\left[\frac{(b^2 + a^2\tan^2\phi)^{3/2}}{2\tan^2\phi\sqrt{1+\tan^2\phi}}\left(\frac{1}{Ka^2}\,\mathrm{Ln}\frac{(1+K)\sqrt{b^2 + a^2\tan^2\phi}}{b\left(K + \sqrt{1+\tan^2\phi}\right)} + \frac{1}{b^2} - \frac{\sqrt{1+\tan^2\phi}}{b^2 + a^2\tan^2\phi}\right) - \frac{a^2}{\sqrt{b^2 + a^2\tan^2\phi}}\right]$$

$$\sigma_\phi, \sigma_\theta = \frac{N_\phi}{t}, \frac{N_\theta}{t}$$

For deflections, refer to Section B7.1.1.4.

Table B7.1.2.2 - 3. Uniform Load over Base Area
Membrane Stress Resultants, Closed Ellipsoidal Dome



Uniform Loading over
Base Area

$$P_\theta = 0$$

$$P_\phi = p \sin\phi \cos\phi$$

$$P_z = p \cos^2\phi$$

$$N_\phi = -\frac{P}{2} \frac{a^2\sqrt{1 + \tan^2\phi}}{\sqrt{b^2 + a^2\tan^2\phi}}$$

$$N = -\frac{a^2 p}{2b^2} \frac{b^2 - a^2\tan^2\phi}{\sqrt{b^2 + a^2\tan^2\phi}\sqrt{1 + \tan^2\phi}}$$

$$\sigma_\phi, \sigma_\theta = \frac{N_\phi}{t}, \frac{N_\theta}{t}$$

For deflections, refer to Section B7.1.1.4.

B7.1.2.3 CASSINI DOMES

This family of shells is useful as bulkheads. The equation of the Cassinian curve as a meridian (see Table B7.1.2.3 - 1) is

$$(r^2 + z^2)^2 + 2a^2(r^2 - z^2) = 3a^4 \quad .$$

The useful property of this shell (zero curvature at $z = 0$, $r = a$) is preserved by making the substitution nz for z, where $n > 1$ but not much greater than 2.

$$(r^2 + n^2z^2)^2 + 2a^2(r^2 - n^2z^2) = 3a^4$$

$$R_1 = \frac{2\,[\,r^2(a^2 + n^2z^2) + n^4z^2(a^2 - r^2)\,]^{3/2}}{3n^2a^3(a^2 - r^2 + n^2z^2)}$$

$$R_2 = \frac{2a\,[\,r^2(a^2 + n^2z^2) + n^4z^2(a^2 - r^2)\,]^{1/2}}{a^2 + r^2 + n^2z^2}$$

This subsection presents solutions for the Cassini dome subjected to uniform pressure loading. Only a closed dome will be considered. The boundaries of the shell must be free to rotate and to deflect normal to the shell middle surface. No abrupt discontinuities in the shell thickness shall be present. Because of the limited usefulness of this shell, a detailed solution is presented for $n = 2$. Nondimensional plots are presented for N_ϕ and N_θ according to the following equations:

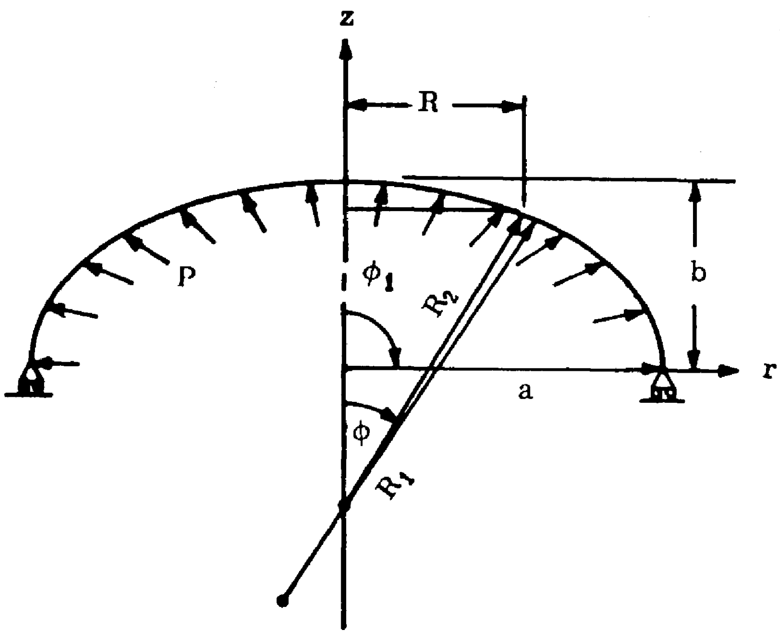
$$\frac{N_\phi}{ap} = \frac{2}{5(4K + 3)}\,[\,5(16K^4 + 24K^3 - 7K^2 + 8K + 3)\,]^{1/2}$$

$$\frac{N_\theta}{ap} = \frac{4(64K^5 + 144K^4 + 44K^3 - 85K^2 - 36K + 23)}{(4K + 3)^2[\,5(16K^4 + 24K^3 - 7K^2 + 8K + 3)\,]^{1/2}}$$

where $K = \left[1 - \frac{15}{16}\left(\frac{R}{a}\right)^2\right]^{1/2}$

Nondimensional plots are also provided for $\bar{w}$ and $\bar{u}$ for t = constant and $\mu = 0.3$.

Table B7.1.2.3 - 1. Uniform Pressure Loading Membrane Stresses and Deflections, Closed Cassini Dome



Special Case, n = 2

$$b = \frac{a}{2}\sqrt{\frac{11}{5}}$$

Uniform Pressure Loading

$$P_\phi = P_\theta = 0$$

$$P_z = p$$

$$N_\phi = \frac{R_2 p}{2}$$

$$N_\theta = \frac{R_2 p}{2}\left(2 - \frac{R_2}{R_1}\right)$$

$$\sigma_\phi , \sigma_\theta = \frac{N_\phi}{t} , \frac{N_\theta}{t}$$

$$\bar{w} = \frac{pR_2^2 \sin\phi}{2Et}\left(2 - \mu - \frac{R_2}{R_1}\right)$$

$$\bar{u} = \bar{w}\cot\phi - \int \frac{R_1(N_\phi - \mu N_\theta) - R_2(N_\theta - \mu N_\phi)}{Et\sin\phi} d\phi + C$$

$$w = \bar{w}\sin\phi - \bar{u}\cos\phi$$

$$u = \bar{w}\cos\phi + \bar{u}\sin\phi$$

Equations for R_1 and R_2 are given in Section B7.1.2.3 .
See Figure B7.1.2.3 - 1 for nondimensional plots of N_ϕ and N_θ .
See Figure B7.1.2.3 - 2 for nondimensional plots of $\bar{w}$ and $\bar{u}$.

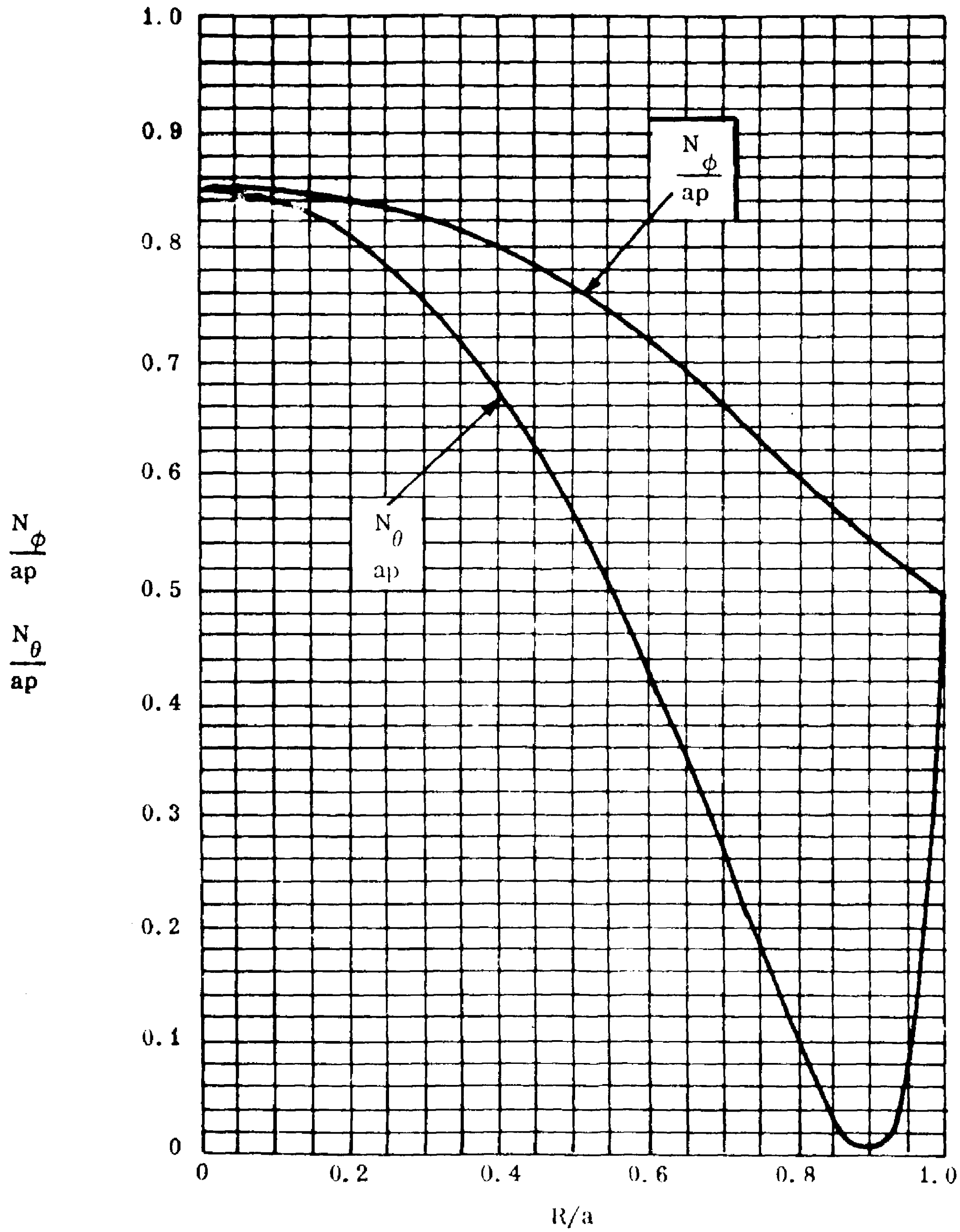


Fig. B7.1.2.3 - 1. Stress Resultant Parameters Cassini Shells (n = 2), Uniform Pressure

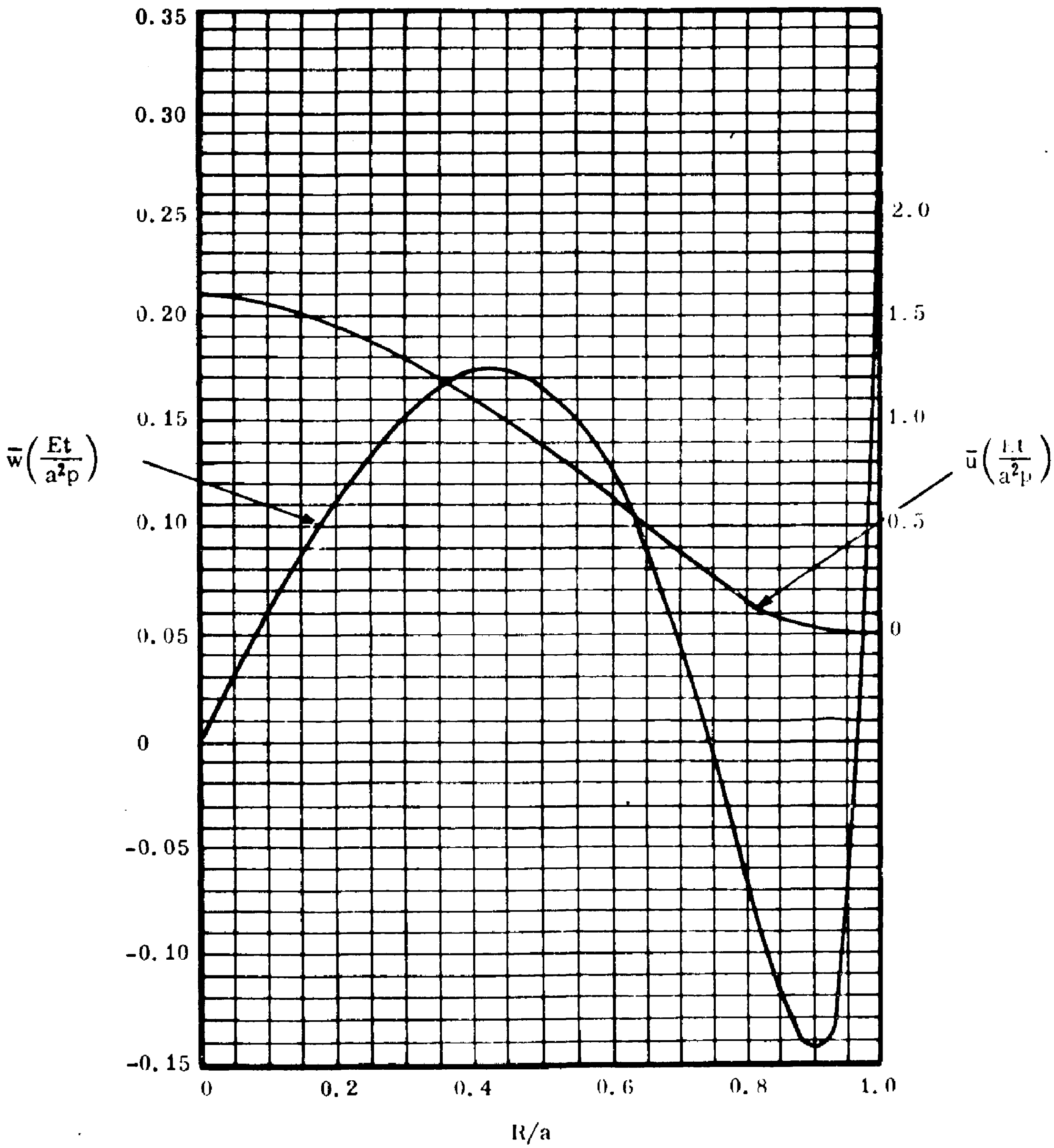


Fig. B7.1.2.3 - 2. Displacement Parameters
Cassini Shells (n = 2), Uniform Pressure

B7.1.2.4 CONICAL DOMES

This subsection presents the solutions for nonshallow conical shells exposed to axisymmetric loading. Both closed and open shells will be considered. The boundaries of the shell must be free to rotate and deflect normal to the shell middle surface. No abrupt discontinuities in the shell thickness shall be present.

Note the special geometry of the conical shell:

$\phi = \alpha =$ constant, $R_1 = \infty$

$R = x\cos\phi$ (Figure B7.1.2.4 - 1)

For convenience, solutions are presented in terms of x instead of R. All other notations are standard for shells of revolution as used in this chapter.

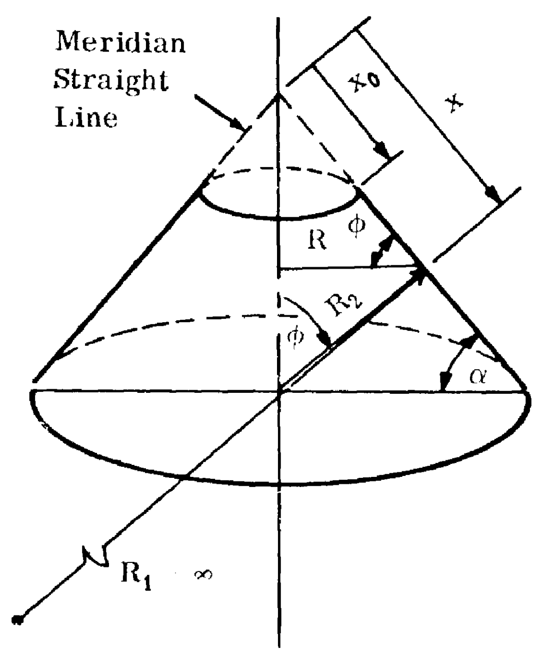
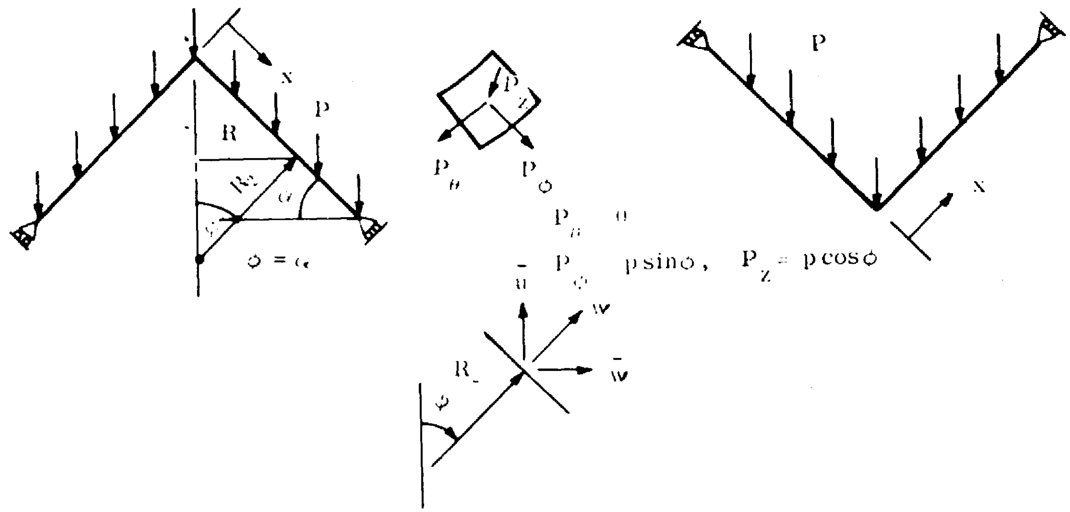


Fig. B7.1.2.4 - 1. Conical Shell Geometry

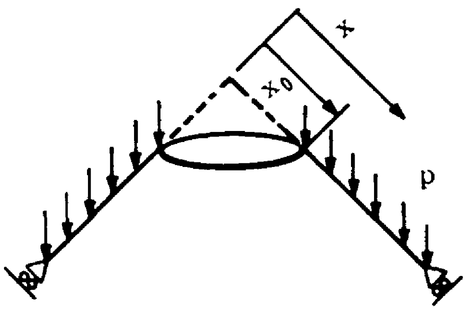
The following loading conditions will be considered: dead weight loading (Table B7.1.2.4 - 1); uniform loading over base area (Table B7.1.2.4 - 2); hydrostatic pressure loading (Table B7.1.2.4 - 3); uniform pressure loading (Table B7.1.2.4 - 4); and lantern loading (Table B7.1.2.4 - 5) . These tables begin on page 53.

Table B7.1.2.4 - 1. Dead Weight Loading Membrane Stresses and Deflections, Closed Conical Domes

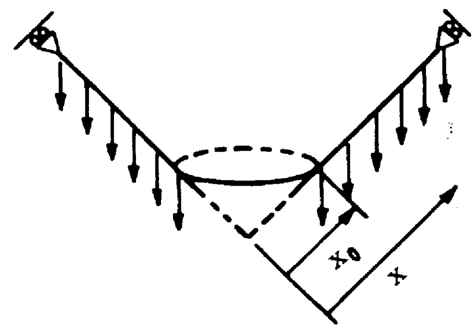


Regular	Inverted
$N_\phi = -\dfrac{px}{2\sin\phi}$	$N_\phi = \dfrac{px}{2\sin\phi}$
$N_\theta = -\dfrac{px\cos^2\phi}{\sin\phi}$	$N_\theta = \dfrac{px\cos^2\phi}{\sin\phi}$
$\sigma_\phi, \sigma_\theta = \dfrac{N_\phi}{t}, \dfrac{N_\theta}{t}$	$\sigma_\phi, \sigma_\theta = \dfrac{N_\phi}{t}, \dfrac{N_\theta}{t}$
$w = -\dfrac{px^2\cot\phi}{Et\sin\phi}\left(\cos^2\phi - \dfrac{\mu}{2}\right)$	$w = \dfrac{px^2\cot\phi}{Et\sin\phi}\left(\cos^2\phi - \dfrac{\mu}{2}\right)$
$\bar{w} = -\dfrac{px^2\cot\phi}{Et}\left(\cos^2\phi - \dfrac{\mu}{2}\right)$	$\bar{w} = \dfrac{px^2\cot\phi}{Et}\left(\cos^2\phi - \dfrac{\mu}{2}\right)$
$\bar{u} = -\dfrac{px^2\cos\phi\,\cot\phi}{Et}\left(\cos^2\phi - \dfrac{\mu}{2}\right)$	$\bar{u} = \dfrac{px^2\cos\phi\,\cot\phi}{Et}\left(\cos^2\phi - \dfrac{\mu}{2}\right)$

Table B7.1.2.4 - 1 (Concluded). Dead Weight Loading Membrane Stresses and Deflections, Open Conical Domes

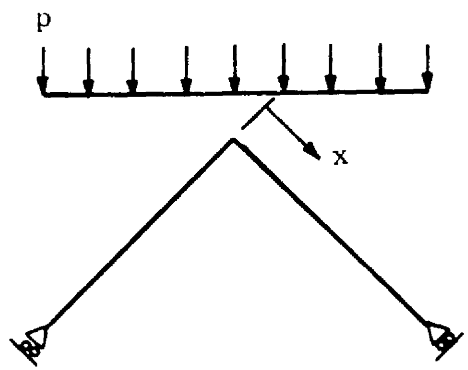


$$P_\theta = 0$$
$$P_\phi = p\sin\phi$$
$$P_z = p\cos\phi$$



Regular	Inverted
$N_\phi = -\dfrac{p}{2\sin\phi}\left(\dfrac{x^2 - x_0^2}{x}\right)$	$N_\phi = \dfrac{p}{2\sin\phi}\left(\dfrac{x^2 - x_0^2}{x}\right)$
$N_\theta = -\dfrac{px\cos^2\phi}{\sin\phi}$	$N_\theta = \dfrac{px\cos^2\phi}{\sin\phi}$
$\sigma_\phi, \sigma_\theta = \dfrac{N_\phi}{t}, \dfrac{N_\theta}{t}$	$\sigma_\phi, \sigma_\theta = \dfrac{N_\phi}{t}, \dfrac{N_\theta}{t}$
$w = -\dfrac{px^2\cot\phi}{2Et\sin\phi}\left[2\cos^2\phi - \mu\left(\dfrac{x^2 - x_0^2}{x^2}\right)\right]$	$w = \dfrac{px\cot\phi}{2Et\sin\phi}\left[2\cos^2\phi - \mu\left(\dfrac{x^2 - x_0^2}{x^2}\right)\right]$
$\bar{w} = w\sin\phi$	$\bar{w} = w\sin\phi$
$\bar{u} = w\cos\phi$	$\bar{u} = w\cos\phi$

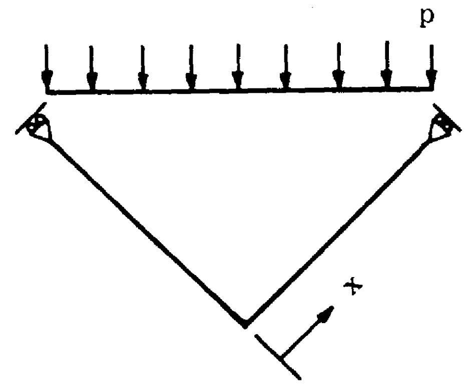
Table B7.1.2.4 - 2. Uniform Loading over Base Area
Membrane Stresses and Deflections, Closed Conical Domes



$P_\theta = 0$

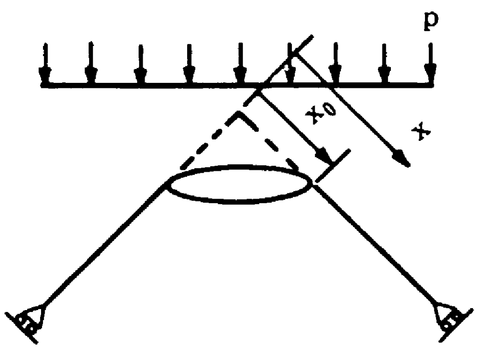
$P_\phi = p \sin\phi \cos\phi$

$P_z = p \cos^2\phi$

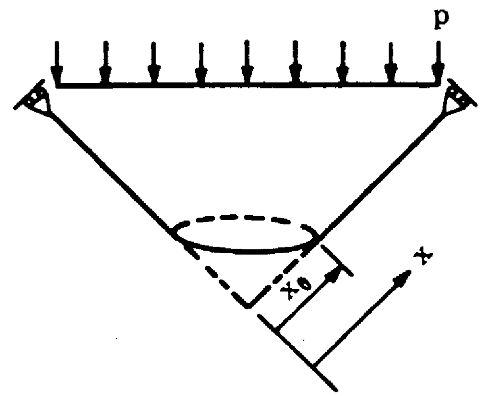


Regular	Inverted
$N_\phi = -\dfrac{px \cot\phi}{2}$	$N_\phi = \dfrac{px \cot\phi}{2}$
$N_\theta = -\dfrac{px \cos^3\phi}{\sin\phi}$	$N_\theta = \dfrac{px \cos^3\phi}{\sin\phi}$
$\sigma_\phi, \sigma_\theta = \dfrac{N_\phi}{t}, \dfrac{N_\theta}{t}$	$\sigma_\phi, \sigma_\theta = \dfrac{N_\phi}{t}, \dfrac{N_\theta}{t}$
$w = -\dfrac{px^2}{Et}\cot^2\phi\left(\cos^2\phi - \dfrac{\mu}{2}\right)$	$w = \dfrac{px^2}{Et}\cot^2\phi\left(\cos^2\phi - \dfrac{\mu}{2}\right)$
$\bar{w} = w \sin\phi$	$\bar{w} = w \sin\phi$
$\bar{u} = w \cos\phi$	$\bar{u} = w \cos\phi$

Table B7.1.2.4 - 2 (Concluded). Uniform Loading over Base Area Membrane Stresses and Deflections, Open Conical Domes

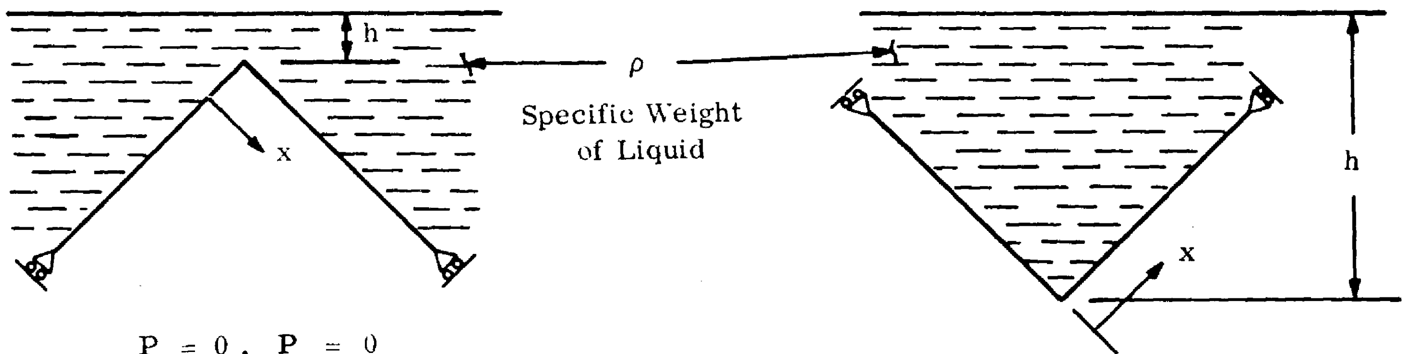


$$P_\theta = 0$$
$$P_\phi = p \sin\phi \cos\phi$$
$$P_z = p \cos^2\phi$$



Regular	Inverted
$N_\phi = -\frac{p \cot\phi}{2} \frac{x^2 - x_0^2}{x}$	$N_\phi = \frac{p \cot\phi}{2} \left(\frac{x^2 - x_0^2}{x}\right)$
$N_\theta = -\frac{px \cos^3\phi}{\sin\phi}$	$N_\theta = \frac{px \cos^3\phi}{\sin\phi}$
$\sigma_\phi , \sigma_\theta = \frac{N_\phi}{t} , \frac{N_\theta}{t}$	$\sigma_\phi , \sigma_\theta = \frac{N_\phi}{t} , \frac{N_\theta}{t}$
$w = -\frac{px^2}{2Et} \cot^2\phi \left[2 \cos^2\phi - \mu \left(\frac{x^2 - x_0^2}{x^2}\right)\right]$	$w = \frac{px^2}{2Et} \cot^2\phi \left[2 \cos^2\phi - \mu \frac{x^2 - x_0^2}{x^2}\right]$
$\bar{w} = w \sin\phi$	$\bar{w} = w \sin\phi$
$\bar{u} = w \cos\phi$	$\bar{u} = w \cos\phi$

Table B7.1.2.4 - 3. Hydrostatic Pressure Loading Membrane Stresses and Deflections, Closed Conical Domes



$P_\theta = 0, \quad P_\phi = 0$

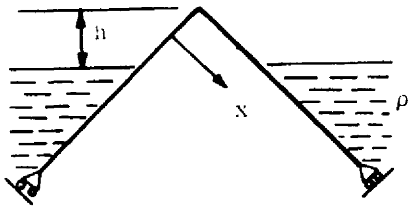
$P_z = \rho(h + x\sin\phi)$

$P_\theta = P_\phi = 0$

$P_z = \rho(h - x\sin\phi)$

Regular		Inverted	
N_ϕ	$-\rho x\cos\phi\left(\frac{h}{2\sin\phi} + \frac{x}{3}\right)$	N_ϕ	$-\rho x\cos\phi\left(\frac{x}{3} - \frac{h}{2\sin\phi}\right)$
N_θ	$-\rho x\cos\phi\left(\frac{h}{\sin\phi} + x\right)$	N_θ	$-\rho x\cos\phi\left(x - \frac{h}{\sin\phi}\right)$
$\sigma_\phi, \sigma_\theta =$	$\frac{N_\phi}{t}, \quad \frac{N_\theta}{t}$	$\sigma_\phi, \sigma_\theta =$	$\frac{N_\phi}{t}, \quad \frac{N_\theta}{t}$
w	$-\frac{\rho x^2}{Et}\cos\phi\cot\phi\left[\frac{h}{\sin\phi}\left(\frac{\mu}{2} - 1\right) - x\left(\frac{\mu}{3} - 1\right)\right]$	w	$-\frac{\rho x^2}{Et}\cos\phi\cot\phi\left[x\left(\frac{\mu}{3} - 1\right) - \frac{h}{\sin\phi}\left(\frac{\mu}{2} - 1\right)\right]$
$\bar{w}$	$w\sin\phi$	$\bar{w}$	$w\sin\phi$
$\bar{u}$	$w\cos\phi$	$\bar{u}$	$w\cos\phi$

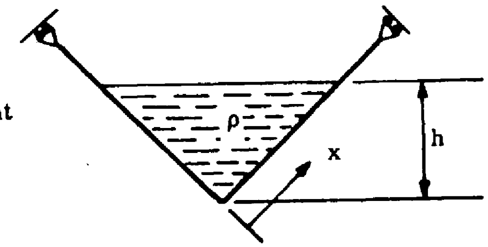
Table B7.1.2.4 - 3 (Continued). Hydrostatic Pressure Loading Membrane Stresses and Deflections, Closed Conical Domes



$P_\theta = P_\phi = 0$

$P_z = \rho(x \sin\phi - h)$

ρ = Specific Weight of Liquid

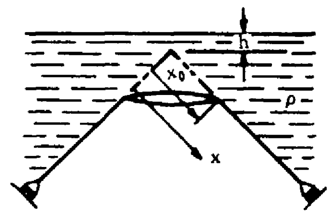


$P_\theta = P_\phi = 0$

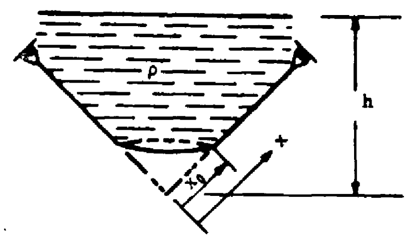
$P_z = \rho(h - x \sin\phi)$

Regular	Inverted
(Above Liquid Level)	(Above Liquid Level)
$N_\phi = 0$	$N_\phi = \dfrac{\rho h^3 \cot\phi}{6x \sin^2\phi}$
(Below Liquid Level)	(Below Liquid Level)
$N_\phi = -\dfrac{\rho}{6x}\left[\dfrac{h^3 \cot\phi}{\sin^2\phi} + x^2(2x\cos\phi - 3h\cot\phi)\right]$	$N_\phi = \dfrac{\rho x}{2}(3h\cot\phi - 2x\cos\phi)$
(Above Liquid Level)	(Above Liquid Level)
$N_\theta = 0$	$N_\theta = 0$
(Below Liquid Level)	(Below Liquid Level)
$N_\theta = -\rho x(x\cos\phi - h\cot\phi)$	$N_\theta = \rho x(h\cot\phi - x\cos\phi)$
$\sigma_\phi, \sigma_\theta = \dfrac{N_\phi}{t}, \dfrac{N_\theta}{t}$	$\sigma_\phi, \sigma_\theta = \dfrac{N_\phi}{t}, \dfrac{N_\theta}{t}$
For Deflections, see Section B7.1.1.4 - IV.	For Deflections, see Section B7.1.1.4 - IV.

Table B7.1.2.4 - 3 (Concluded). Hydrostatic Pressure Loading Membrane Stresses and Deflections, Open Conical Domes



ρ = Specific Weight of Liquid



$P_\phi = P_\theta = 0$

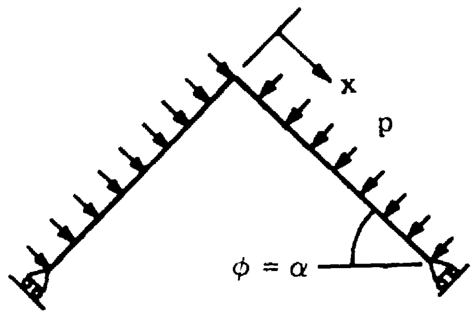
$P_z = \rho(h + x\sin\phi)$

$P_\phi = P_\theta = 0$

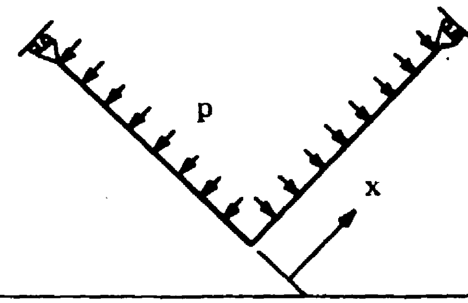
$P_z = \rho(h - x\sin\phi)$

Regular	Inverted
$N_\phi = -\rho h \frac{x^2 - x_0^2}{2x}\cot\phi - \rho\frac{x^3 - x_0^3}{3x}\cos\phi$	$N_\theta = \rho h \frac{x^2 - x_0^2}{2x}\cot\phi - \rho\frac{x^3 - x_0^2}{3x}\cos\phi$
$N_\theta = -\rho x\left(h\cot\phi + x\cos\phi\right)$	$N_\theta = -\rho x\left(x\cos\phi - h\cot\phi\right)$
$\sigma_\phi, \sigma_\theta = \frac{N_\phi}{t}, \frac{N_\theta}{t}$	$\sigma_\phi, \sigma_\theta = \frac{N_\phi}{t}, \frac{N_\theta}{t}$
$w = \frac{\rho x^2}{Et}\cos\phi\ \cot\phi\left[\frac{\mu h}{2\sin\phi}\left(\frac{x^2 - x_0^2}{x^2}\right) + \frac{\mu}{3}\left(\frac{x^3 - x_0^3}{x^2}\right) - \frac{h}{\sin\phi} - x\right]$	$w = \frac{\rho x^2}{Et}\cos\phi\ \cot\phi\left[\frac{\mu}{3}\left(\frac{x^3 - x_0^3}{x^2}\right) - \frac{\mu h}{2\sin\phi}\left(\frac{x^2 - x_0^2}{x^2}\right) + \frac{h}{\sin\phi} - x\right]$
$\bar{w} = w\sin\phi$	$\bar{w} = w\sin\phi$
$\bar{u} = w\cos\phi$	$\bar{u} = w\cos\phi$

Table B7.1.2.4 - 4. Uniform Pressure Loading
Membrane Stresses and Deflections, Closed Conical Domes

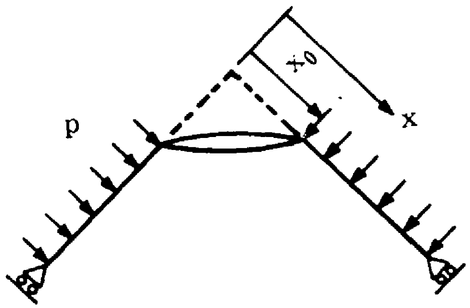


$$P_\theta = P_\phi = 0$$
$$P_z = P$$



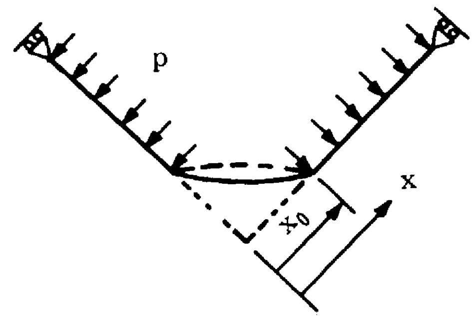
Regular	Inverted
$N_\phi = -\frac{px}{2}\cot\phi$	$N_\phi = \frac{px}{2}\cot\phi$
$N_\theta = -px\cot\phi$	$N_\theta = px\cot\phi$
$\sigma_\phi, \sigma_\theta = \frac{N_\phi}{t}, \frac{N_\theta}{t}$	$\sigma_\phi, \sigma_\theta = \frac{N_\phi}{t}, \frac{N_\theta}{t}$
$w = \frac{px^2}{Et}\cot^2\phi\left(1 - \frac{\mu}{2}\right)$	$w = \frac{px^2}{Et}\cot^2\phi\left(1 - \frac{\mu}{2}\right)$
$\bar{w} = \frac{px^2}{Et}\cos\phi\ \cot\phi\left(1 - \frac{\mu}{2}\right)$	$\bar{w} = \frac{px^2}{Et}\cos\phi\ \cot\phi\left(1 - \frac{\mu}{2}\right)$
$\bar{u} = \frac{px^2}{Et}\cos\phi\ \cot^2\phi\left(1 - \frac{\mu}{2}\right)$	$\bar{u} = \frac{px^2}{Et}\cos\phi\ \cot^2\phi\left(1 - \frac{\mu}{2}\right)$

Table B7.1.2.4 - 4 (Concluded). Uniform Pressure Loading Membrane Stresses and Deflections, Open Conical Domes



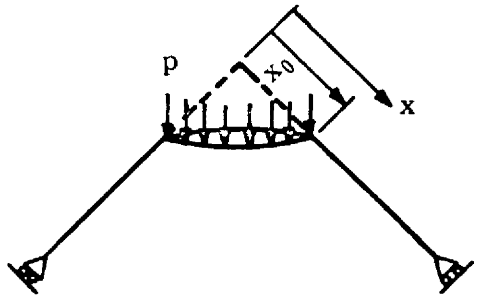
$P_\phi = P_\theta = 0$

$P_z = P$

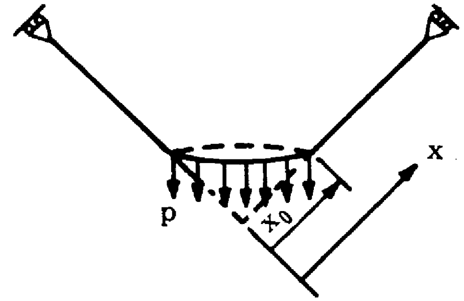


Regular	Inverted
$N_\phi = -\frac{p\cot\phi}{2}\left(\frac{x^2 - x_0^2}{x}\right)$	$N_\phi = \frac{p\cot\phi}{2}\left(\frac{x^2 - x_0^2}{x^2}\right)$
$N_\theta = -px\cot\phi$	$N_\theta = px\cot\phi$
$\sigma_\phi, \sigma_\theta = \frac{N_\phi}{t}, \frac{N_\theta}{t}$	$\sigma_\phi, \sigma_\theta = \frac{N_\phi}{t}, \frac{N_\theta}{t}$
$w = \frac{px^2}{Et}\cot^2\phi\left[1 - \frac{\mu}{2}\left(\frac{x^2 - x_0^2}{x^2}\right)\right]$	$w = \frac{px^2}{Et}\cot^2\phi\left[1 - \frac{\mu}{2}\left(\frac{x^2 - x_0^2}{x^2}\right)\right]$
$\bar{w} = \frac{px^2}{Et}\cot\phi\ \cot\phi\left[1 - \frac{\mu}{2}\left(\frac{x^2 - x_0^2}{x^2}\right)\right]$	$\bar{w} = \frac{px^2}{Et}\cot\phi\ \cot\phi\left[1 - \frac{\mu}{2}\left(\frac{x^2 - x_0^2}{x^2}\right)\right]$
$\bar{u} = \frac{px^2}{Et}\cos\phi\ \cot^2\phi\left[1 - \frac{\mu}{2}\left(\frac{x^2 - x_0^2}{x^2}\right)\right]$	$\bar{u} = \frac{px^2}{Et}\cos\phi\ \cot^2\phi\left[1 - \frac{\mu}{2}\left(\frac{x^2 - x_0^2}{x^2}\right)\right]$

Table B7.1.2.4 - 5. Lantern Loading
Membrane Stresses and Deflections of Conical Domes



$$P_\phi = P_\theta = P_z = 0$$



Regular	Inverted
$N_\phi = -\dfrac{px_0}{x\sin\phi}$	$N_\phi = \dfrac{px_0}{x\sin\phi}$
$N_\theta = 0$	$N_\theta = 0$
$\sigma_\phi, \sigma_\theta = \dfrac{N_\phi}{t}, \dfrac{N_\theta}{t}$	$\sigma_\phi, \sigma_\theta = \dfrac{N_\phi}{t}, \dfrac{N_\theta}{t}$
$w = \dfrac{\mu p x_0}{Et}\left(\dfrac{\cot\phi}{\sin\phi}\right)$	$w = -\dfrac{\mu p x_0}{Et}\left(\dfrac{\cot\phi}{\sin\phi}\right)$
$\bar{w} = \dfrac{\mu p x_0}{Et}\cot\phi$	$\bar{w} = -\dfrac{\mu p x_0}{Et}\cot\phi$
$\bar{u} = \dfrac{\mu p x_0}{Et\sin\phi}$	$\bar{u} = -\dfrac{\mu p x_0}{Et\sin\phi}$

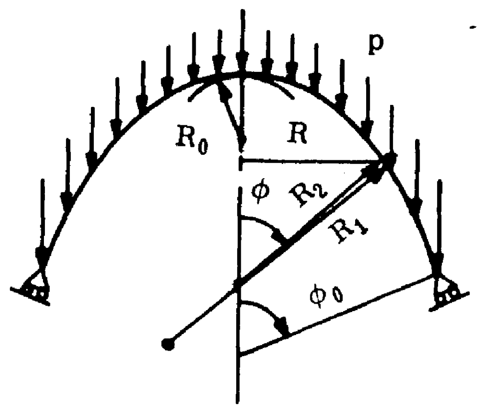
B7.1.2.5 PARABOLIC DOMES

This subsection presents the solutions for nonshallow parabolic shells exposed to axisymmetric loading. Only closed shells will be considered. The boundaries of the shell must be free to rotate and deflect normal to the shell middle surface. No abrupt discontinuities in the shell thickness shall be present.

Note that because of the geometry of the parabolic meridian, the solutions simplify by use of the radius of curvature at the vertex R_0 where $\phi = 0$. For the parabolic shell at $\phi = 0$, $R_1 = R_2 = R_0 =$ twice the focal distance.

The following loading conditions will be considered: dead weight loading (Table B7.1.2.5 - 1); uniform loading over base area (Table B7.1.2.5 - 2); hydrostatic pressure loading (Table B7.1.2.5 - 3); and uniform pressure loading (Table B7.1.2.5 - 4). These tables begin on page 64.

Table B7.1.2.5-1. Dead Weight Loading Membrane Stress Resultants for Closed Parabolic Domes



$R_0 =$ z (Focal Distance)

$$P_\theta = 0 \,, \quad P_z = p\cos\phi \,, \quad P_\phi = p\sin\phi$$

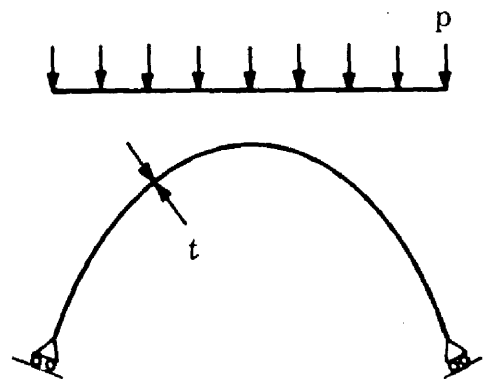
$$N_\phi = -\frac{pR_0}{3}\left(\frac{1-\cos^3\phi}{\sin^2\phi\,\cos^2\phi}\right)$$

$$N_\theta = -\frac{pR_0}{3}\left(\frac{2-3\cos^2\phi+\cos^3\phi}{\sin^2\phi}\right)$$

$$\sigma_\phi\,,\ \sigma_\theta = \frac{N_\phi}{t}\,,\quad \frac{N_\theta}{t}$$

For Deflections, see Section B7.1.1.4-IV.

Table B7.1.2.5 - 2. Uniform Loading over Base Area
Membrane Stress Resultants for Closed Parabolic Domes



$R_0 = z$ (Focal Distance)

$P_\theta = 0$, $P_z = p\cos^2\phi$, $P_\phi = p\cos\phi\ \sin\phi$

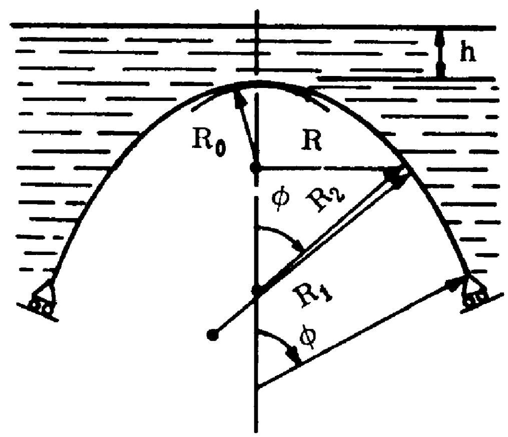
$$N_\phi = -\frac{pR_0}{2\cos\phi}$$

$$N_\theta = -\frac{pR_0\cos\phi}{2}$$

$$\sigma_\phi , \sigma_\theta = \frac{N_\phi}{t} , \frac{N_\theta}{t}$$

For Deflections, see Section B7.1.1.4 - IV.

Table B7.1.2.5 - 3. Hydrostatic Pressure Loading Membrane Stress Resultants for Closed Parabolic Domes



ρ = Specific Weight of Liquid

$$P_\theta = P_\phi = 0 \ , \quad P_z = \rho\left(h + \frac{R_0}{2}\tan^2\phi\right)$$

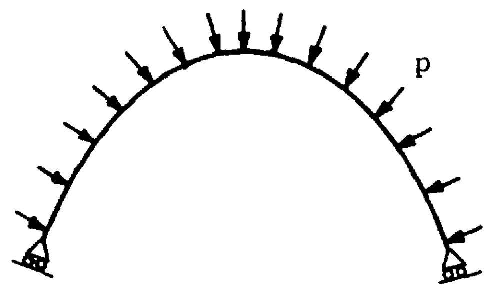
$$N_\phi = -\frac{\rho R_0}{2\cos\phi}\left(h + \frac{R_0}{4}\tan^2\phi\right)$$

$$N_\theta = -\frac{\rho R_0 \cos\phi}{2}\left[h\left(2\tan^2\phi + 1\right) + R_0\tan^2\phi\left(\tan^2\phi + \frac{3}{4}\right)\right]$$

$$\sigma_\phi \, , \, \sigma_\theta = \frac{N_\phi}{t} \quad , \quad \frac{N_\theta}{t}$$

For Deflections, see Section B7.1.1.4 - IV .

Table B7.1.2.5 - 4. Uniform Pressure Loading
Membrane Stress Resultants for Closed Parabolic Domes



$$P_\theta = P_\phi = 0 \ , \ P_z = P$$

$$N_\phi = -\frac{pR_0}{2\cos\phi}$$

$$N_\theta = -\frac{pR_0}{2}\left(\frac{1+\sin^2\phi}{\cos\phi}\right)$$

$$\sigma_\phi \, , \, \sigma_\theta = \frac{N_\phi}{t} \ , \ \frac{N_\theta}{t}$$

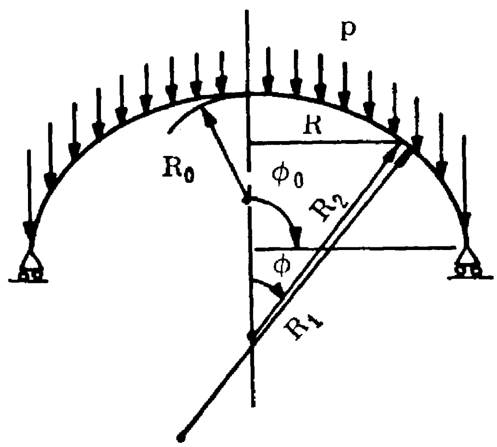
For Deflections, see Section B7.1.1.4 - IV

B7.1.2.6 CYCLOIDAL DOMES

This subsection presents the solutions for nonshallow cycloidal shells exposed to axisymmetric loading. Only closed shells will be considered. The boundaries of the shell must be free to rotate and deflect normal to the shell middle surface. No abrupt discontinuities in the shell thickness shall be present.

The following loading conditions will be considered: dead weight loading (Table B7.1.2.6 - 1) and uniform loading over base area (Table B7.1.2.6 - 2). These tables begin on page 69.

Table B7.1.2.6 - 1. Dead Weight Loading
Membrane Stress Resultants for Cycloidal Domes



$$P_\theta = 0, \quad P_\phi = p\sin\phi, \quad P_z = p\cos\phi$$

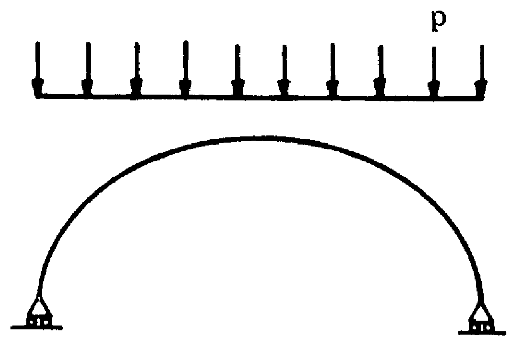
$$N_\phi = -2pR_0\left(\frac{\phi\sin\phi + \cos\phi - \frac{1}{3}\cos^3\phi - \frac{2}{3}}{}\right)$$

$$N_\theta = -pR_0\left[\frac{1}{3}\left(\frac{1-\cos^3\phi}{\sin^2\phi\cos\phi}\right) - \frac{\phi}{2}\tan\phi - \frac{1}{2}\sin^2\phi\right]$$

$$\sigma_\phi, \sigma_\theta = \frac{N_\phi}{t}, \quad \frac{N_\theta}{t}$$

For Deflections, see Section B7.1.1.4 - IV.

Table B7.1.2.6 - 2. Uniform Loading over Base Area
Membrane Stress Resultants for Cycloidal Domes



$$P_\theta = 0, \quad P_\phi = p \sin\phi \cos\phi, \quad P_z = p\cos^2\phi$$

$$N_\phi = -\frac{pR_0}{8}\left(\frac{2\phi + \sin 2\phi}{\sin\phi}\right)$$

$$N_\theta = -\frac{pR_0}{16}\left(\frac{2\phi + \sin 2\phi}{\sin\phi}\right)\left(4\cos^2\phi - \frac{2\phi}{\sin 2\phi} - 1\right)$$

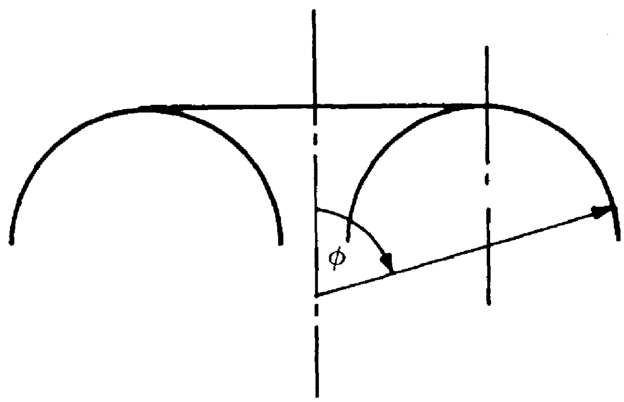
$$\sigma_\phi, \sigma_\theta = \frac{N_\phi}{t} \quad , \quad \frac{N_\theta}{t}$$

For Deflections, see Section B7.1.1.4 - IV.

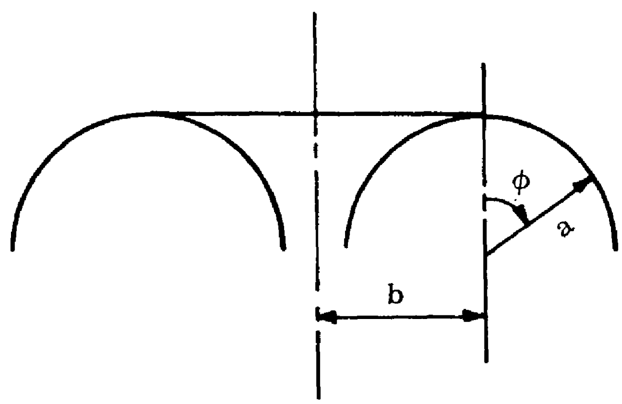
B7.1.2.7 TOROIDAL DOMES

This subsection presents the solutions for toroidal shells exposed to axisymmetric loading. Both closed and open shells will be considered. The boundaries of the shell must be free to rotate and deflect normal to the shell middle surface. No abrupt discontinuities in the shell thickness shall be present.

Note in Figure B7.1.2.7 - 1 that, because of the geometry of the toroidal shell, the definition of the angle ϕ is changed (for this subsection only) to increase the useful range of the solutions.



Useful Range $35^\circ \leq \phi \leq 90^\circ$



ϕ Definition (Section B7.7.1.2.7 - 1 only)

Fig. B7. 1. 2. 7. - 1. Toroidal Shell Geometry

There is also a special type of toroidal shell where the axis of revolution bisects the cross section. This type of shell is referred to as a pointed dome, and the angle ϕ is defined in Figure B7.1.2.7 - 2.

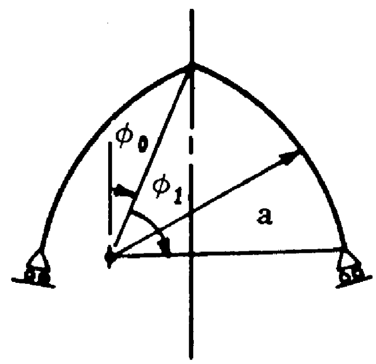


Fig. B7.1.2.7 - 2. Pointed Dome

The following loading conditions will be considered for regular toroidal shells and pointed domes.

Regular Toroidal Shells:

Dead Weight Loading (Table B7.1.2.7 - 1)

Hydrostatic Loading (Table B7.1.2.7 - 2)

Uniform Pressure Loading (Table B7.1.2.7 - 3)

Lantern Loading (Table B7.1.2.7 - 4)

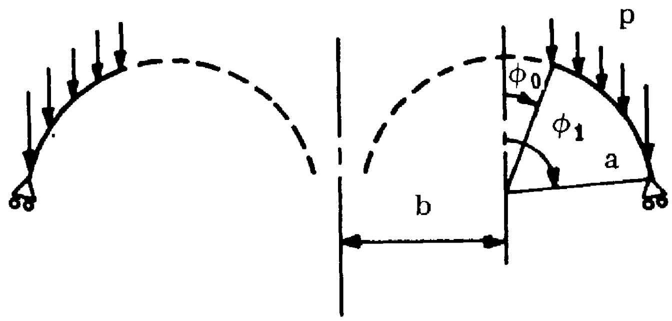
Pointed Domes:

Dead Weight Loading (Table B7.1.2.7 - 5)

Uniform Loading Over Base Area (Table B7.1.2.7 - 6)

These tables begin on page 73.

Table B7.1.2.7 - 1. Dead Weight Loading Membrane Stresses, Regular Toroidal Shells



$P_\phi = P_\theta = 0$

$P_z = p$

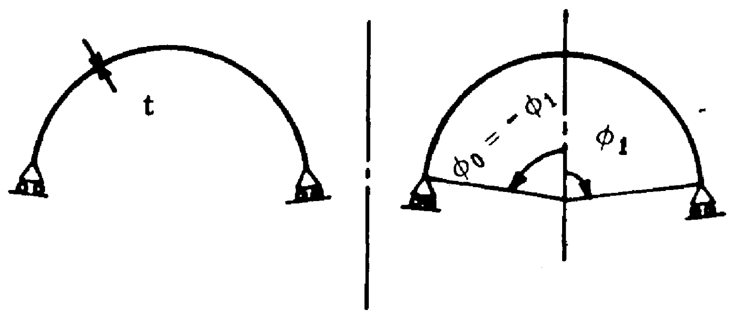
$$N_\phi = -p\frac{ab(\phi - \phi_0) + a^2(\cos\phi_0 - \cos\phi)}{(b + a\sin\phi)\sin\phi}$$

$$N_\theta = -\frac{p}{\sin\phi}\left[(b + a\sin\phi)\cos\phi - b(\phi - \phi_0) - a(\cos\phi_0 - \cos\phi)\right]$$

$$\sigma_\phi, \sigma_\theta = \frac{N_\phi}{t}, \quad \frac{N_\theta}{t}$$

Table B7.1.2.7 - 1 (Concluded). Dead Weight Loading Membrane Stresses, Regular Toroidal Shells

For Symmetrical Cross Section ($\phi_0 = -\phi_1$)



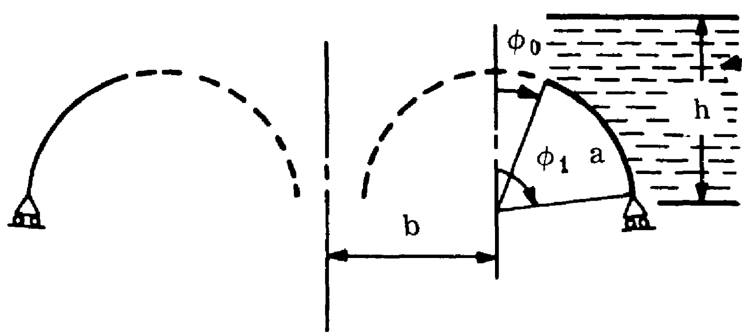
Loading Same as Above

$$N_\phi = -p \frac{ba\phi + a^2(1 - \cos\phi)}{(b + a\sin\phi)\sin\phi}$$

$$N_\theta = -\frac{p}{\sin\phi}\left[(b + a\sin\phi)\cos\phi - b\phi - a(1 - \cos\phi)\right]$$

$$\sigma_\phi, \sigma_\theta = \frac{N_\phi}{t}, \frac{N_\theta}{t}$$

Table B7.1.2.7 - 2. Hydrostatic Pressure Loading Membrane Stresses, Regular Toroidal Shells



ρ = Specific Weight of Liquid

$P_\phi = P_\theta = 0$

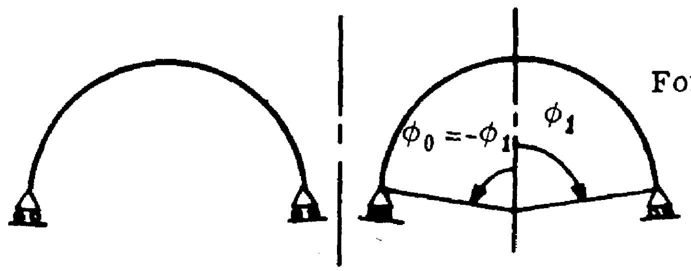
$P_z = \rho(h - a\cos\phi)$

$$N_\phi = -\frac{\rho a}{(b + a\sin\phi)\sin\phi}\left[-bh(\sin\phi_0 - \sin\phi) + \frac{ah}{2}(\cos^2\phi_0 - \cos^2\phi) + \frac{ba}{2}(\sin\phi_0\cos\phi_0 - \sin\phi\cos\phi - \phi + \phi_0) - \frac{a^2}{3}(\cos^3\phi_0 - \cos^3\phi)\right]$$

$$N_\theta = -\frac{\rho}{\sin^2\phi}\left[(h - a\cos\phi)(b + a\sin\phi)\sin\phi + bh(\sin\phi_0 - \sin\phi) - \frac{ah}{2}(\cos^2\phi_0 - \cos^2\phi) - \frac{ba}{2}(\sin\phi_0\cos\phi_0 - \sin\varphi\cos\phi - \phi + \phi_0) + \frac{a^2}{3}(\cos^3\phi - \cos^3\phi)\right]$$

Table B7.1.2.7 - 2 (Concluded). Hydrostatic Pressure Loading Membrane Stresses, Regular Toroidal Shells

For Symmetrical Cross Section ($\phi_0 = \phi_1$)



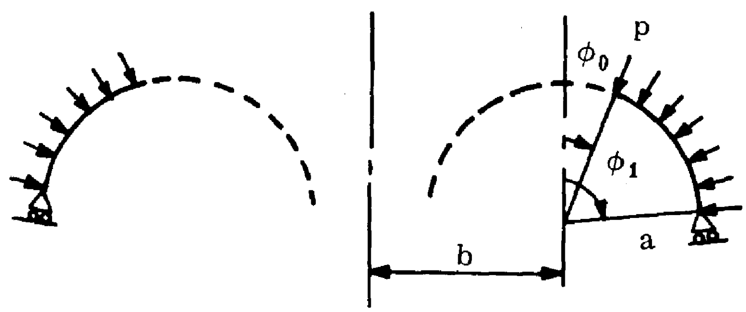
For Symmetrical Cross Section $\phi_0 = -\phi_1$

Same Loading as Above

$$N_\phi = -\frac{\rho a}{(b + a\sin\phi)\sin\phi}\left[bh\sin\phi + \frac{ah}{2}\sin^2\phi - \frac{ba}{2}(\sin\phi\cos\phi + \phi) - \frac{a^2}{3}(1 - \cos^3\phi)\right]$$

$$N_\theta = -\frac{\rho}{\sin\phi}\left[\frac{ah}{2}\sin^2\phi - \frac{ba}{2}(\sin\phi\cos\phi - \phi) - a^2\left(\cos\phi\sin^2\phi - \frac{1 - \cos^3\phi}{3}\right)\right]$$

Table B7.1.2.7 - 3. Uniform Pressure Loading
Membrane Stresses, Regular Toroidal Shells



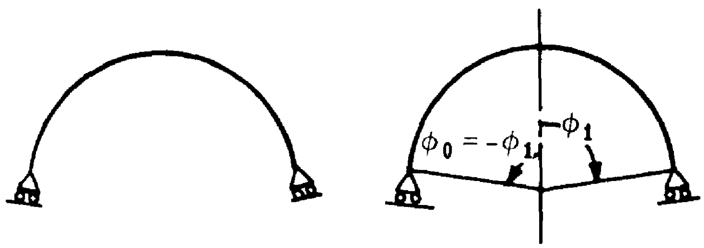
$P_\phi = P_\theta = 0$
or $P_z = P$

$$N_\phi = -\frac{p}{2(b + a\sin\phi)\sin\phi}\left[(b + a\sin\phi)^2 - (b + a\sin\phi_0)^2\right]$$

$$N_\theta = -\frac{p}{2\sin^2\phi}\left[2b\sin\phi_0 + a(\sin^2\phi_0 + \sin^2\phi)\right]$$

Table B7.1.2.7 - 3 (Concluded). Uniform Pressure Loading Membrane Stresses, Regular Toroidal Shells

For Symmetrical Cross Section ($\phi_0 = -\phi_1$)

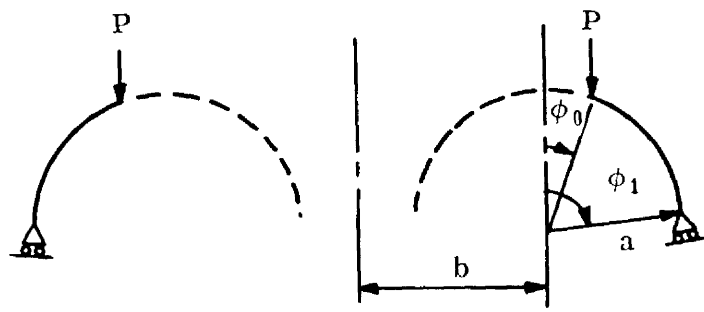


Same Loading as Above

$$N_\phi = -\frac{pa}{2}\,\frac{2b + a\sin\phi}{b + a\sin\phi}$$

$$N_\theta = -\frac{pa}{2}$$

Table B7.1.2.7 - 4. Lantern Loading
Membrane Stresses, Regular Toroidal Shells



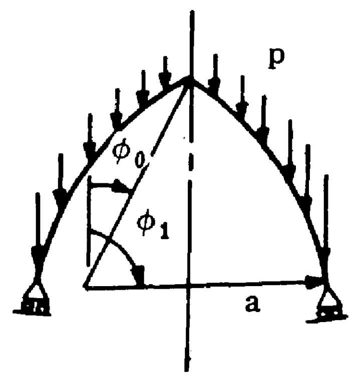
$$P_\theta = P_\phi = P_z = 0$$

$$N_\phi = -\frac{P(b + a \sin\phi)}{(b + a \sin\phi)\sin\phi}$$

$$N_\theta = -\frac{P}{a}\left(\frac{b + a \sin\phi_0}{\sin^2\phi}\right)$$

$$\sigma_\phi , \sigma_\theta = \frac{N_\phi}{t} , \frac{N_\theta}{t}$$

Table B7.1.2.7 - 5. Dead Weight Loading Membrane Stresses, Pointed Toroidal Dome



$$P_\theta = 0$$

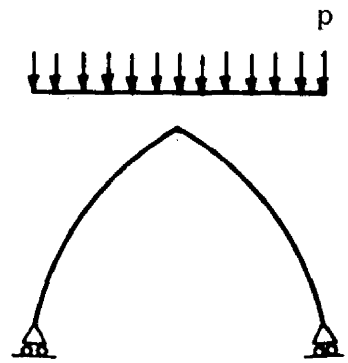
$$P_\phi = p\sin\phi \ , \quad P_z = p\cos\phi$$

$$N_\phi = -pa\left[\frac{\cos\phi_0 - \cos\phi - (\phi - \phi_0)\sin\phi_0}{(\sin\phi - \sin\phi_0)\sin\phi}\right]$$

$$N_\theta = -\frac{pa}{\sin^2\phi}\left[(\phi - \phi_0)\sin\phi_0 - (\cos\phi_0 - \cos\phi) + (\sin\phi - \sin\phi_0)\sin\phi\cos\phi\right]$$

$$\sigma_\phi , \sigma_\theta = \frac{N_\phi}{t} , \frac{N_\theta}{t}$$

Table B7.1.2.7 - 6. Uniform Loading over Base Area Membrane Stresses, Pointed Toroidal Dome



$$P_\theta = 0$$

$$P_\phi = p \sin\phi \cos\phi \;, \quad P_z = p \cos^2\phi$$

$$N_\phi = -\frac{pa}{2}\left(1 - \frac{\sin\phi_0}{\sin\phi}\right)$$

$$N_\theta = -\frac{pa}{2}\left(\cos 2\phi - 2\sin\phi \sin\phi_0 - \frac{\sin^2\phi_0}{\sin^2\phi}\right)$$

$$\sigma_\phi, \sigma_\theta = \frac{N_\phi}{t} \;, \quad \frac{N_\theta}{t}$$

B7.1.3.0 CYLINDER ANALYSIS

Many types of cylinders can be analyzed using membrane theory. However, only the circular cylinder falls into the category of shells of revolution being discussed in this chapter.

B7.1.3.1 CIRCULAR CYLINDERS

The circular cylinder is a special type of surface of revolution. If the standard shell-of-revolution nomenclature is applied to the cylinder geometry, (Figure B7.1.3.1 - 1) analysis is straightforward.

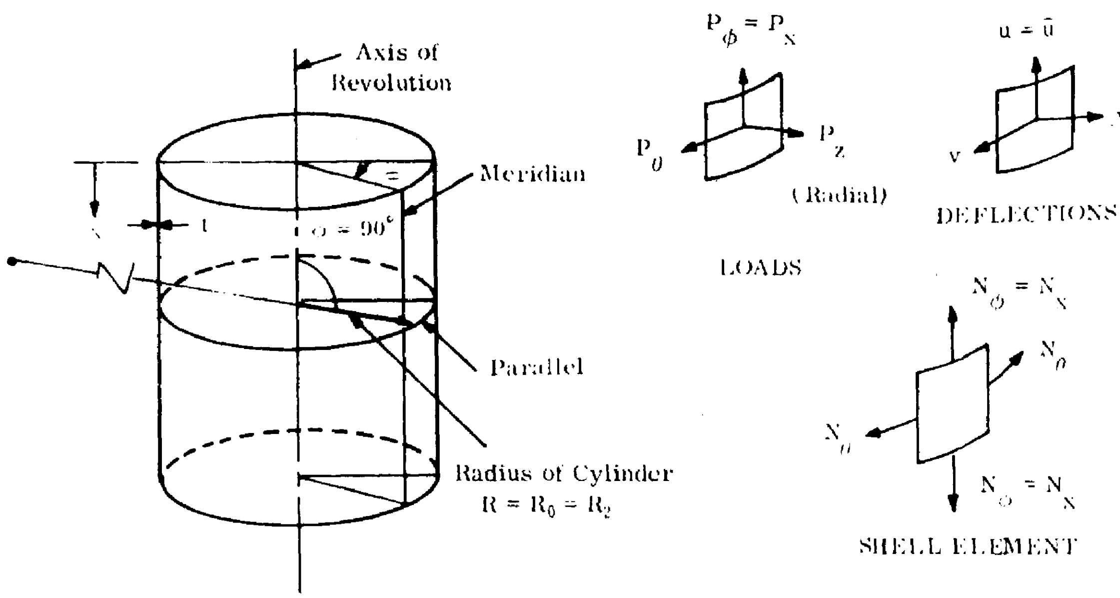


Fig. B7.1.3.1 - 1. Circular Cylinder Geometry

The following loading conditions will be considered for circular cylinders:

Linear Loading (Table B7.1.3 - 1)

Trigonometric Loading (Table B7.1.3 - 2)

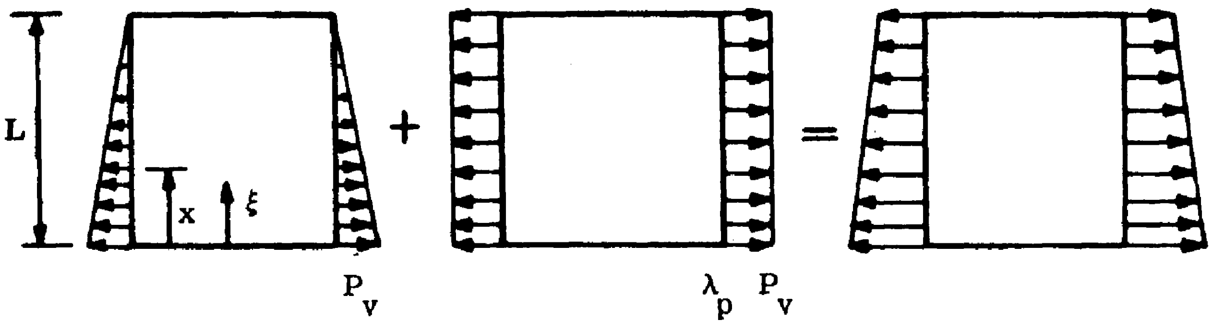
Dead Weight Loading (Table B7.1.3 - 3)

Circumferential Loading (Table B7.1.3 - 4)

Axial Loading (Table B7.1.3 - 5) .

These tables begin on page 84.

Table B7.1.3 - 1. Linear Loading



$$P_z = P_v(1 + \lambda_p - \xi)$$

$$\text{where} \quad \xi = \frac{x}{L}$$

λ_p = coefficient defining ratio of uniform load to maximum value of linear load

$$N_\theta = P_v R (1 + \lambda_p - \xi)$$

$$N_\phi = 0$$

$$N_{\phi\theta} = 0$$

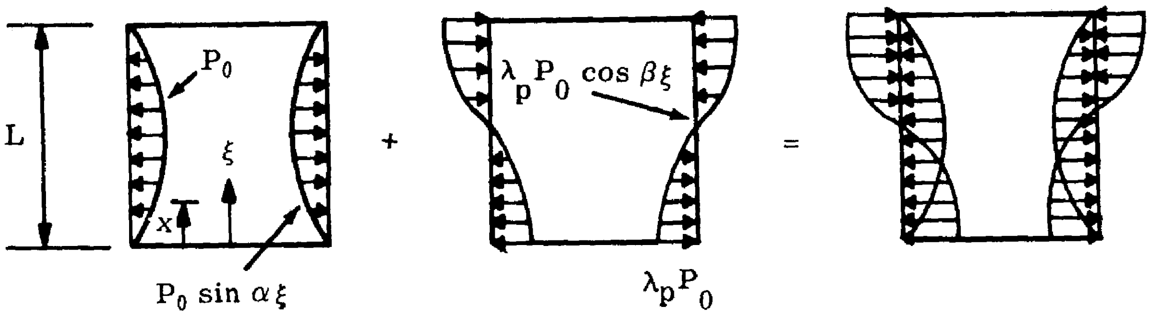
$$\sigma_\theta = \frac{N_\theta}{t}$$

$$\bar{u} = \frac{1}{Et}\left[-\mu P_v R L \xi (1 + \lambda_p - \frac{1}{2}\xi)\right]$$

$$v = 0$$

$$\bar{w} = \frac{1}{Et}\left[P_v R^2 (1 + \lambda_p - \xi)\right]$$

Table B7.1.3 - 2. Trigonometric Loading



$$P_z = -p_0(\sin\alpha\xi + \lambda_p \cos\beta\xi)$$

where $\xi = \dfrac{x}{L}$

α, β = coefficients defining shape of sin and cos curves

λ_p = coefficients defining ratio of maximum amplitude of cos loading to maximum amplitude of sin loading

$N_\theta = P_0 R(\sin\alpha\xi + \lambda_p \cos\beta\,\xi)$

$N_\phi = 0$

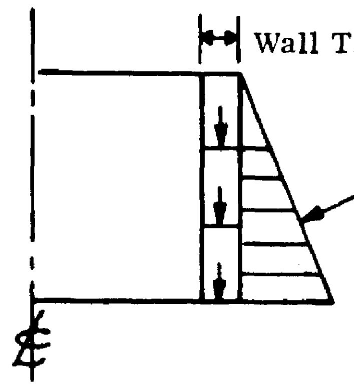
$N_{\phi\theta} = 0$

$\bar{u} = \dfrac{\mu}{Et} P_0 R L\left(\dfrac{\cos\alpha\xi}{\alpha} - \lambda_p \dfrac{\sin\beta\,\xi}{\beta}\right)$

$v = 0$

$\bar{w} = \dfrac{1}{Et} P_0 R^2(\sin\alpha\xi + \lambda_p \cos\beta\,\xi)$

Table B7.1.3 - 3. Dead Weight Loading



$$P_\phi = P_{\phi_0}(1 - \xi)$$

$$N_\theta = 0$$

$$N_\phi = P_{\phi_0} L\left(\frac{1}{2} - \xi + \frac{\xi^2}{2}\right)$$

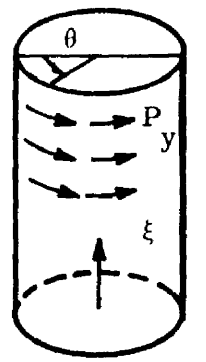
$$N_{\phi\theta} = 0$$

$$\bar{u} = \frac{1}{Et}\left[P_{\phi_0} L^2 \xi\left(\frac{1}{2} - \frac{\xi}{2} + \frac{\xi^2}{6}\right)\right]$$

$$v = 0$$

$$\bar{w} = \frac{1}{Et}\mu P_{\phi_0} \alpha\, L\left(-\frac{1}{2} + \xi - \frac{\xi^2}{2}\right)$$

Table B7.1.3 - 4. Circumferential Loading



$$P_y = P_\theta$$

$$N_\theta = 0$$

$$N_\phi = 0$$

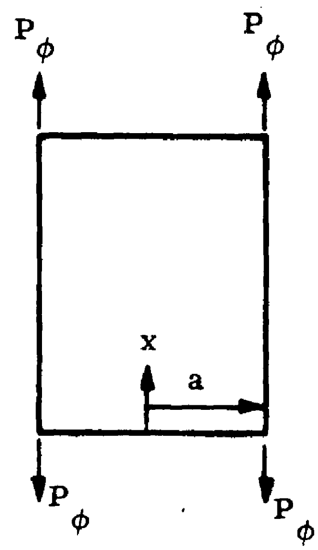
$$N_{\theta\phi} = P_\theta L (1 - \xi)$$

$$\bar{u} = 0$$

$$v = \frac{1}{Et}\left[2(1+\mu)\ P_\theta\ L^2\left(\xi - \frac{\xi^2}{2}\right)\right]$$

$$\bar{w} = 0$$

Table B7.1.3 - 5. Axial Load



$$N_\phi = P_\phi$$

$$N_\theta = 0$$

$$N_{\phi\theta} = 0$$

$$\bar{u} = \frac{N_\phi x}{Et} + C$$

$$\bar{w} = -\frac{\mu a N_\phi}{Et}$$

$$v = 0$$