POLITECNICO DI MILANO





Estimation in Aerospace

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School of Industrial and Information Engineering

Aeronautical Engineering

- General information on the course.
- Overview of the course programme.

General information

- Marco Lovera
  - Dipartimento di Scienze e Tecnologie Aerospaziali
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  - email marco.lovera@polimi.it

- Meeting hours: Tuesday 14.30-16.30

- Course web page: accessible from

  http://www.aero.polimi.it/lovera

Course schedule

- Schedule:
  - Monday 14.30-16.15 BL27.12
  - Tuesday 9.30-11.15 BL27.17
  - Thursday 9.30-11.15 L0.2
  - Thursday 16.30-18.15 BL27.15

- Composition of the course:

  50/55 lecture hours

  20/25 exercises and worked examples

  for a total of 8 CFU.

Exam

Two options available for the exam:

- oral exam
- a project.

There is no mid-semester test.

Prerequisites

- Main prerequisite: Fondamenti di Automatica or a similar course covering
  - System modeling: modelling concepts, state space models, examples.
  - Dynamic behavior: differential equations, qualitative analysis, stability.
  - Linear systems: matrix exponential, input/output response, linearisation.
  - Transfer functions: frequency domain modelling, transfer function, block diagrams, Bode plots, Laplace transform.
  - Frequency domain analysis: loop transfer function, Nyquist criterion, stability margins, generalised gain and phase.

Prerequisites

- Main prerequisite: basic background in probability and statistics
  - Random variable, probability, probability distribution and density functions.
  - Functions of random variables, mean, variance and moments.
  - Gaussian random variables.

- Teaching material
  - Slides are available for some (but not yet for all) topics
  - References to some specific textbooks and selected book chapters will be provided.

Motivation and objectives: parameter estimation and model identification

- The Fondamenti di Automatica course presents:
  - the basic methods and tools for mathematical modelling of dynamic systems
  - the fundamental concepts underlying the operation of feedback control systems
  - an introduction to the controller synthesis problem.

- The design of a control system however is a much more complex activity.

- In particular, the design approach studied in FdA is *model-based*, *i.e.*, it assumes that a model of the system exists.

Motivation and objectives: parameter estimation and model identification

Where can such a model come from?

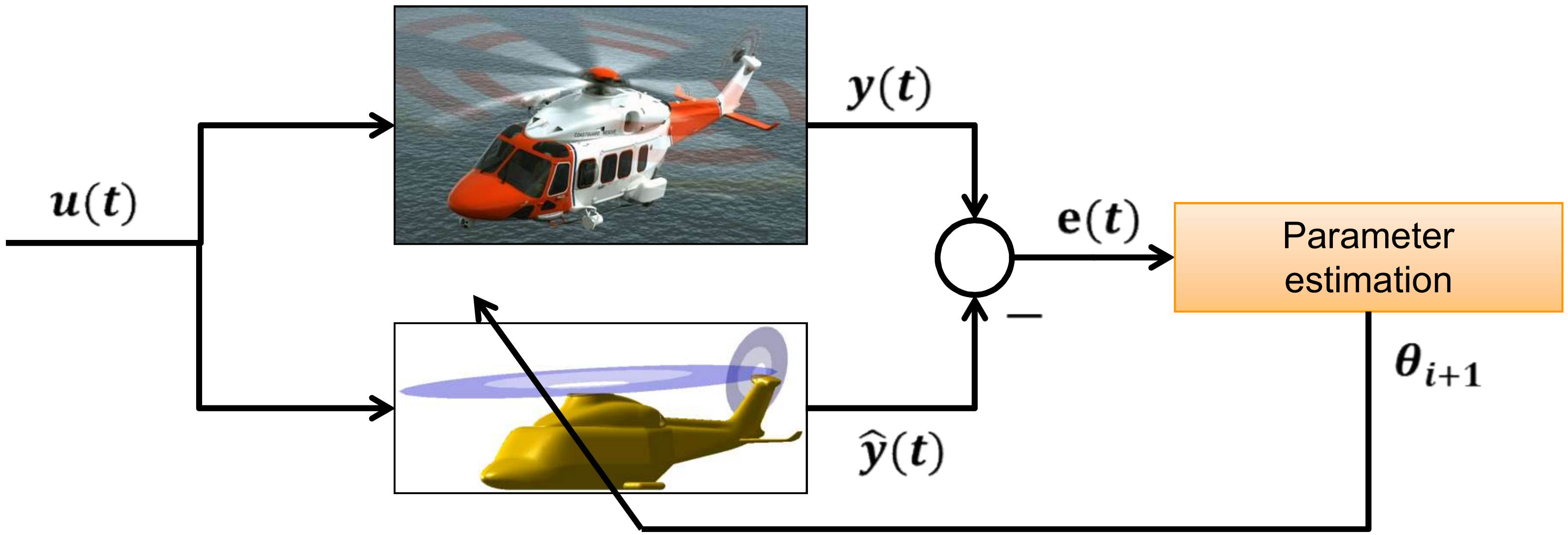
Three typical situations that may occur:

- First principles modelling: basic laws of physics allow to derive a complete mathematical description of the system (white box modelling).
- First principle modelling with uncertain parameters: basic laws of physics allow to derive a complete model but some parameters are *uncertain*, *e.g.*, damping ratios in a structure, aerodynamic coefficients *etc* (grey box modelling).
- No prior knowledge is available (black-box modelling).

(many “shades of grey” can be defined between “grey” and “black”.)

In the second and third cases, DATA collected on the system can be used to obtain additional information and complete the model.

The question is: how does one find *optimal* models?



Example: sensor calibration

- A (stationary) accelerometer will measure Earth's gravitational field due to the reaction force on the mass within the MEMS structure
- A magnetometer will measure the Earth's magnetic field compounded with local magnetic interference
- For a calibrated sensor, the measured magnitude will be constant for all orientations

$$m = ||Ku - b|| = constant$$

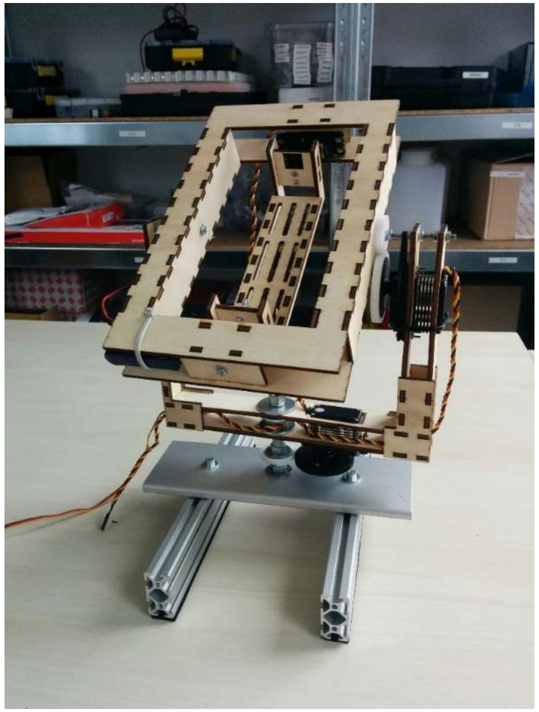
m: magnitude of the sensor's respective field

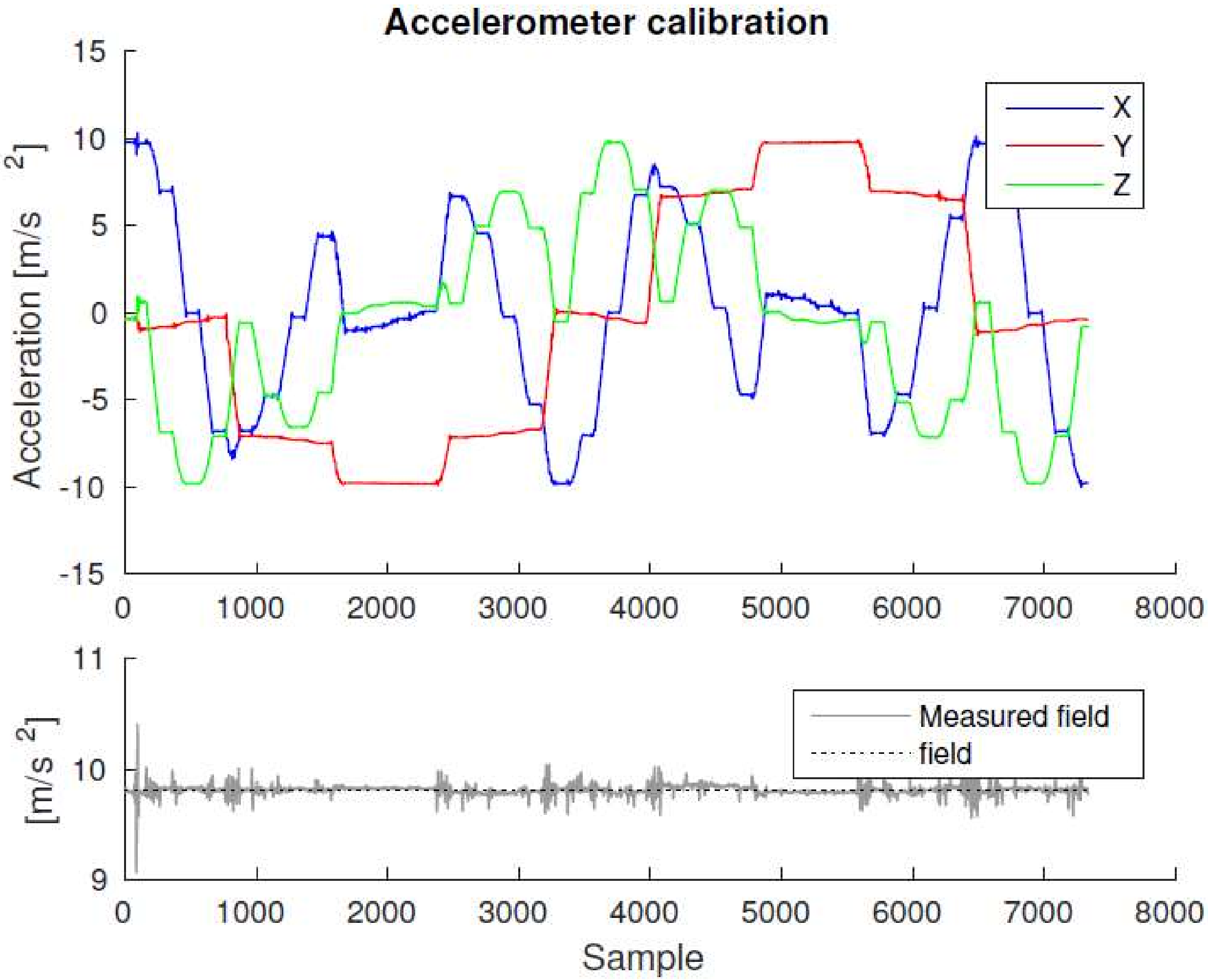
u: raw sensor measurement

K: matrix of calibration parameters

b: vector of biases

Example: sensor calibration

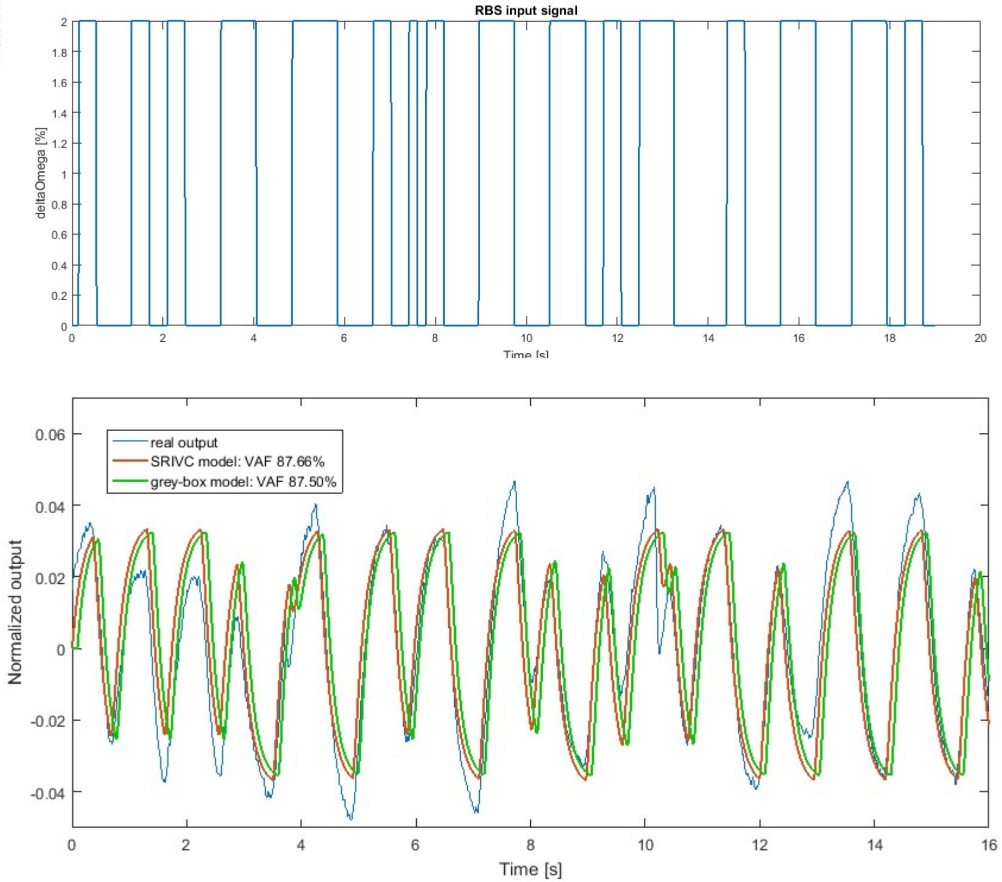




Example: single axis UAV attitude dynamics

- Simplified linearized roll dynamics: $\delta\dot{p} = \frac{1}{I_{xx}}\left[4K_t b\delta\Omega + \frac{\delta L}{\delta p}\delta p\right].$
- Parameters to be identified:

  Roll Inertia I_{xx} , Stability derivative $\frac{\delta L}{\delta p}$



Example: rotorcraft model identification

- Consider the problem of characterizing helicopter dynamics near a given trim condition
- For helicopters the six-DOF state-space model must be considered, because of the strong inter-axis couplings
- Furthermore, body roll and rotor flapping are coupled as well
- This leads to a very complex model with many parameters

Example: rotorcraft model identification

6-DOF dynamics → $x = [u \quad v \quad w \quad p \quad q \quad r \quad \phi \quad \theta]$

Forward flight at 80 knots → $U_0 = 40\,\tfrac{m}{s} \quad V_0 = 3\,\tfrac{m}{s} \quad W_0 = -5\,\tfrac{m}{s} \quad \Theta_0 = 0$

State space model

$$\dot{u} = X_u u + X_v v + X_w w + X_p p + (X_q - W_0)q + (X_r + V_0)r - g\theta + X_{\delta_{lat}}\delta_{lat} + X_{\delta_{lon}}\delta_{lon} + X_{\delta_{ped}}\delta_{ped} + X_{\delta_{col}}\delta_{col}$$

$$\dot{v} = Y_u u + Y_v v + Y_w w + (Y_p + W_0)p + Y_q q + (Y_r - U_0)r + g\phi + Y_{\delta_{lat}}\delta_{lat} + Y_{\delta_{lon}}\delta_{lon} + Y_{\delta_{ped}}\delta_{ped} + Y_{\delta_{col}}\delta_{col}$$

$$\dot{w} = Z_u u + Z_v v + Z_w w + (Z_p - V_0)p + (Z_q + U_0)q + Z_r r + Z_{\delta_{lat}}\delta_{lat} + Z_{\delta_{lon}}\delta_{lon} + Z_{\delta_{ped}}\delta_{ped} + Z_{\delta_{col}}\delta_{col}$$

$$\dot{p} = L_u u + L_v v + L_w w + L_p p + L_q q + L_r r + L_{\delta_{lon}}\delta_{lon} + L_{\delta_{ped}}\delta_{ped} + L_{\delta_{col}}\delta_{col}$$

$$\dot{q} = M_u u + M_v v + M_w w + M_p p + M_q q + M_r r + M_{\delta_{lat}}\delta_{lat} + M_{\delta_{ped}}\delta_{ped} + M_{\delta_{col}}\delta_{col}$$

$$\dot{r} = N_u u + N_v v + N_w w + N_p p + N_q q + N_r r + N_{\delta_{lat}}\delta_{lat} + N_{\delta_{lon}}\delta_{lon} + N_{\delta_{ped}}\delta_{ped} + N_{\delta_{col}}\delta_{col}$$

$$\dot{\phi} = p$$

$$\dot{\theta} = q$$

Example: rotorcraft model identification

Choice of outputs

$$y = [u \quad v \quad w \quad p \quad q \quad r \quad \phi \quad \theta]^T$$

State space model (with time delays

$$M\dot{x}(t) = Fx(t) + Gu(t - \tau)$$
$$y(t) = H_1 x(t) + H_2 \dot{x}(t)$$

$$\tau = \begin{bmatrix} \tau_{lat} \\ \tau_{lon} \\ \tau_{ped} \\ \tau_{col} \end{bmatrix}$$

Motivation and objectives: parameter estimation and model identification

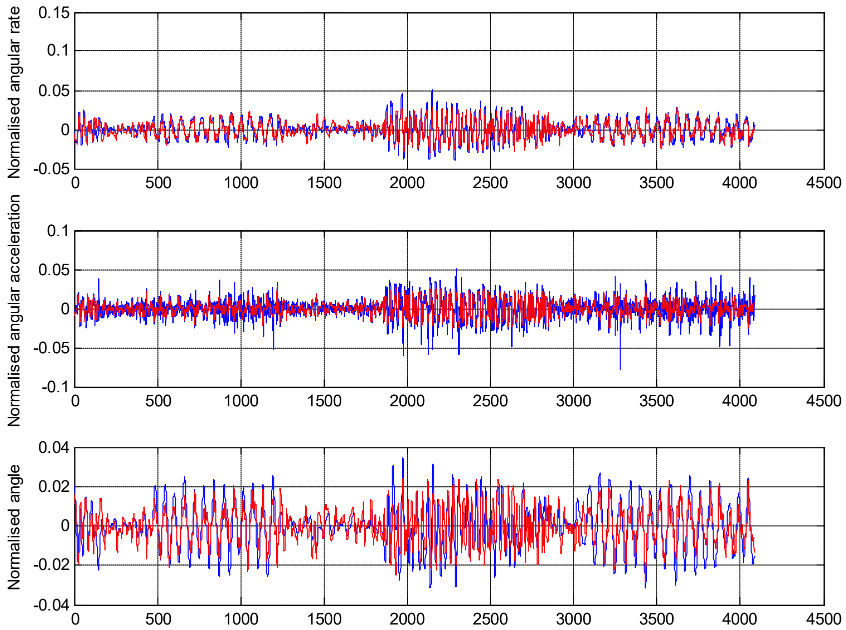
Typical questions to be faced:

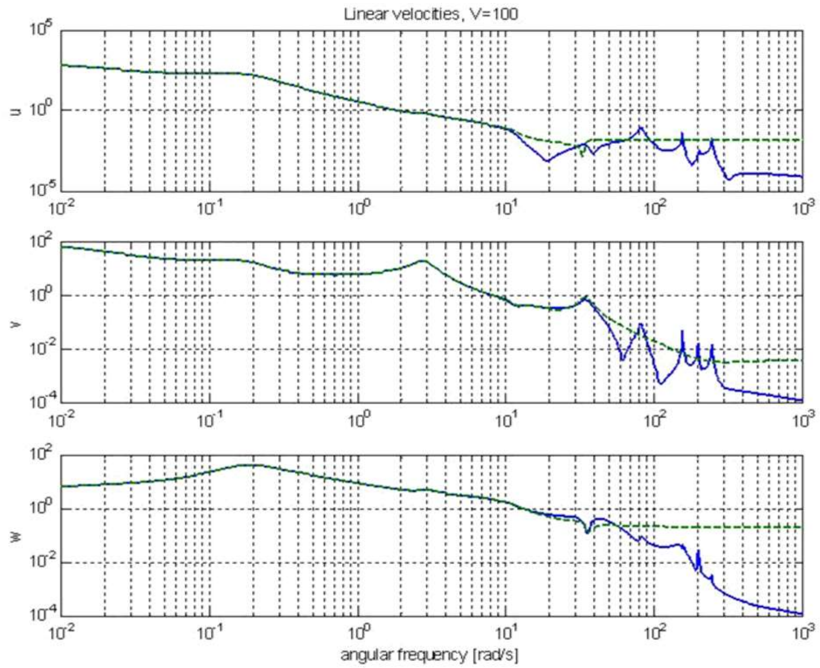
- Grey box modelling: how do I derive a parametric model and how do I guarantee that the values of the parameters can be determined from data?

- Black-box modelling: how do I define the structure of the model and find the values of its parameters from data?

Motivation and objectives: parameter estimation and model identification

In the linear case problems can be formulated either in the *time-domain* or in the *frequency-domain*:

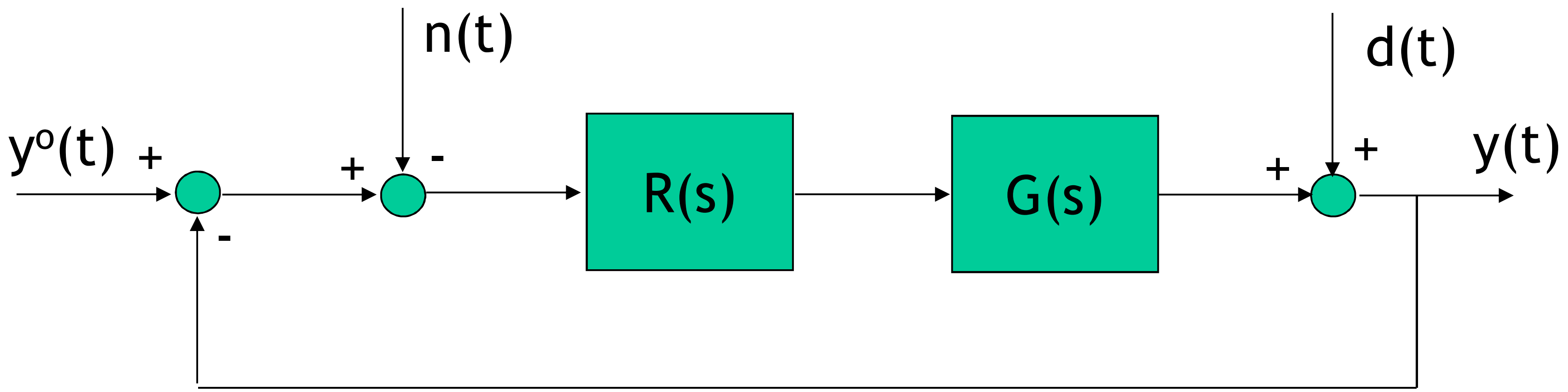




Both problems will be considered and extensions to nonlinear models will be studied (time-domain only).

Motivation and objectives: state estimation

In the elementary formulation of control design problems, it is assumed that the measured variable coincides with the controlled variable:



Motivation and objectives: state estimation

This however is not always true in practice.

Actually, it is hardly ever true in aerospace applications.

Typical situation:

design of flight control laws which aim at regulating

- attitude
- position

of an aircraft/rotorcraft/spacecraft using measurements of

- acceleration
- angular rate
- other variables such as magnetic field *etc.*

Motivation and objectives: state estimation

By referring, for the sake of simplicity, to a linear model, we have

$$\dot{x} = Ax + Bu$$
$$y_1 = C_1 x + D_1 u$$
$$y_2 = C_2 x + D_2 u$$

where y_1 is the measured output and y_2 is the (unmeasured) output to be controlled.

The estimation problem is then to compute an estimate of y_2 using measurements of u and y_1 and the model relating the variables.

If y_2=x this is known as *state estimation*.

Example: attitude estimation for UAVs

Typically available measurements are provided by

- a tri-axial gyroscope,
- a tri-axial accelerometer and
- a tri-axial magnetometer

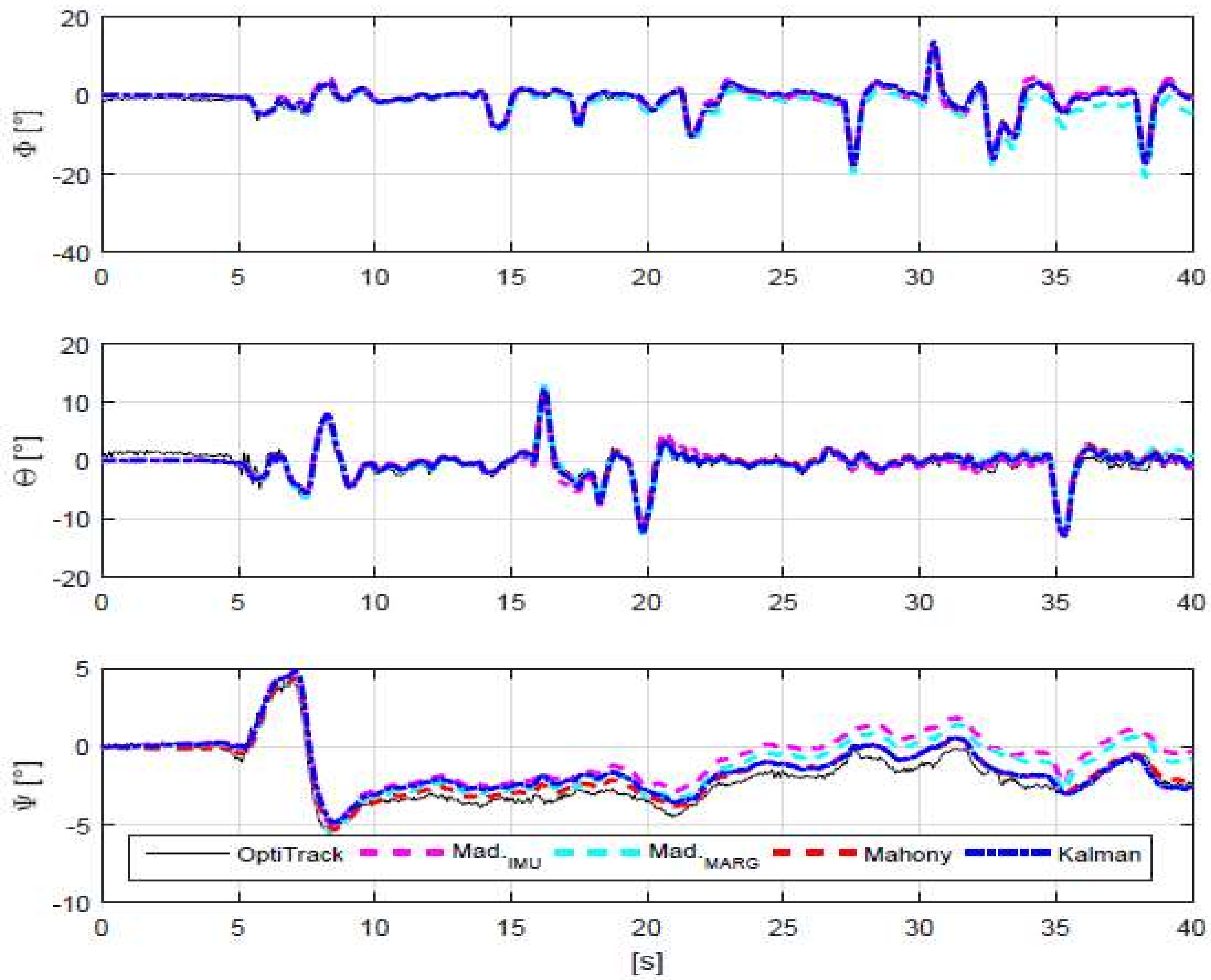
mounted on the platform.

Problems to be faced:

- Write a model for the kinematics of the UAV
- Write a measurement model for each sensor
- Work out an algorithm – to be run on-board – which estimates the state variables in real time given the measurements.

Example: attitude estimation for UAVs

Sample results using different state estimation methods



Motivation and objectives: fault detection

Another critical problem in aerospace engineering is the detection of faults in, *e.g.*, control systems sensors.

The most common approach is *physical redundancy, i.e.,* using two or more functionally equivalent devices the outputs of which can be compared to improve reliability.

Alternative approach: *analytical redundancy*, *i.e.,* measured outputs are compared to model-based estimates and discrepancies are used to detect faults (or changes) in the system.

An example

Consider the linear, discrete-time system given by

$$x_k = \Phi x_{k-1} + w_{k-1}$$
$$z_k = H x_k + v_k$$

$$\Phi = \begin{bmatrix} 0 & 1 \\ -0.4 & 0.6 \end{bmatrix}, \quad H = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix},$$
$$Q = \sigma_w^2 I_2, \quad R = \sigma_v^2, \quad \sigma_w = 0.1, \quad \sigma_v = 0.01$$

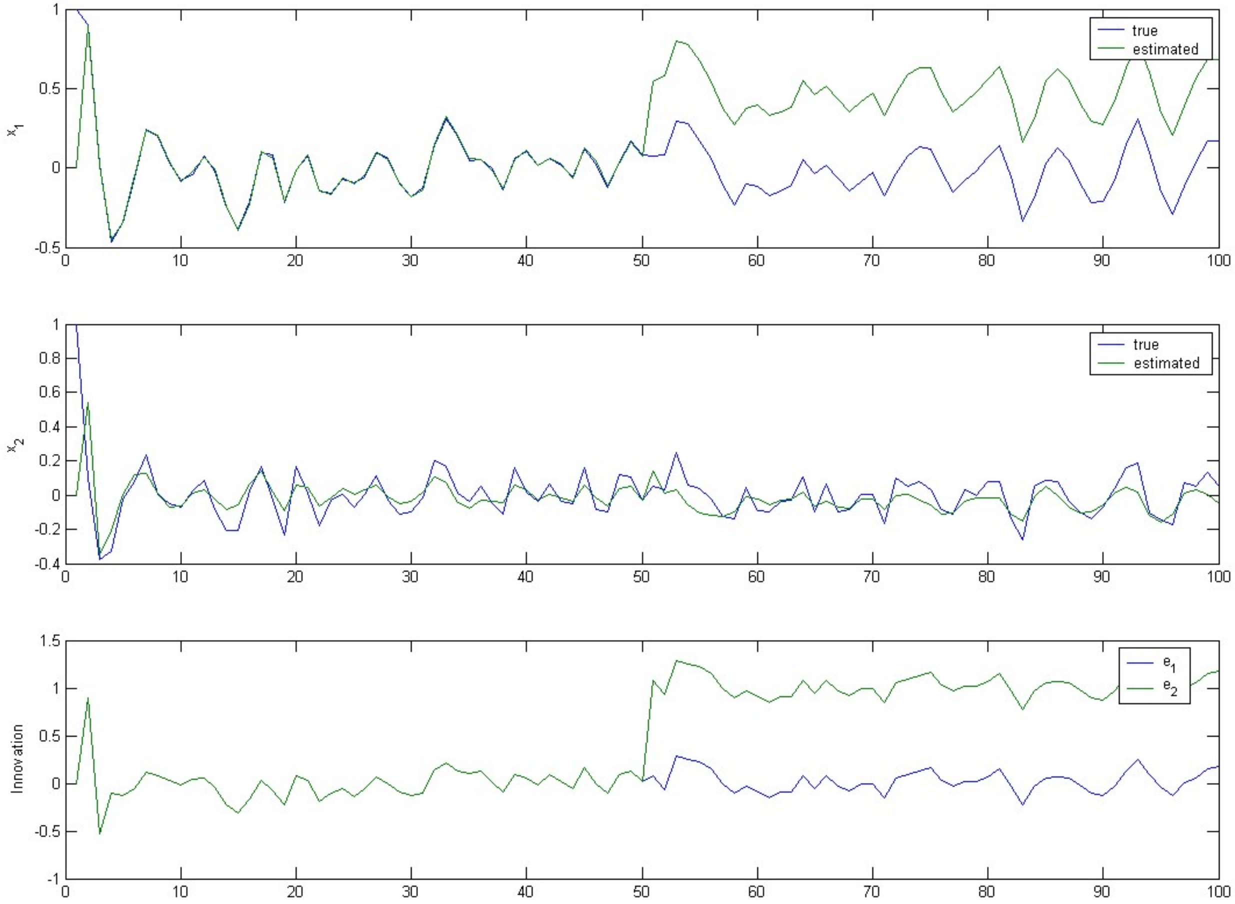
Now we have two sensors measuring x_1.

At time k=50, the second sensor becomes biased:

$$\{z_k\}_2 = \begin{bmatrix} 1 & 0 \end{bmatrix} x_k + v_k + 1, \quad k > 50$$

An example (2)

Simulation results



An example (3)

The problem with sensor 2 can be detected by monitoring $\{e_k\}_2$;

The faulty sensor can then be switched off;

If needed, a warning can be sent to a supervision system.

Motivation and objectives

In view of this, the Estimation in Aerospace course has the following objectives:

- to provide an introductory exposition to estimation theory, with specific reference to parameter estimation, state estimation and model identification.

- To study in greater depth some specific topics, namely:
  - Time-domain and frequency-domain parameter estimation for linear systems
  - State estimation for linear systems
  - Extensions to some classes of nonlinear systems.

- To illustrate the above by means of detailed case studies from aeronautics and space engineering.

Course programme

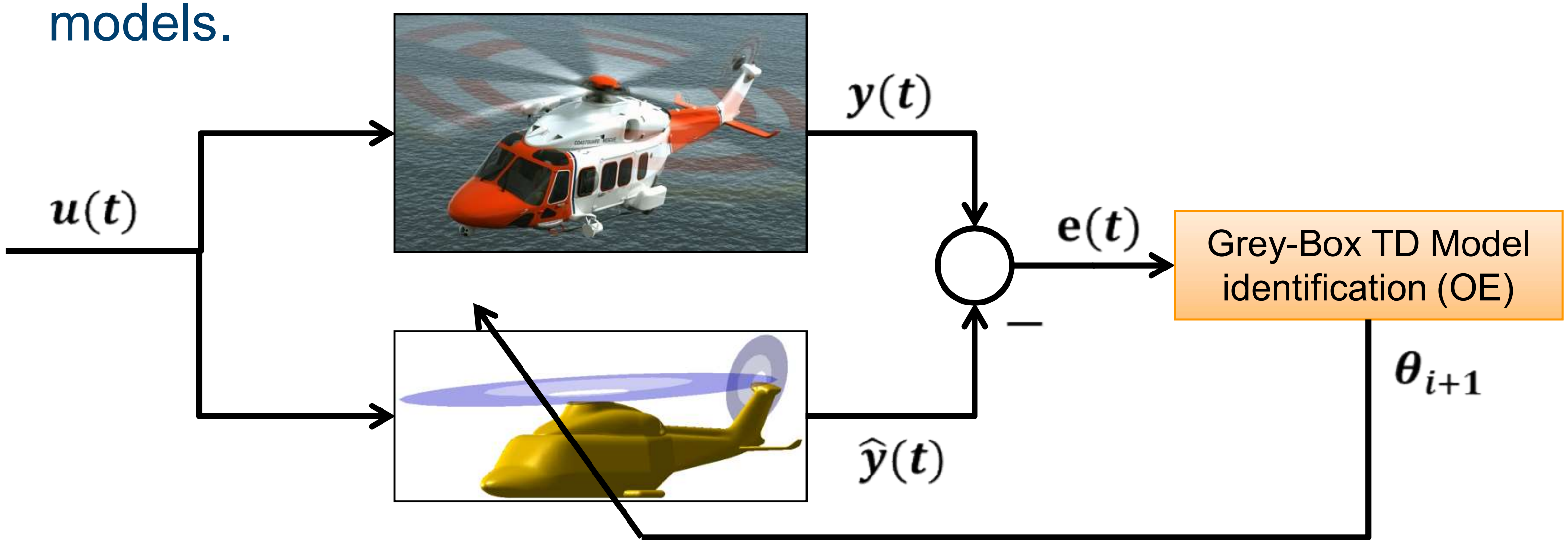
Part 1: introduction to estimation in aerospace.

- Overview of estimation problems in aerospace: sensor calibration, parameter estimation, model identification, state estimation, navigation, fault detection, fault tolerant control.
- Introduction to the theory of estimation
- Introduction to model identification: problem statement; grey vs black box models; linear vs nonlinear models; the notions of structural and experimental identifiability.
- The model identification process: from experiment design to model validation.

Course programme

Part 2: parameter estimation and output error model identification

- Estimation theory: the maximum likelihood method; least squares estimation.
- Time-domain output error identification of linear state space models.
- Frequency-domain output error identification of linear state space models.
- Time-domain output error identification of nonlinear state space models.



Course programme

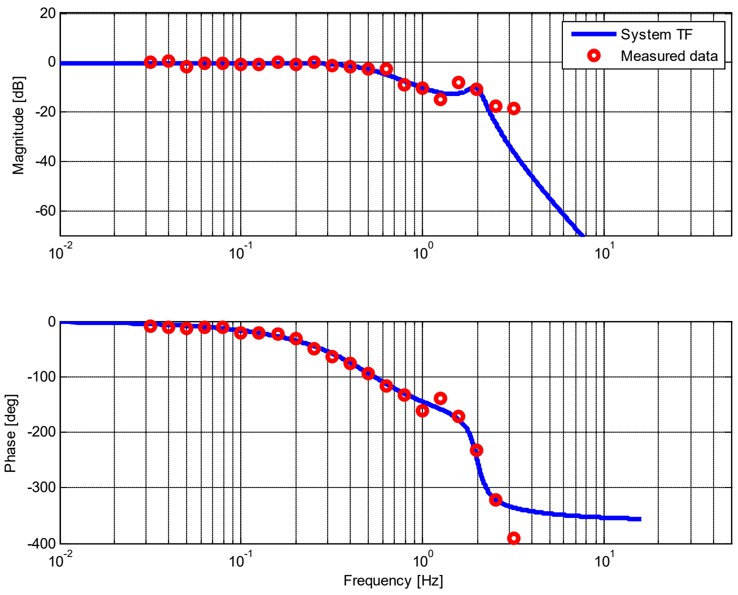
Part 3: state estimation and equation error model identification

- Estimation theory: introduction to Bayesian estimation.
- Optimal state estimation for linear systems: the Kalman filter.
- Time-domain equation error identification of linear state space models.
- State estimation for nonlinear systems: the Extended Kalman filter; overview of more general estimation schemes.
- Kalman filters: implementation issues.

Course programme

Part 4: black-box linear model identification

- Problem statement: structure selection vs parameter estimation.
- Time- and frequency-domain identification of SISO linear models.
- Identification of MIMO linear models: introduction to subspace methods.



Course programme

Part 5: case studies

- Identification of control-oriented models for helicopter flight mechanics.

- Attitude determination for a quadrotor UAV.
- Model-based control law design for small-scale and full-scale rotorcraft.