POLITECNICO DI MILANO

Model-based control law design for small-scale and full-scale rotorcraft.

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Outline

- Introduction and motivations
- Attitude dynamics identification
- Robust attitude control

Introduction

The interest in quadrotors as platforms for both research and commercial Unmanned Aerial Vehicle (UAV) applications is steadily increasing:

- ✓ surveillance & security
- ✓ environment monitoring & remote sensing
- ✓ buildings & industrial plants monitoring
- ✓ photogrammetry

Advantages w.r.t. classical helicopter architecture:

- ✓ simpler rotor articulation (no swash plate, no cyclic command)
- ✓ weak DoFs coupling → easier to control
- ✓ possibility of rotors protection (shrouding) → safer

Possible quadrotor architectures:

- variable RPM (fixed blade pitch) → simple and light rotors hub
- variable pitch (fixed RPM) → avoid performance limitation due to bandwidth of motors dynamics

Some of the envisaged applications lead to tight performance requirements on the attitude control system → this calls for increasingly accurate dynamics models of the vehicle's response to which advanced controller synthesis approaches can be applied

Motivations

Development of an integrated, highly automated, control design tool chain aimed at fast and reliable deployment of vehicle's attitude control system (applied to pitch DoF)

Identification experiments indoor on proper test-bed, avoiding risky and time consuming in-flight test campaign

→ Identification of LTI attitude response models

→ Attitude controller tuning solving structured H_∞ robust design problem

→ GOAL: demonstrate that it guarantees acceptable performance also in flight near hover conditions

ANTEOS prototype

- ✓ Variable collective pitch (fixed RPM)
- ✓ MTOW = 5 kg
- ✓ Rotors diameter = 54 cm

Attitude dynamics identification
State of the art

- It is apparent from literature that quadrotor mathematical models are easy to establish as far the kinematics and dynamics of linear and angular rigid body motion are concerned

- Characterizing aerodynamic effects and additional dynamics such as, *e.g.*, due to actuators and sensors, is far from trivial → increasing interest in experimental characterization of quadrotor dynamics response through system identification

- System identification is actually a well established approach for the development of control-oriented LTI models in the rotorcraft field:
  - ✓ Frequency-domain approaches (*e.g.* NASA CIFER tool)
  - ✓ Iterative time-domain approaches (*e.g.* OE, EE, etc.)
  - ✓ NON-iterative time-domain approaches (*e.g.* subspace methods)

- The application to full scale rotorcraft is fairly mature but less experience has been gathered on small-scale vehicles

Attitude dynamics identification
Identification experiments (1)

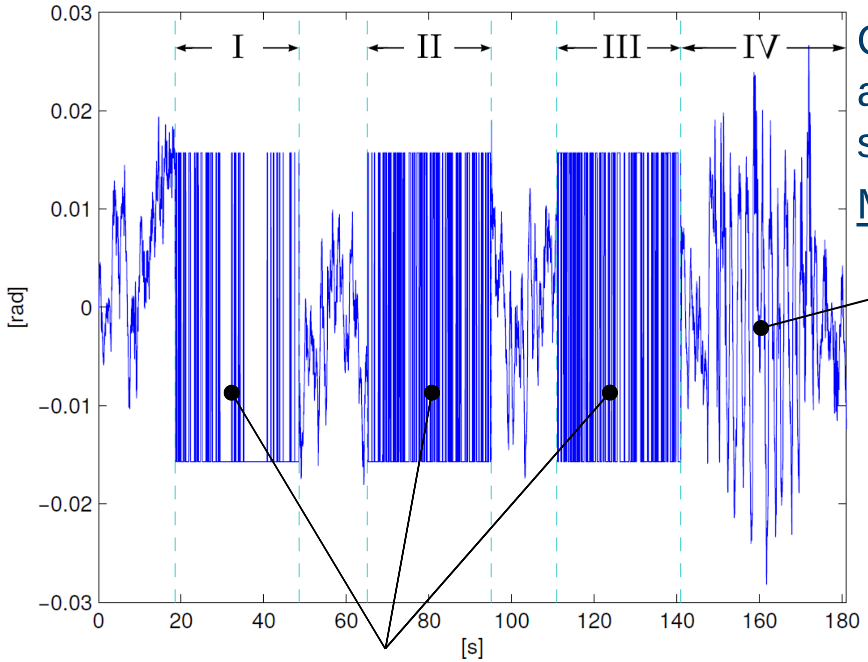
- The identification experiments have been carried out in laboratory conditions, using a test-bed that constrains all DoFs except pitch rotation
- Similar experiments have been carried out in flight to ensure that indoor set-up is representative of actual attitude dynamics in near hovering

- Pseudo Random Binary Sequences (PRBS) were selected as excitation signal
- Experiments have been carried out in quasi open-loop conditions:
  - nominal attitude and position controllers were disabled
  - a supervision task enforcing attitude limits (±20°) was left active (inherently fast instability)

Attitude dynamics identification:
Identification experiments (2)

Input signal (collective pitch difference between opposite rotor) in identification test

The parameters of the PRBS were tuned to obtain an excitation spectrum consistent with the expected dominant attitude dynamics (4 to 8 rad/s)



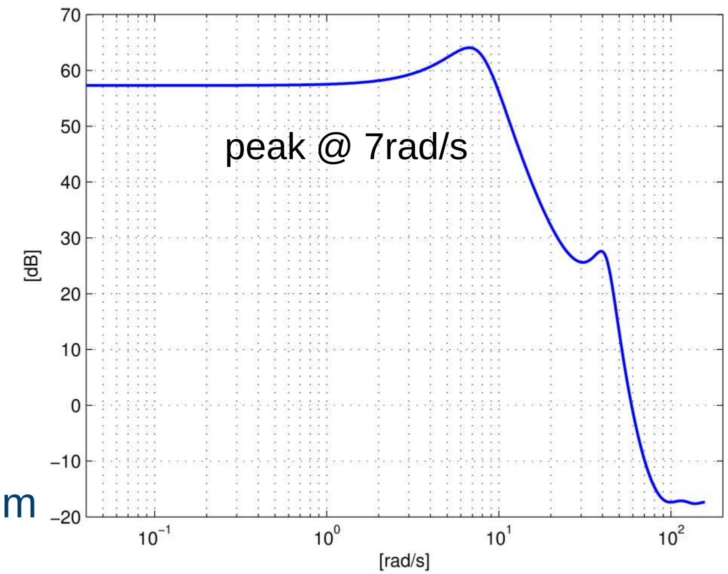
Closed-loop on nominal attitude controller, angle sp variation imposed

Model validation dataset

PRBS + supervision task feedback

Dataset used for identification

Excitation spectrum



Attitude dynamics identification
Considered algorithms

BLACK-BOX METHODS: gives unstructured model, non-physically motivated state space

- Subspace Model Identification (SMI) algorithms, non-iterative (efficient computation):
  - PI-MOESP (Past Inputs - Multivariable Output Error State sPace realization)
  - PBSID (Predictor Based System IDentification)

both providing LTI SISO state space model of the pitch rate response to control input

- On-line implementation of the Least Mean Squares (LMS) algorithm:
  - updates recursively on-board an estimate of the SISO impulse response of pitch angular velocity in the form of Finite Impulse Response (FIR) model
  - state space model for the pitch dynamics recovered via Kung's realization

GREY-BOX METHODS: structure imposed a-priori defining a first-principle model for pitch dynamics

- Output Error (OE) Maximum Likelihood (ML) estimation
- H_∞ approach (model matching problem, non-convex & non-smooth optimization)

both determine the unknown physical parameter of structured LTI model via iterative (time consuming) procedure

SMI was proposed about 25 years ago to handle black-box MIMO problems in a numerically stable and efficient way (numerical linear algebra tools) and has proved extremely successful in a number of industrial applications

PI-MOESP (*Verhaegen & Dewilde,1991*) assumes feeding data gathered in open-loop operations

PBSID (*Chiuso,2007*) is a more advanced and recent algorithm respect to MOESP, suitable for dealing with data generated in closed-loop operations

Attitude dynamics identification

Grey-box approach: model structure and OE

- Quadrotor is modeled as a rigid body with rotors aerodynamics terms from closed-form BET
- LTI first-principle model of pitch attitude dynamics on test-bed (all other DoFs constrained) includes a rotational mass-spring-damper to modeling IMU vibration damping system through which the device is connected to vehicle

$$A_s = \begin{bmatrix} 1/I_{yy}\,\partial M/\partial q & -c/I_{yy} & -k/I_{yy} & k/I_{yy} \\ c/J & -c/J & k/J & -k/J \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad B_s = \begin{bmatrix} 1/I_{yy}\,\partial M/\partial u \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad C_s = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}^T \quad D_s = [0]$$

state vector: $x(t) = [q(t)\ \ q_P(t)\ \ \theta(t)\ \ \theta_P(t)]^T$ where P discern IMU from vehicle quantities

The unknown parameters of structured model are: $\Theta = \left(\frac{\partial M}{\partial q}, \frac{\partial M}{\partial u}, J, k, c\right)$

Stability and control derivatives of pitch moment ($\frac{\partial M}{\partial q}, \frac{\partial M}{\partial u}$)

IMU damping system model param. (J, k, c)

Given sampled I/O dataset $\{u_t, y_t\}$ the OE ML estimate is equal to the value of Θ that maximizes the likelihood function, defined as the probability density function of y given Θ $\rightarrow$ $\mathbb{L}(y,\Theta) = P(y|\Theta)$

- If $P(y)$ is Gaussian, as the measurement noise, the ML estimator minimizes a positive cost function of the prediction error: minimum search via iterative Newton-Raphson
- The convergence towards absolute minimum is not guaranteed: multiple executions varying initial guess of Θ are needed

Attitude dynamics identification
Grey-box H_∞ approach

- SMI methods are more attractive than OE because of their non-iterative nature
- On SMI model is not possible to enforce a-priori knowledge of model structure, naturally allowed by grey-box approach

Novel identification procedure (*Bergamasco & Lovera, 2013)* bridging the gap between structured and unstructured model

H_∞ model matching problem in frequency-domain, relating black-box model from SMI to structured one from first principle approach

Unstructured LTI DT from SMI → convert to CT (zero order hold)

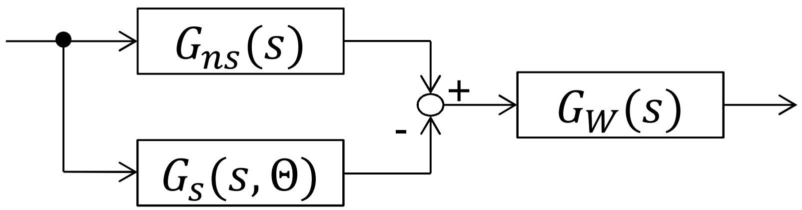
$$\dot{x}(t) = A_{ns}x(t) + B_{ns}u(t)$$
$$y(t) = C_{ns}x(t) + D_{ns}u(t)$$

→ convert to tf in Laplace domain → $G_{ns}(s)$

Structured LTI CT from first principle

$$\dot{x}(t) = A_s(\Theta)x(t) + B_s(\Theta)u(t)$$
$$y(t) = C_s(\Theta)x(t) + D_s(\Theta)u(t)$$

→ $G_s(s,\Theta)$



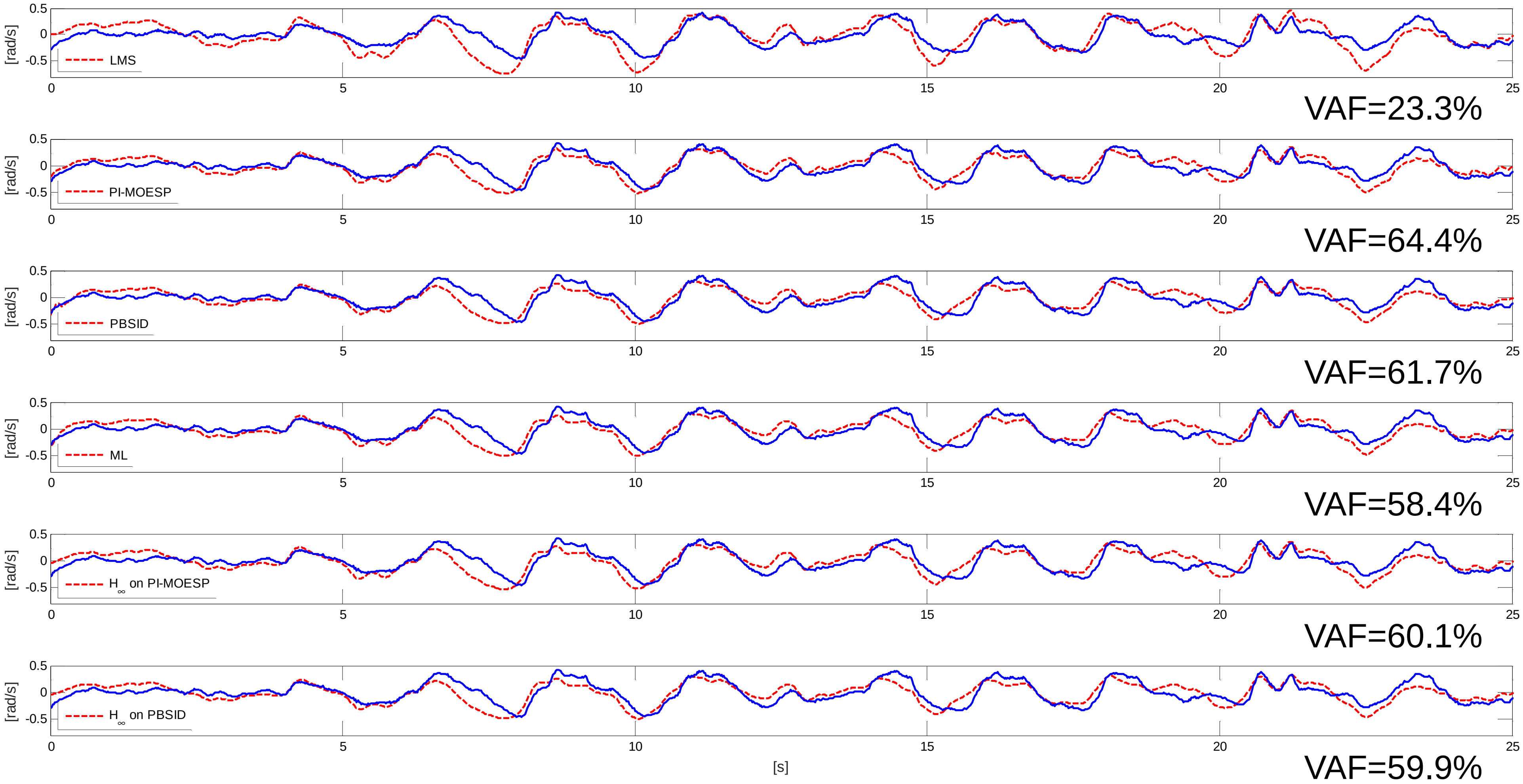
$$\hat{\Theta} = \arg\min_{\Theta} \| G_W(s)(G_{ns}(s) - G_s(s,\Theta)) \|_\infty$$

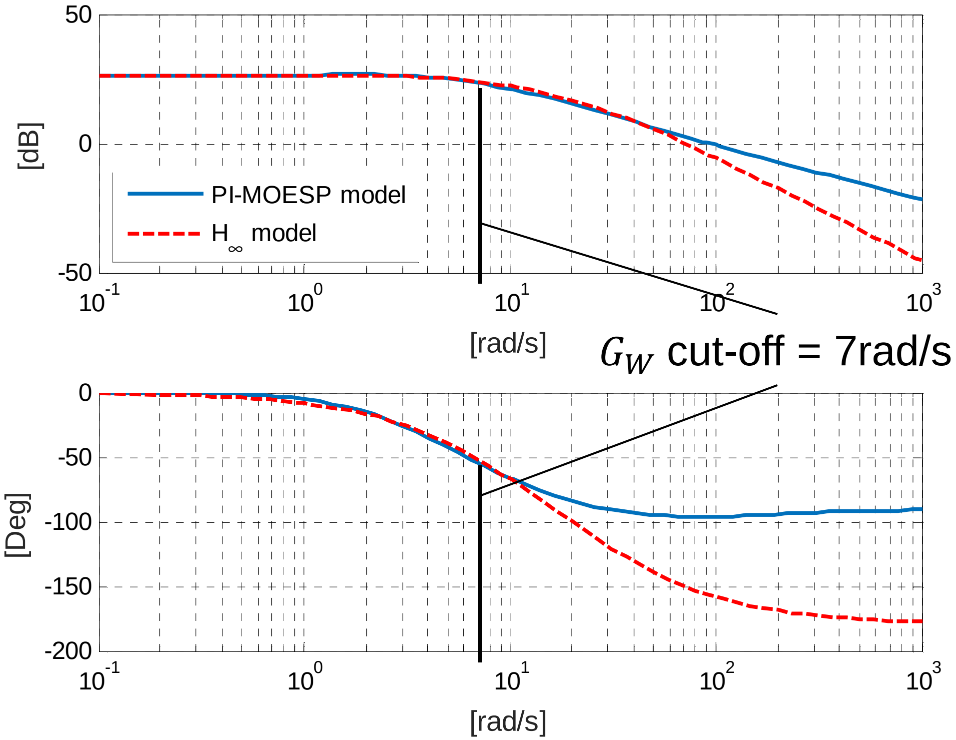
non-convex, non-smooth optimization problem
computational tool (*Apkarian & Noll, 2006*)
available in Matlab from R2012a

G_W is a suitable filter to focus the matching in the frequency range where $G_{ns}(s)$ well describes the real system, then in PRBS excitation spectrum

Attitude dynamics identification
Results on closed-loop validation dataset

blue lines: measured pitch rate ; red dashed lines: models simulation





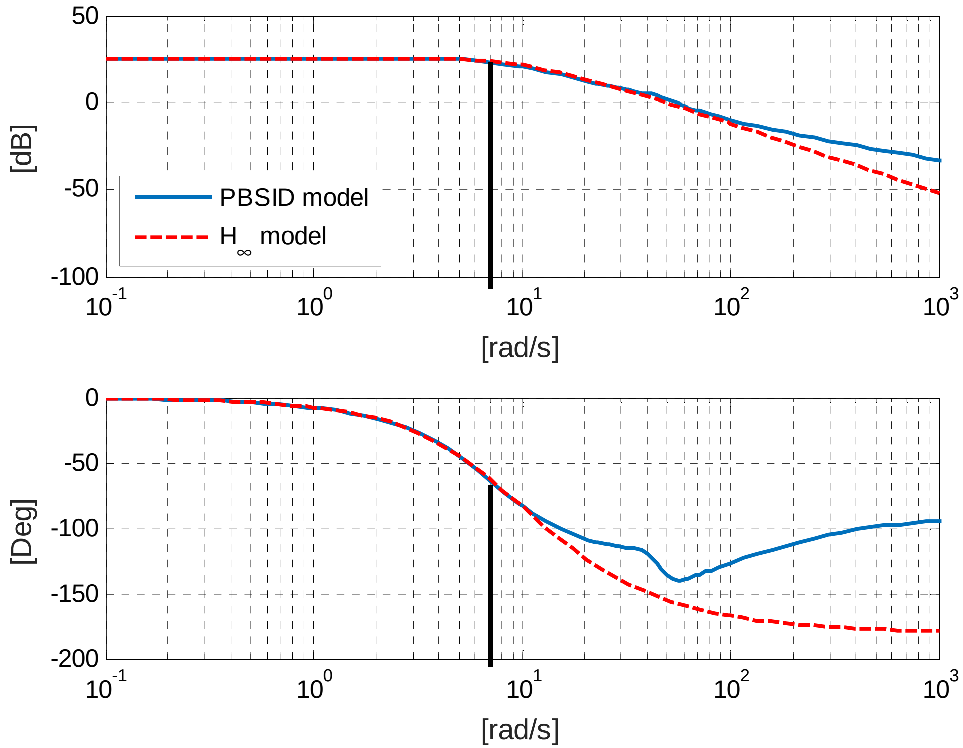
SMI algorithms parameter tuned to obtain higher VAF on cross-validation dataset

PI-MOESP

Model order $n = 4$; Hankel I/O matrices rows n° $p = 40$

PBSID

Model order $n = 5$; past / future window size $p / f = 11 / 7$



Filter G_W tuned to reach higher VAF on cross-validation dataset: adopted a 15th order low pass Butterworth, cut-off complies with excitation spectrum peak

- H_∞ model *vs.* PI-MOESP, VAF = 96.9%
- H_∞ model *vs.* PBSID VAF = 99.6%

- Introduction and motivations
- Attitude dynamics identification
- **Robust attitude control**
- Wind tunnel testing on isolated rotor
- One rotor fault condition
- Longitudinal control enabling rotors RPM variations
- Concluding remarks and further works

Robust attitude control
Introduction

Quadrotor control synthesis has been studied extensively in the literature, adopting several approaches:

- PID architecture and LQ synthesis
- Robust control design
- Backstepping and Sliding mode
- Trajectory planning & tracking (e.g., adaptive control, dynamic inversion, flatness-based control, trajectory smoothing using motion primitives)

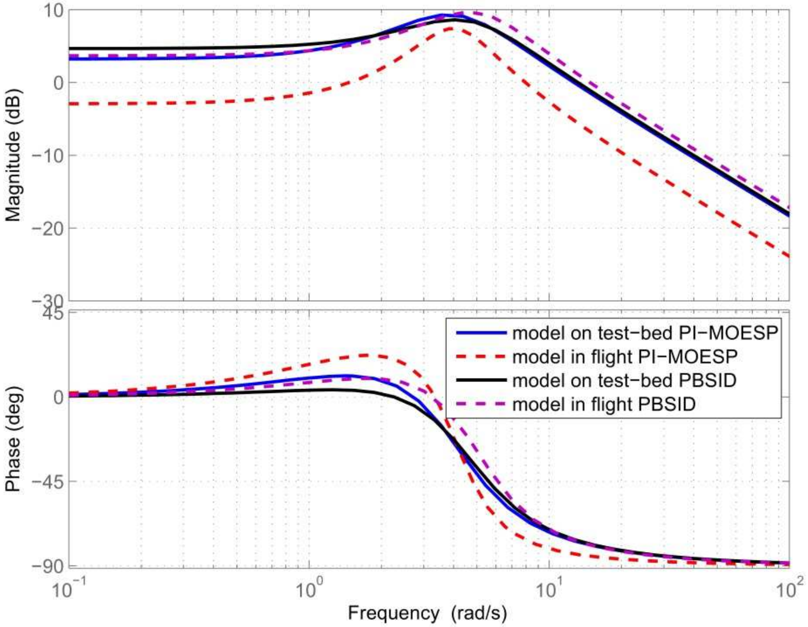
Concerning the control design part of the developed tool chain it was preferred to maintain the pre-existing on-board attitude controller scheme (cascade PID loops), in order to work in continuity and accelerating implementation process → structured H_∞ synthesis

The work focuses on near hovering condition:

- ✓ quadrotors mainly operate in this regime during typical missions
- ✓ in this operating mode the tighter handling qualities performance are required
- ✓ the attitude dynamics in hover can be replicated operating on a proper test-bed

Robust attitude control
Adopted identified models

Pitch angular velocity models comparison:
on test-bed *vs.* in flight (near hover), PI-MOESP *vs.* PBSID



- ✓ PI-MOESP: poor agreement test-bed *vs.* flight
- ✓ Feedback action of supervision task added to PRBS is more invasive during in flight identification (quasi open-loop): avoid attitude angle limit overcoming in presence of wind disturbances
- ✓ PI-MOESP assumes feeding data gathered operating in open-loop

PBSID is able to deal with data generated in closed-loop

- ✓ PBSID: good agreement test-bed *vs.* flight
- ✓ PBSID close to PI-MOESP on test-bed data, when feedback action is less invasive (operations are nearest to be in open-loop)

PBSID on test-bed

$$\frac{q}{u} = \frac{12.517(s + 2.906)}{s^2 + 5.583s + 21.3}$$

n=2, p=35, f=6

PBSID in-flight

$$\frac{q}{u} = \frac{13.8194(s + 2.761)}{s^2 + 5.198s + 25.04}$$

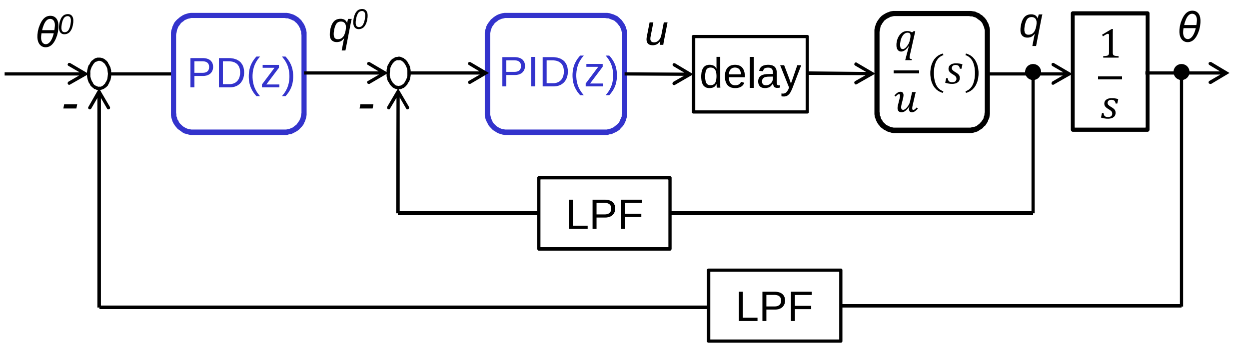
n=2, p=f=5

Test-bed set-up is representative of the pitch attitude dynamics in hovering flight

Algorithm	ν-gap metric test-bed *vs.* flight	VAF test-bed	VAF flight
PI-MOESP	0.3405	65.8%	20.1%
PBSID	0.0741	65.1%	21.4%

Robust attitude control

Controller architecture & H_∞ synthesis requirements



- Implemented accurate replica of pre-existing on-board controller in Simulink
- Cycle time 0.02 s

Pre-existing tuning from experimental trial & error manual process:

- ✓ guarantees adequate performance in terms of set-point tracking
- ✓ needs improvement in terms of wind gust rejection

Define proper requirements for H_∞ synthesis on fixed-structure controller

Performance channel

- Crossover frequency of each loop into specified bandwidth: 3.5→14 rad/s
- Set-point tracking target response time: 0.5 s
- Set-point tracking target maximum steady-state error: 0.001%

Robustness channel

- From process noise (wind gust) to control variable
- Disturbance rejection specified assigning maximum gain constraint function: high pass filter (gust is a low frequency noise), first order, gain 40 dB, cutting freq. 10 rad/s

Given

$P(s)$: real rational transfer matrix, PLANT

$K(\vartheta)$: STRUCTURED controller depending smoothly on a design parameter vector ϑ varying in space $\mathbb{R}^n$

Solve the optimization program

minimize $\|T_{w\to z}(P, K(\vartheta))\|_\infty$

subject to $\vartheta \in \mathbb{R}^n : K(\vartheta)$ stabilizes P internally

$T_{w\to z}(P, K(\vartheta))$: closed loop transfer function on considered I/O channel $w \to z$ on which requirements (performance and robustness) are defined

$P(s)$ regroups the process and the filter functions in loop shaping context

Resulting non-convex, non-smooth optimization problem is solved explointing computational tool developed by *Apkarian & Noll, 2006*

Available in Matlab Robust Control Toolbox from R2012a → <hinfstruct>

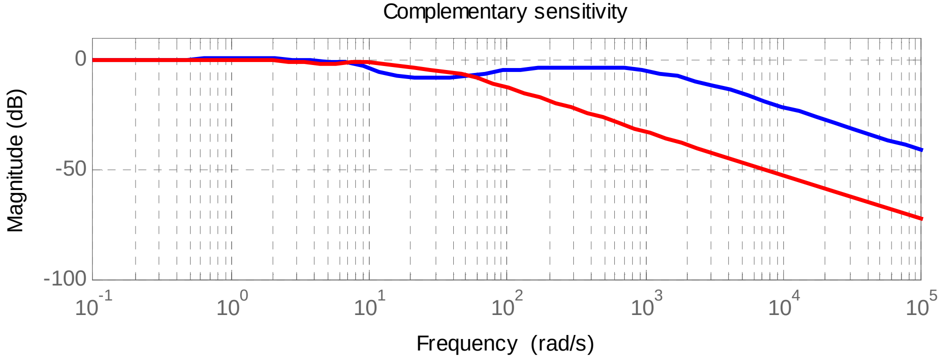
Robust attitude control
Optimal tuning parameters

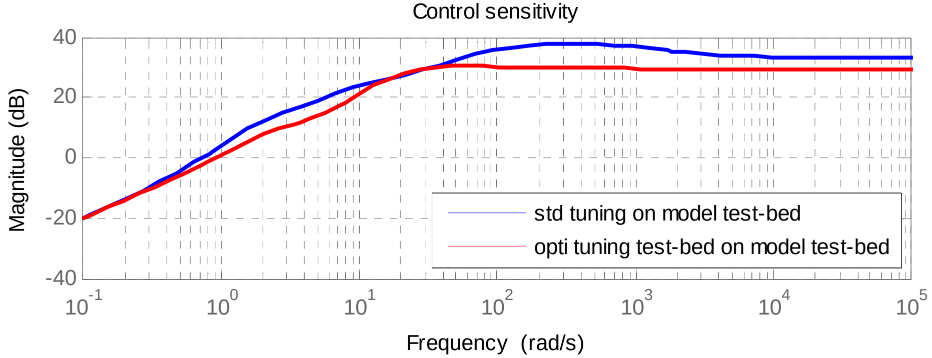
For the assigned controller structure, applied on identified models, the structured H_∞ algorithm finds the locally optimal parameters ϑ of the PIDs to satisfy desired requirements

	Controller parameter	Standard tuning	Optimal tuning: test-bed model	Optimal tuning: flight model
Outer loop on θ	K_p PD	9.26	5.4631	6.0491
	K_d PD	1.11	0.9376	1.0320
	T_f PD	0.03	0.0380	0.0377
Inner loop on q	K_p PID	0.257	0.3539	0.2986
	K_i PID	0.643	1.8562	1.6150
	K_d PID	0.0231	0.0084	0.0075
	T_f PID	0.0225	0.0430	0.0415

The standard tuning obtained through trial & error empirical procedure done manually was used as starting guess for the optimization procedure

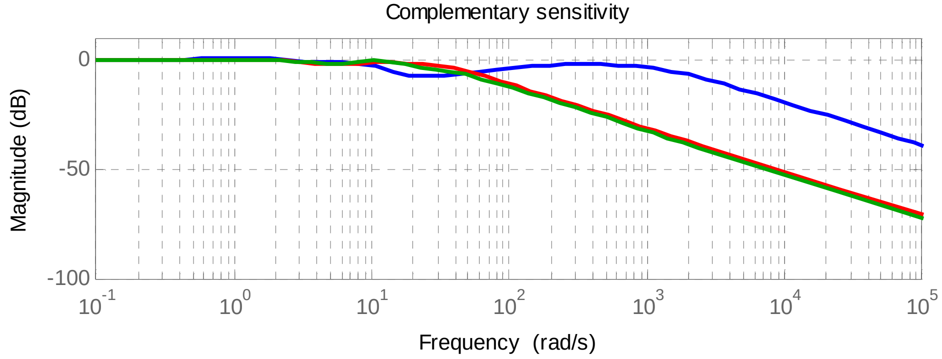
Robust attitude control
Closed-loop functions

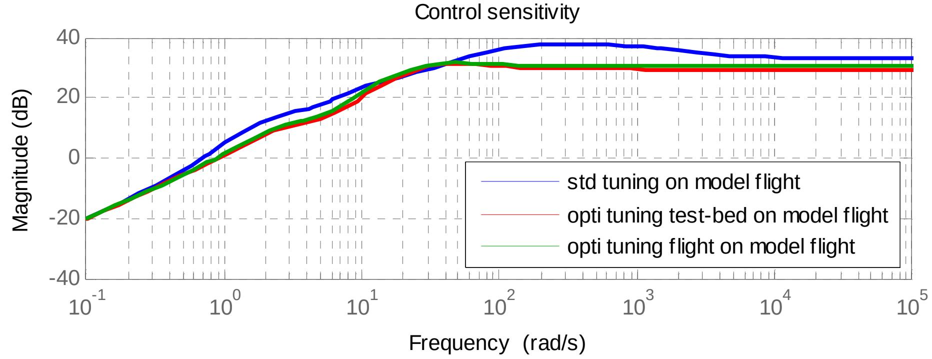




On test-bed model

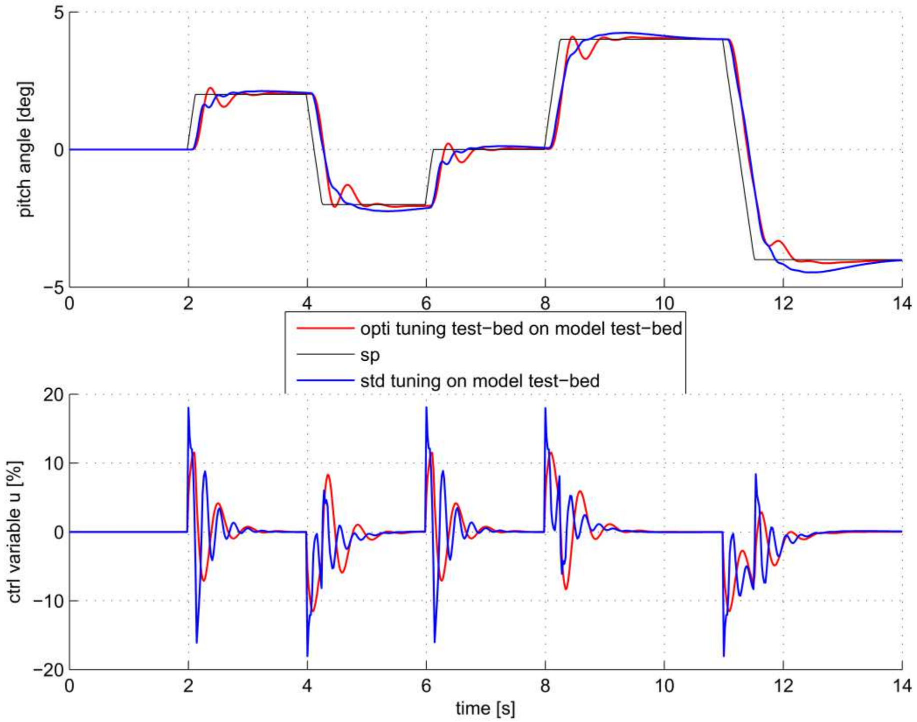
In-flight model





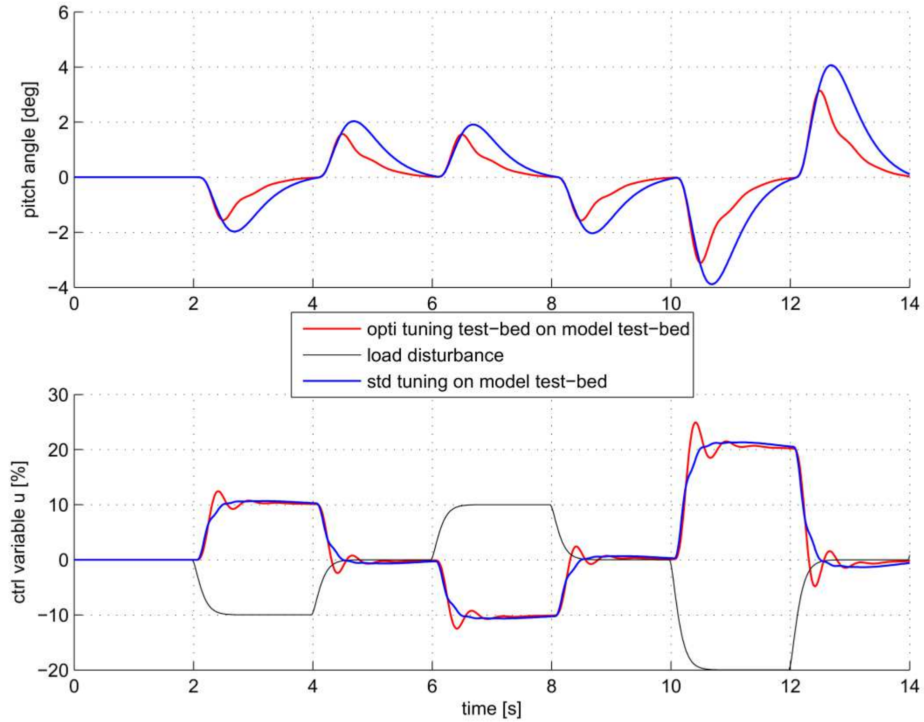
Robust attitude control

Simulation results: on test-bed model in the loop



Pitch control variable saturation = ±30%

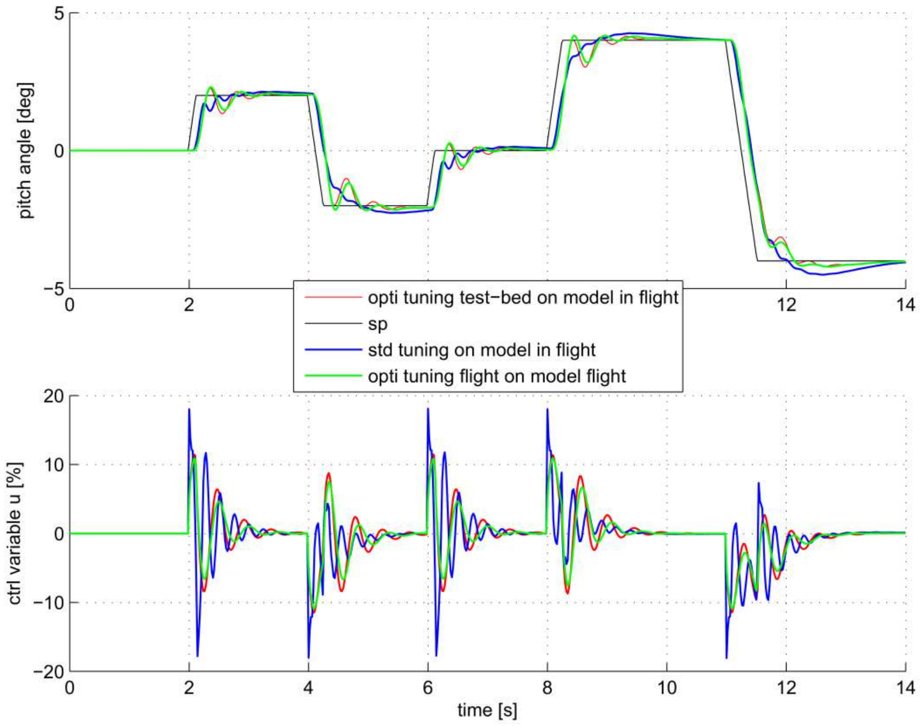
- ✓ Angular sp variation requested
- ✓ No process/measures noise
- ✓ Optimal tuning guarantees control effort reduction with similar/better tracking respect to standard



- ✓ Process disturbance, typical wind gust
- ✓ Angular sp null
- ✓ Optimal tuning guarantees angular drift reduction

Robust attitude control

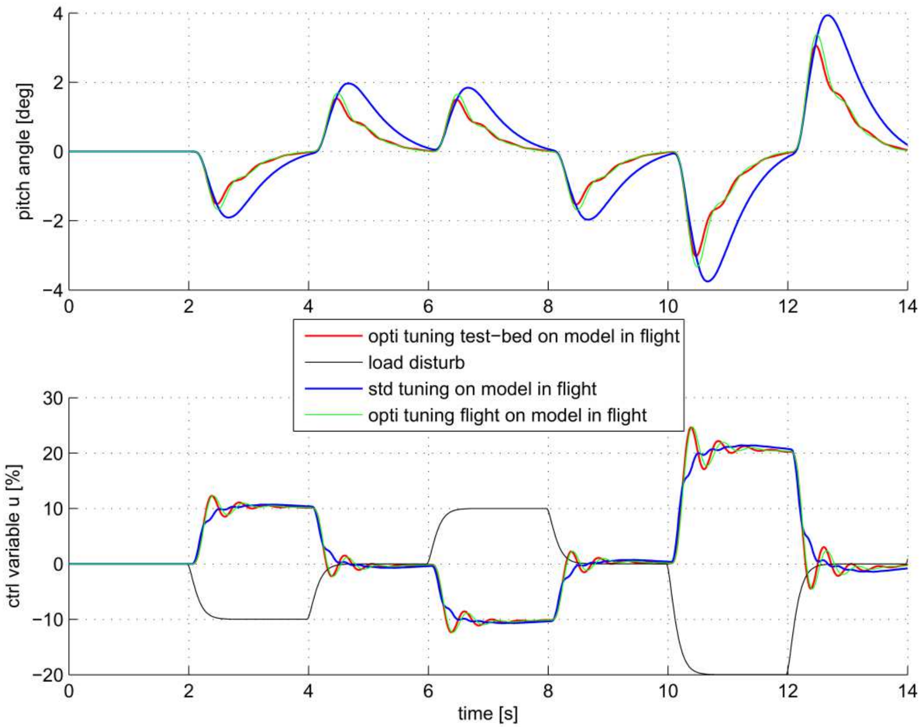
Simulation results: in-flight model in the loop



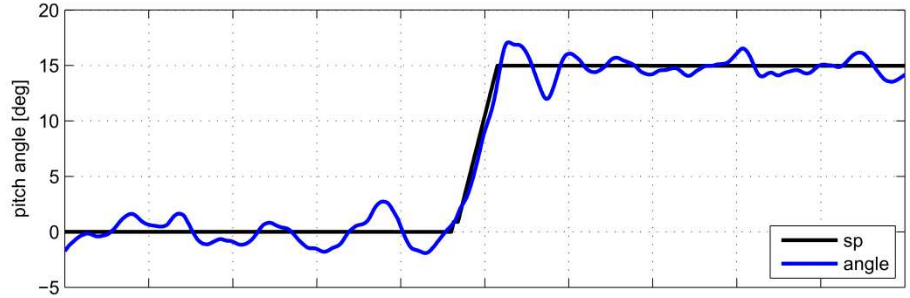
- ✓ Process disturbance, typical wind gust
- ✓ Angular sp null
- ✓ Optimal tuning flight guarantees angular drift reduction
- ✓ Optimal tuning test-bed is close to flight one

Pitch control variable saturation = ±30%

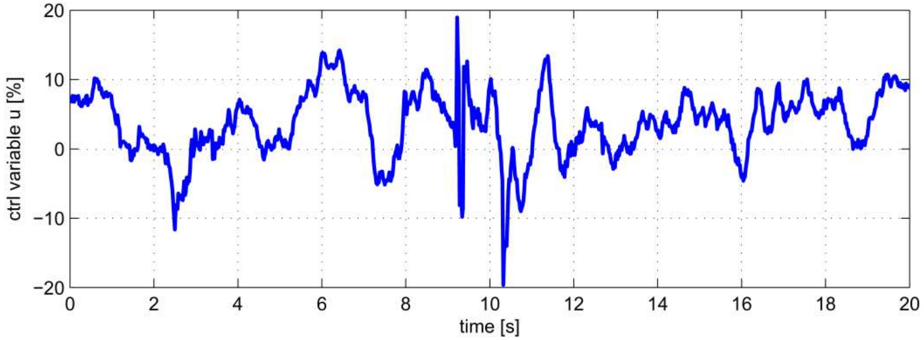
- ✓ Angular sp variation requested
- ✓ No process/measures noise
- ✓ Optimal tuning guarantees control effort reduction with similar/better tracking respect to standard
- ✓ Optimal tuning test-bed is close to flight one



Robust attitude control
Experimental results on test-bed

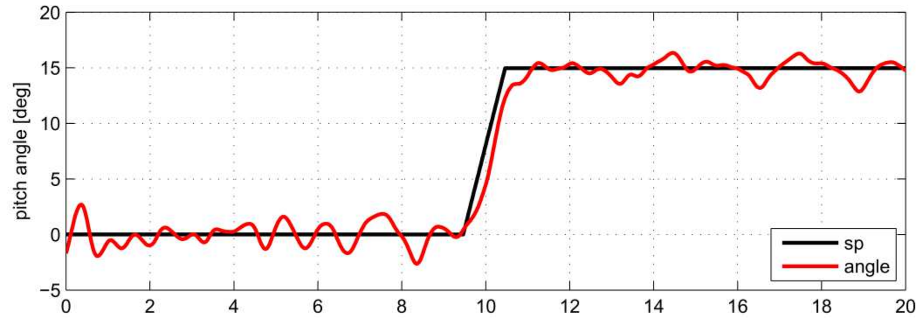


Standard tuning

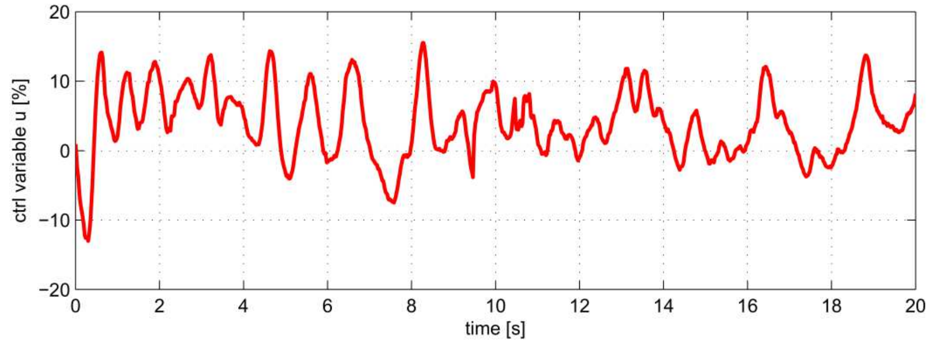


Pitch control variable saturation = ±30%

- ✓ Base throttle = 60% (hovering value)
- ✓ Angular sp variation requested
- ✓ Aerodynamic disturbances due to rotors wake recirculation in closed indoor test area



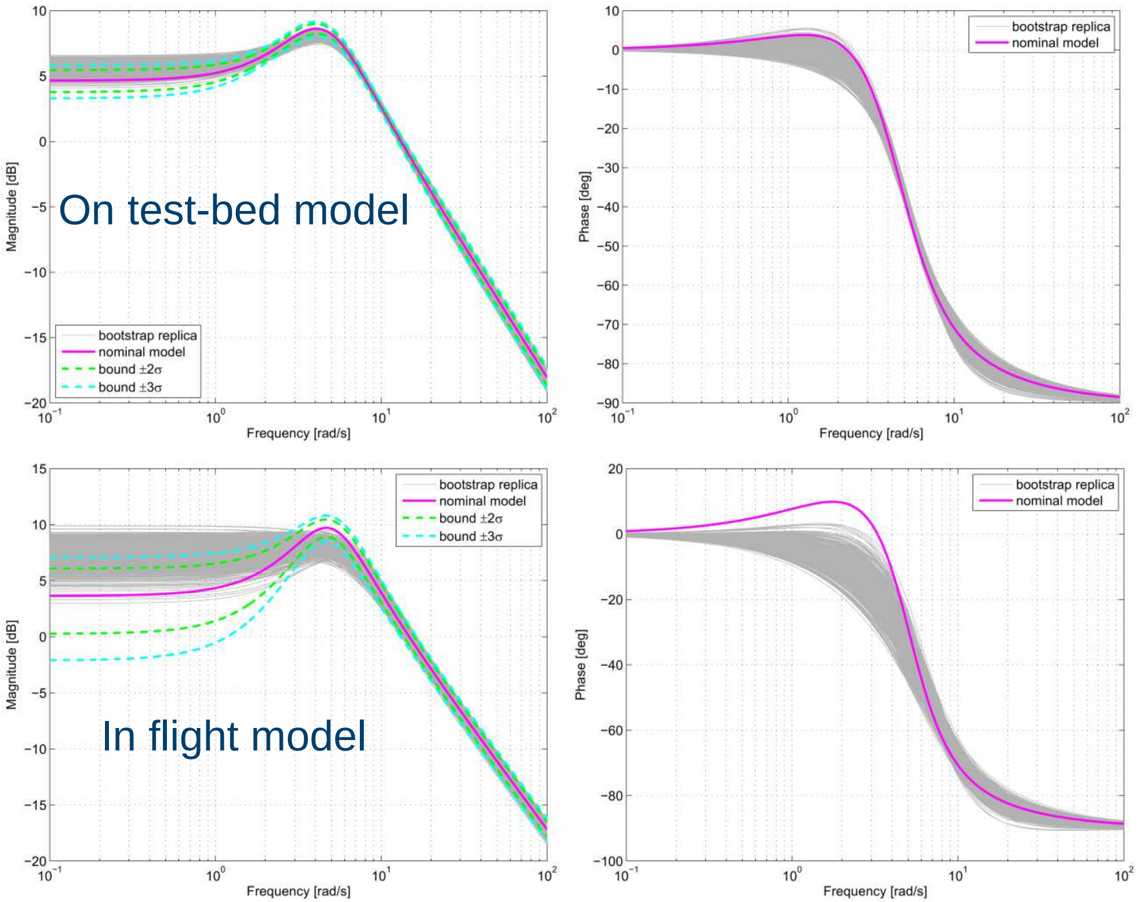
Optimal tuning test-bed



- ✓ Optimal tuning guarantees similar tracking performance with a reduction in control effort (confirming behavior from simulation)

Robust attitude control
PBSID models uncertainty

Bootstrap based approach, 1000 replications

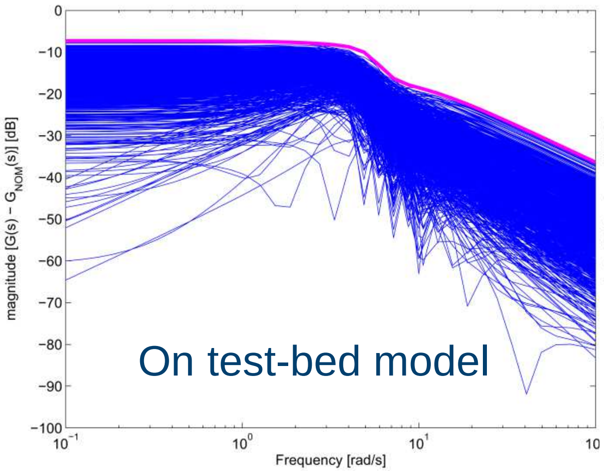


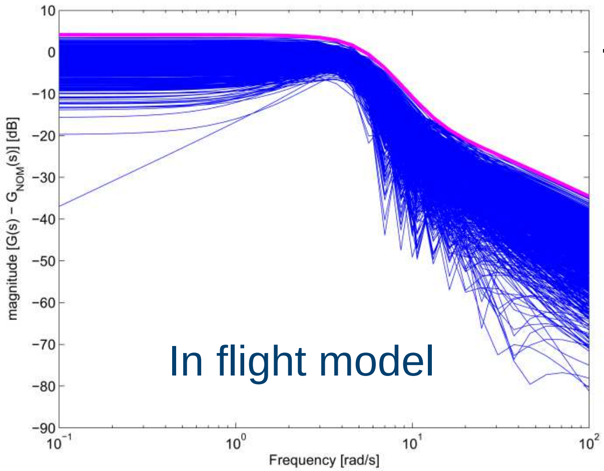
- ✓ On test-bed attitude pitch dynamics captured with very good accuracy
- ✓ Limited uncertainty band on all considered frequency range
- ✓ Especially narrow around PRBS excitation cut-off (7 rad/s)

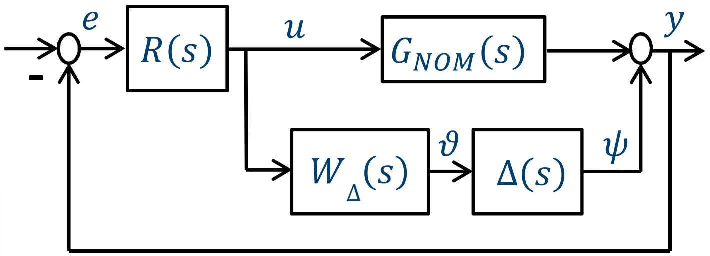
- ✓ Identified model in flight presents a wider uncertainty band
- ✓ In design control bandwidth (3.5→14 rad/s) level of uncertainty can be considered acceptable

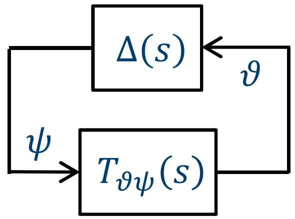
- ✓ The presence of wind gust implies a less repeatable test conditions w.r.t. identification in laboratory on test-bed
- ✓ In flight quadrotor attitude pitch dynamics is coupled with longitudinal one during the PRBS excitation, while on test-bed only the pitch rotation is allowed

Robust attitude control
Robust stability analysis









Additive uncertainty

$\mathcal{G} \coloneqq \{G(s) = G_{NOM}(s) + W_{\Delta}(s)\Delta(s), \quad \|\Delta\|_\infty < h\}$

$\Delta(s)$: uncertainty LTI SISO random dynamics (with assigned peak gain)

$W_{\Delta}(s)$: stable, minimum phase, shaping filter, order 3

$if\ \|\Delta\|_\infty < 1, \quad |G(j\omega) - G_{NOM}(j\omega)| = |\Delta(j\omega)W_\Delta(j\omega)| < |W_\Delta(j\omega)|, \forall\omega$

Tuning	Process	h_{lim}
Standard	test-bed	0.370
	flight	0.114
Optimal on test-bed	test-bed	0.600
	flight	0.189
Optimal in flight	flight	0.173

The control system can be represented by the *two level scheme*

$$T_{\vartheta\psi}(s) = \frac{W_\Delta(s)R(s)}{1 + R(s)G_{NOM}(s)} = W_\Delta(s)V_{NOM}(s)$$

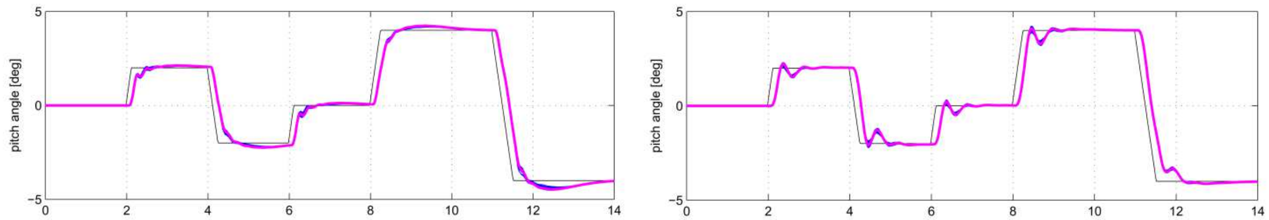
From *small gain theorem* the c.l.s. is stable $\forall\ \Delta(s) \in \mathcal{H}_\infty$ (*i.e.* a stable t.f) with $\|\Delta\|_\infty < \left(\|T_{\vartheta\psi}\|_\infty\right)^{-1}$, hence

$$h_{lim} = (\|W_\Delta(s)V_{NOM}(s)\|_\infty)^{-1}$$

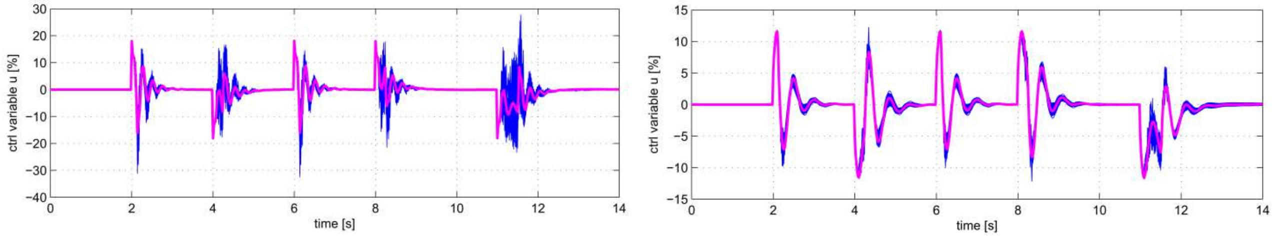
Robust attitude control
Monte Carlo simulation results – on test-bed model

Uncertainty block with imposed peak gain equal to robust stability limit h_{lim}, randomly sampled to generate 1000 Monte Carlo simulations

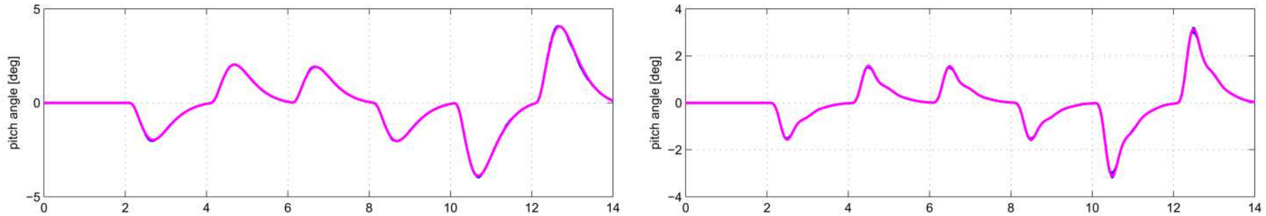
Set-point change



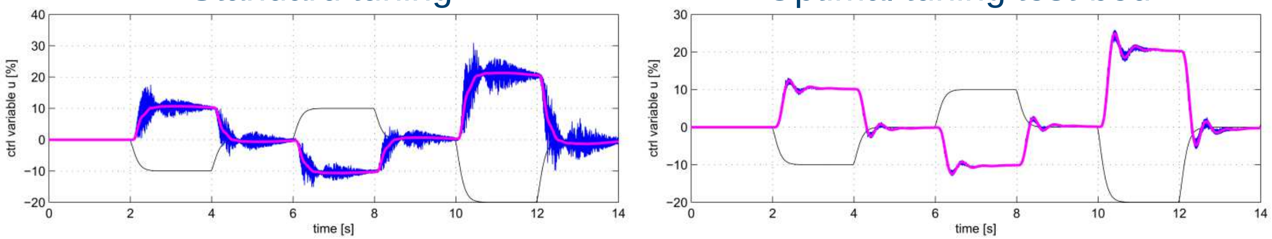
Standard tuning | Optimal tuning test-bed



Load change

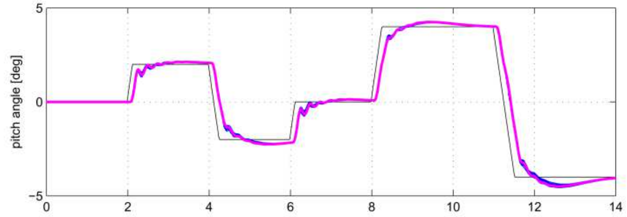


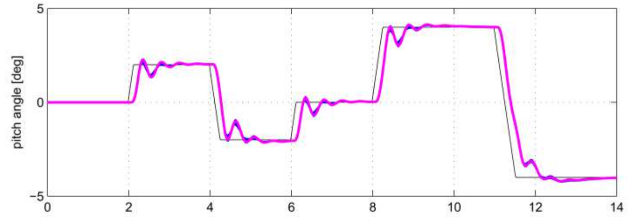
Standard tuning | Optimal tuning test-bed

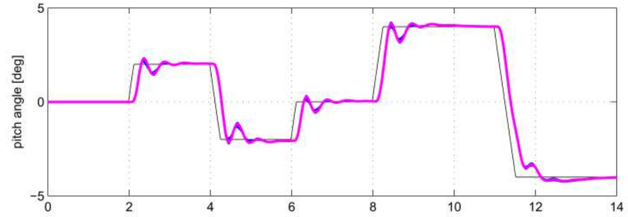


Robust attitude control
Monte Carlo simulation results – in flight model

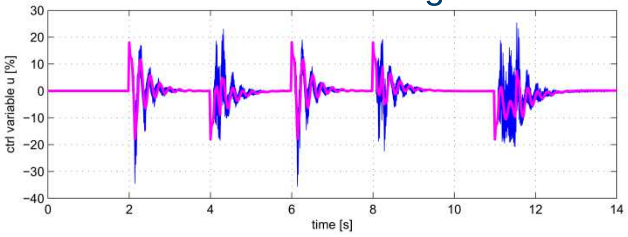
Set-point change

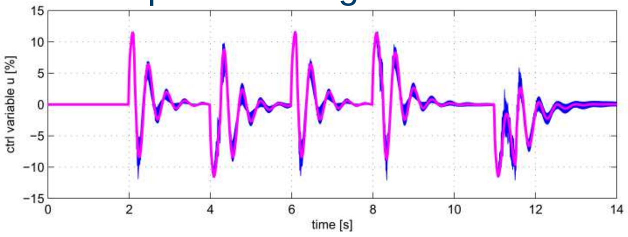


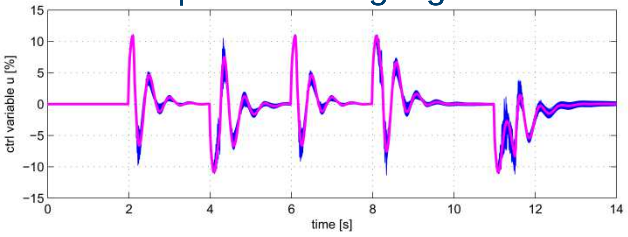




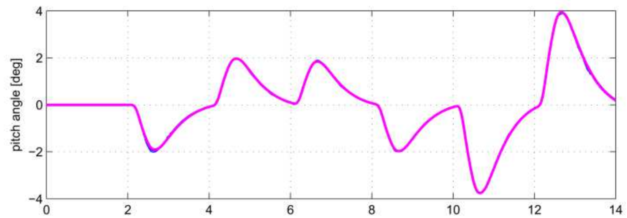
Standard tuning | Optimal tuning test-bed | Optimal tuning flight

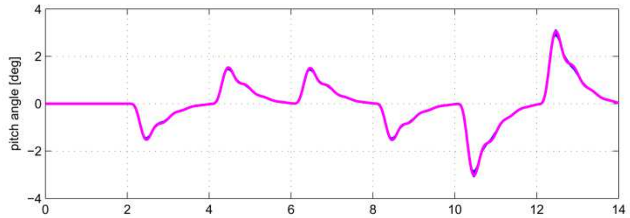


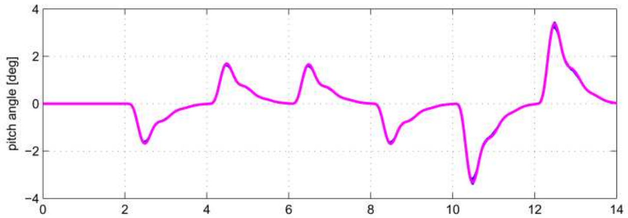




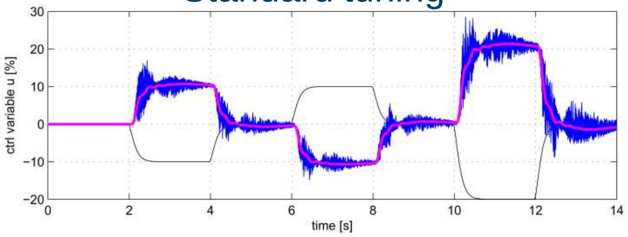
Load change

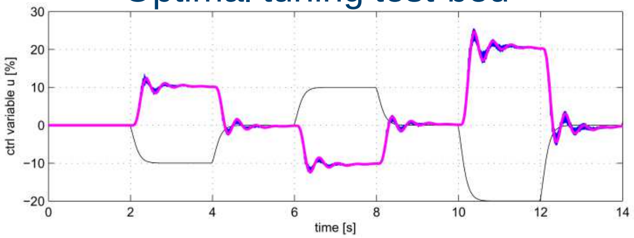


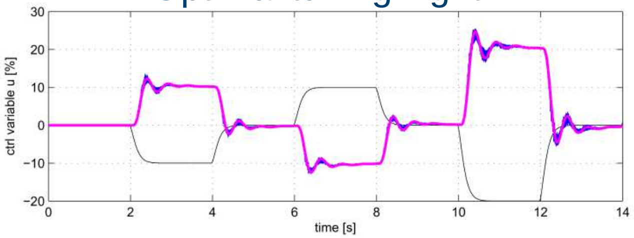




Standard tuning | Optimal tuning test-bed | Optimal tuning flight







Concluding remarks

- The proposed attitude control design procedure, specifically addressed to near hovering condition, was developed and successfully applied to the real case of considered quadrotor pitch DoF: it would be included in AERMATICA control development process

- Simulations demonstrate that structured H_∞ optimal tuning obtained with test-bed model in the loop can be applied also in flight with a non-significant loss in control performance, hence the attitude controller tuning can be achieved using models obtained in safe, faster and more repeatable identification experiments executed indoor