POLITECNICO DI MILANO

Identification of the attitude dynamics for a variable-pitch quadrotor UAV

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Motivation: rotorcraft model identification

- The dynamics of a rotorcraft in steady flight (e.g., hover, forward flight) can be described using a MIMO LTI CT model

$$\dot{x}(t) = A(\theta)x(t) + B(\theta)u(t)$$
$$y(t) = C(\theta)x(t) + D(\theta)u(t)$$

- Intrinsic limitations in physical modelling call for full or partial resort to empirical modelling → increasing attention given to system identification

Problem statement

Identification of the pitch attitude dynamics for the Aermatica Anteos RPA, enabling fast and reliable deployment of control system

Longitudinal control

Pitch angle

Pitch rate

Pitch acceleration

- Variable collective blade pitch – fixed rotor RPM architecture
- MTOW=5kg

State of the art

- Frequency-domain approaches (e.g., the NASA CIFER tool)
  - Advantage: computationally efficient
  - Advantage: easier to deal with unstable systems
  - Drawback: long experiments (frequency sweeps)

- Iterative time-domain approaches (e.g., OE, EE, etc.)
  - Advantage: shorter experiments (e.g., 3211 sequences)
  - Drawback: computationally inefficient
  - Drawback: some “tricks” are needed to deal with unstable systems

- NON-iterative time-domain approaches (e.g., subspace methods)
  - Advantage: computationally efficient and robust
  - Advantage: shorter experiments (3211 sequences)
  - Drawback: no control on state space basis of identified models

Experimental set-up

- The identification experiments have been carried out in laboratory conditions, using a test-bed that constrains all DoFs except pitch rotation
- Similar experiments have been carried out in flight to ensure that indoor setup is representative of actual attitude dynamics in near hovering

Identification experiments (1)

- Pseudo Random Binary Sequences (PRBS) were selected as excitation signal

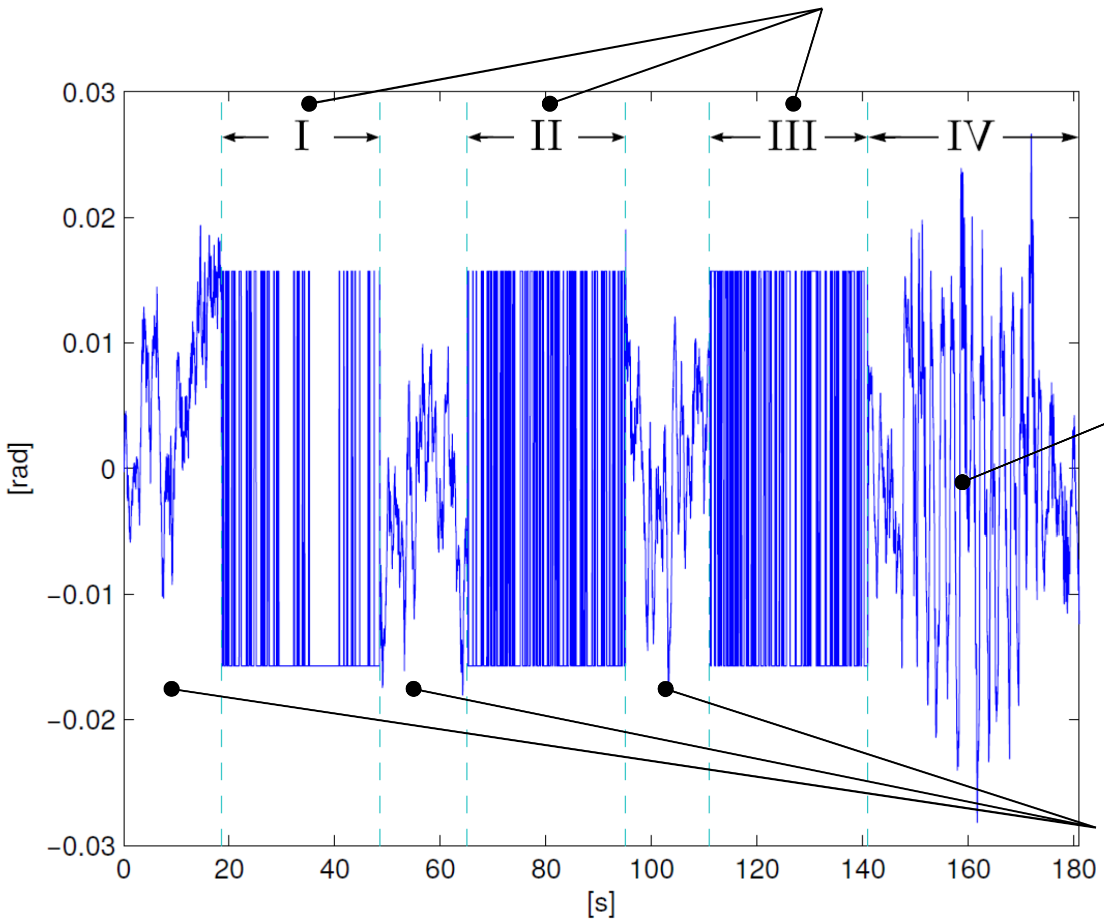
- Experiments have been carried out in quasi open-loop conditions:
  - nominal attitude and position controllers were disabled
  - a supervision task enforcing attitude limits was left active (inherently fast instability)

- The parameters of the PRBS sequence (signal amplitude and min/max switching interval) were tuned to obtain an excitation spectrum consistent with the expected dominant attitude dynamics (3 to 6 rad/s)

Identification experiments (2)

Input signal (collective pitch difference between opposite rotor) in identification test

PRBS excitation + feedback action of supervision task , quasi open-loop
Dataset used for identification

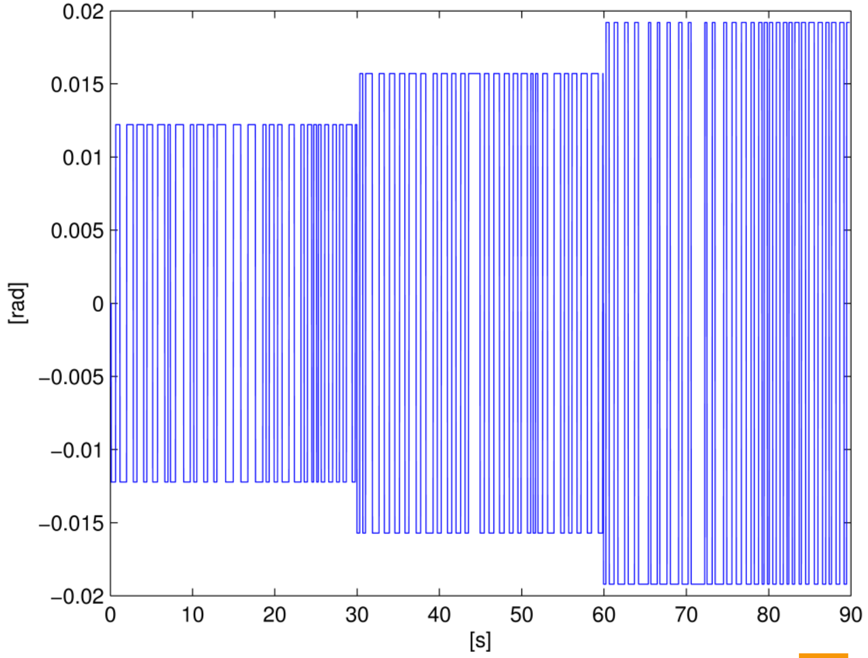


Closed-loop on nominal attitude controller
Imposed pitch angle set-point variations
Model validation dataset

Closed-loop on nominal attitude controller
Pitch angle set-point null
Non-relevant information

Identification experiments (3)

Concatenating 3 excitation segments from different test with ascending amplitude in order to average out mild non linearities from identification process



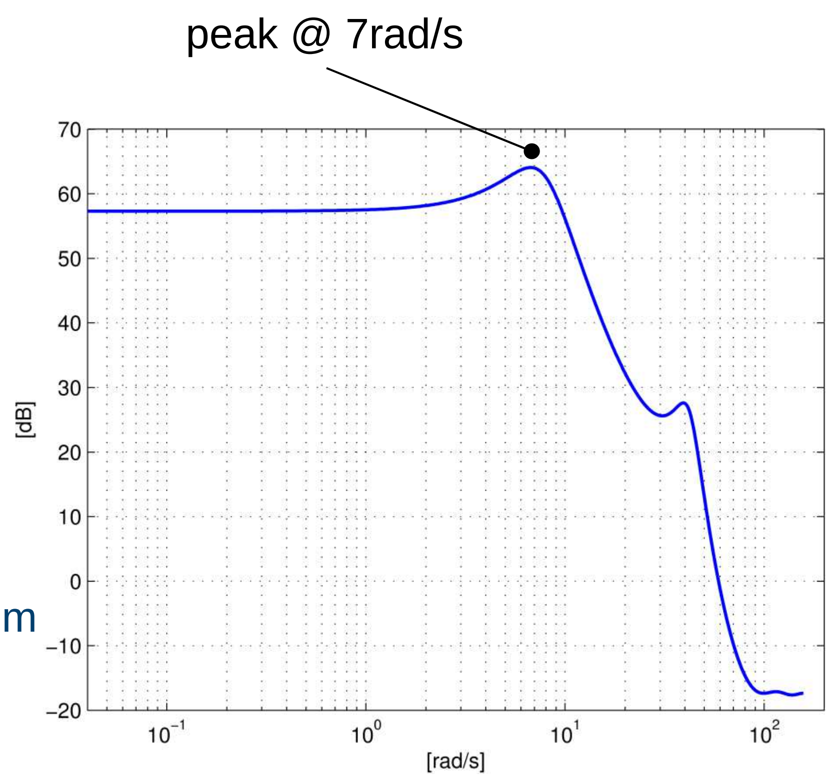
Estimate quality degradation caused by data concatenation is negligible due to large duration of PRBS and small n° of concatenation

Data logged during test @ 50Hz

Input: pitch attitude control variable

Output: $\theta, q, \dot{q}$ from on-board IMU

Excitation spectrum



Considered identification approaches

In this study a number of identification methods have been compared, covering structured and unstructured models, as well as on- and off-line estimation

- Black-box models:
  - State-space: Subspace Model Identification (SMI) → PI-MOESP algorithm (Multivariable Output Error State sPace realization)
  - Input-output: Least Mean-Squares (LMS) algorithm

- Grey box models:
  - Output Error (OE) Maximum Likelihood (ML) estimation
  - H_∞ approach

Black-box models: SMI PI-MOESP algorithm (1)

Consider the discrete time LTI state space model, with $y_t = \tilde{y}_t + v_t$

$$\begin{aligned} x_{t+1} &= Ax_t + Bu_t \\ \tilde{y}_t &= Cx_t + Du_t \end{aligned}$$

- STEP1: estimation of coloumn space of extended observability matrix

$$\Gamma = \left[C^T \ \ (CA)^T \ \ (CA^2)^T \ \ \ldots \ \ \left(CA^{i-1}\right)^T\right]^T$$

from measured input-output sample $\{u_t, y_t\}$, through the data equation

$$Y_{t,i,j} = \Gamma X_{t,j} + HU_{t,i,j}$$

relating (block) Hankel matrices constructed from I/O samples as

$$Y_{t,i,j} = \begin{bmatrix} y_t & \cdots & y_{t+j-1} \\ \vdots & \ddots & \vdots \\ y_{t+i-1} & \cdots & y_{t+i+j-2} \end{bmatrix} \quad U_{t,i,j} = \begin{bmatrix} u_t & \cdots & u_{t+j-1} \\ \vdots & \ddots & \vdots \\ u_{t+i-1} & \cdots & u_{t+i+j-2} \end{bmatrix}$$

$$X_{t,j} = [x_t \ \ x_{t+1} \ \ldots \ x_{t+j-1}]$$

and the block-Toeplitz matrix

$$H = \begin{bmatrix} D & 0 & \ldots & 0 \\ CB & D & \ldots & 0 \\ CAB & CB & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ CA^{i-2}B & CA^{i-3}B & \ldots & D \end{bmatrix}$$

Black-box models: SMI PI-MOESP algorithm (2)

From the data equation the PI-MOESP algorithm (Verhaegen & Dewilde 1991) considers the RQ factorization

$$\begin{bmatrix} U_{t+1,i,j} \\ U_{t,i,j} \\ Y_{t+1,i,j} \end{bmatrix} = \begin{bmatrix} R_{11} & 0 & 0 \\ R_{21} & R_{22} & 0 \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix}$$

and a consistent estimate of the coloumn space of Γ is obtained via SVD of matrix R_{32} under assumption that v_t is Gaussian measurement noise, zero mean, uncorrelated with u_t

- STEP2: from the Γ estimate, matrices A and C of the model can be determined exploiting the invariance of observability subspace
- STEP3: solve a linear least square problem to determine B and D matrices

- SMI was proposed about 25 years ago to handle black-box MIMO problems in a numerically stable way and has proved extremely successful in a number of industrial applications

- Main downside: impossibility to impose a fixed basis to the state space representation, i.e., the identified models are unstructured

Black-box models:
Least Mean Squares algorithm

Alternative black-box model identification method adopted: on-line implementation of the Least Mean Squares (LMS) algorithm

Updates recursively on board an estimate of the SISO impulse response of pitch angular velocity in the form of the Finite Impulse Response (FIR) model

$$y_t = w_1 u_{t-1} + w_2 u_{t-2} + \cdots + w_r u_{t-r}$$

State space model for the pitch dynamics recovered from estimated impulse response w_i $(i = 1, \dots, r)$ via suitable realization techniques (Kung's algorithm)

Remark: time delay in system dynamics (from IMU measurements to servo actuation of rotors collective pitch $\hat{\tau}$ = 0.06s = 3 samples) leads to a non-minimum phase zero in identified model via PI-MOESP → applied forward shift of 3 samples on input signal before model identification and reintroduced as delay in model simulations

Grey-box models: model structure

Grey-box model have structured parametrization derived from first principle approach: quadrotor modeled as rigid body with rotors aerodynamic terms from combined momentum theory and BET

Pitch attitude dynamics on test-bed (all other DoF's constrained) is defined as

$$I_{yy}\dot{q}(t) = \frac{\partial M}{\partial q} q(t) + \frac{\partial M}{\partial u} u(t-\hat{\tau})$$
$$\dot{\theta}(t) = q(t)$$

where stability and control derivative of pitch moment M in hovering trim are

$$\frac{\partial M}{\partial q} = -2\rho A(\Omega R)^2 \frac{\partial C_T}{\partial q} d \quad ; \quad \frac{\partial C_T}{\partial q} = \frac{C_l/\alpha}{8}\frac{\sigma}{\Omega R} \quad ; \quad \frac{\partial M}{\partial u} = \rho A_b (\Omega R)^2 \frac{\partial\, C_T/\sigma}{\partial \theta_R} d$$

Adding a rotational mass-spring-damper to modeling IMU vibration damping system through which the device is connected to vehicle, equations becomes

$$I_{yy}\dot{q}(t) = \frac{\partial M}{\partial q} q(t) + \frac{\partial M}{\partial u} u(t-\hat{\tau}) + k(\theta_P(t) - \theta(t)) + c(q_P(t) - q(t))$$
$$J\dot{q}_P(t) = -k\big(\theta_P(t) - \theta(t)\big) - c(q_P(t) - q(t))$$
$$\dot{\theta}(t) = q(t)$$
$$\dot{\theta}_P(t) = q_P(t)$$

Grey box models: Output Error Maximum Likelihood

Rewriting the equations in state space form, obtain continuous time LTI model

$$\dot{x}(t) = A(x(t) + Bu(t-\hat{\tau})$$
$$y(t) = Cx(t) + Du(t-\hat{\tau}) + v(t)$$

where the state vector is $x(t) = [q(t) \;\; q_P(t) \;\; \theta(t) \;\; \theta_P(t)]^T$ and matrices are

$$A = \begin{bmatrix} 1/I_{yy}\,\partial M/\partial q & -c/I_{yy} & -k/I_{yy} & k/I_{yy} \\ c/J & -c/J & k/J & -k/J \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1/I_{yy}\,\partial M/\partial u \\ 0 \\ 0 \\ 0 \end{bmatrix}$$
$$C = [0 \quad 1 \quad 0 \quad 0] \qquad D = [0]$$

The quadrotor inertia was obtained from previously dedicated identification procedure, then the unknown parameters of structured model are

$$\Theta = \left(\frac{\partial M}{\partial q}, \frac{\partial M}{\partial u}, J, k, c\right)$$

Given sampled I/O dataset $\{u_t, y_t\}$ the ML estimate is equal to the value of Θ that maximizes the likelihood function, defined as the probability density function of y given $\Theta \;\rightarrow\; \mathbb{L}(y,\Theta) = P(y|\Theta)$

If $P(y)$ is Gaussian, as the measurement noise $v(t)$, the ML estimator minimize a positive function of the prediction error

Grey box models: H_∞ approach rationale

- SMI methods are more attractive than OE because of non-iterative nature
- On black-box model obtained via SMI is not possible to enforce a-priori knowledge of model structure and then recover numerical values of physical parameters, naturally allowed by grey-box approach
- Not possible to initialize the OE iteration using model from SMI because the state space basis of black-box model is different and non physical

Need: bridge the gap between structured and unstructured model

H_∞ model matching problem in frequency domain, relating black-box unstructured model from SMI to structured one (with unknown parameters to be determined) from first principle approach

Novel identification procedure proposed by Bergamasco & Lovera 2013

Resulting non-convex, non-smooth optimization problem is solved exploiting computational tool developed by Apkarian & Noll 2006 (available in Matlab from R2012a)

Unstructured black-box LTI discrete time model from PI-MOESP

convert to continuous time (zero order hold)

$$\dot{x}(t) = A_{ns}x(t) + B_{ns}u(t)$$
$$y(t) = C_{ns}x(t) + D_{ns}u(t)$$

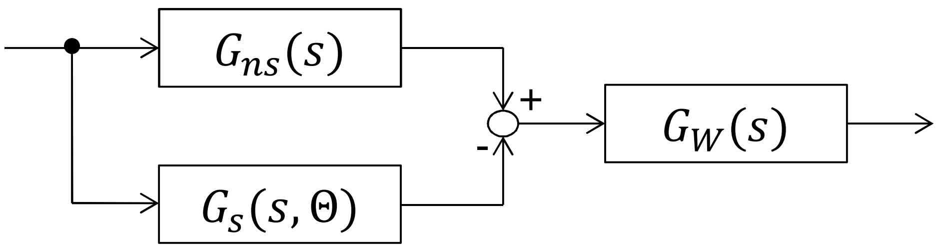
convert to tf in Laplace domain

$$G_{ns}(s)$$

Structured LTI time continuous model from first principle

$$\dot{x}(t) = A_s(\Theta)x(t) + B_s(\Theta)u(t)$$
$$y(t) = C_s(\Theta)x(t) + D_s(\Theta)u(t)$$

$$G_s(s,\Theta)$$



$$\hat{\Theta} = \arg\min_{\Theta} \| G_W(s)(G_{ns}(s) - G_s(s,\Theta)) \|_\infty$$

Introduced a suitable filter G_W to focus the matching in the frequency range where $G_{ns}(s)$ well describes the real system, then in PRBS exitation spectrum

Since the time delay in system dynamics was removed before SMI, in structured model was set $\hat{\tau} = 0$ for the model matching (and reintroduced later in validation)

Results and validation: introduction

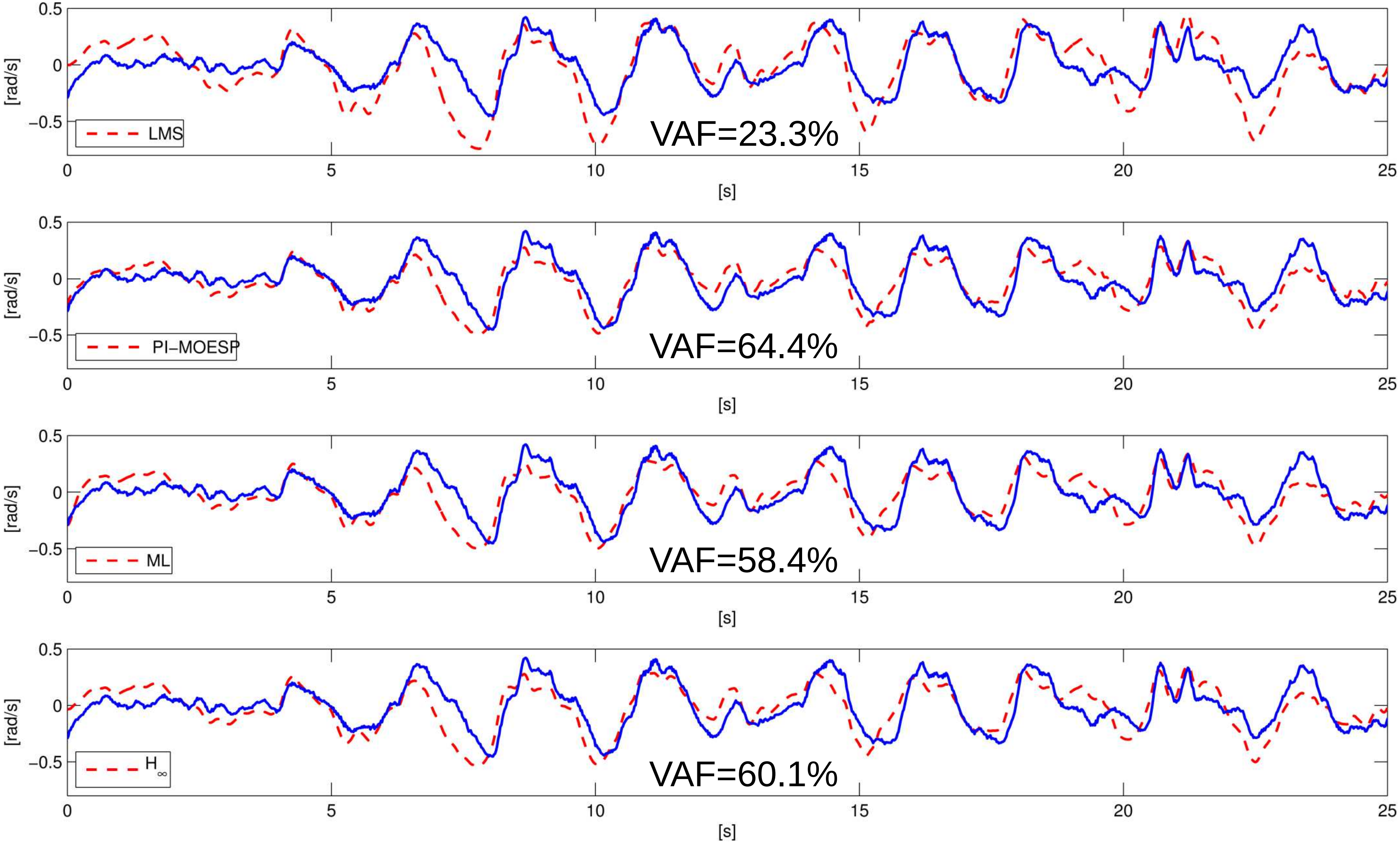
- PI-MOESP
  - applied to identify SISO model of quadrotor pitch angular rate
  - algorithm parameters model order *n* and rows n° of Hankel I/O matrices *p* tuned to obtain best identification results on cross-validation dataset portion in terms on Variance Accounted For (VAF) → *n=5, p=40*
- H_∞ approach
  - filter G_W tuned to reach best VAF performance on cross-validation dataset: adopted a 15th order low pass Butterworth, cut-off of 7rad/s (complies with excitation spectrum peak)

Considered 2 different dataset for validation and models performance comparison:

- Normal closed-loop operation of pitch attitude control system representing typical flight condition: selected randomly one of the final phase between the available identification tests
- Cross-validation: single PRBS excitation phase not employed in the identification process (dataset used for algorithms parameters tuning)

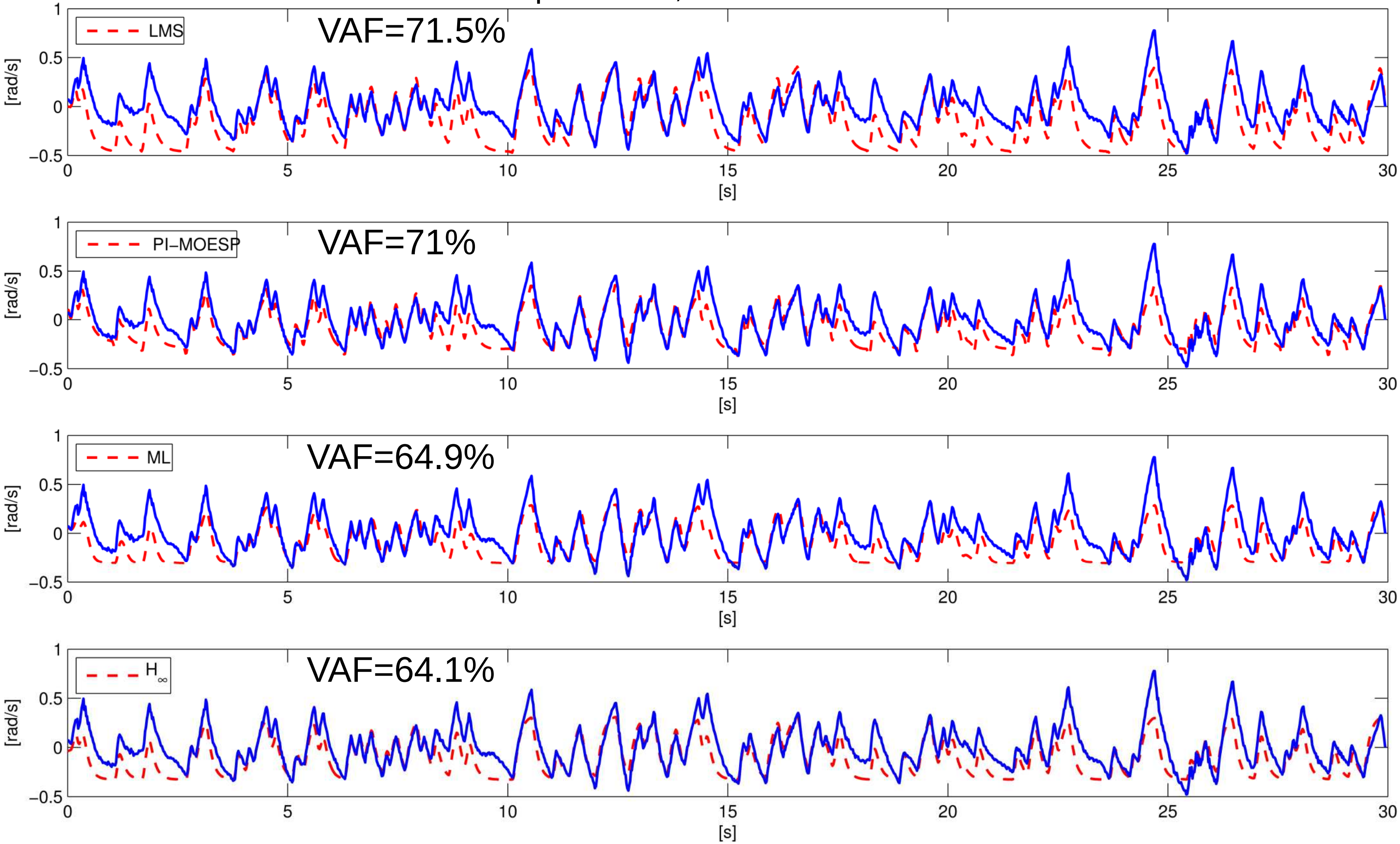
Results and validation: normal closed-loop operation validation dataset

blue lines: measured pitch rate ; red dashed lines: models simulation

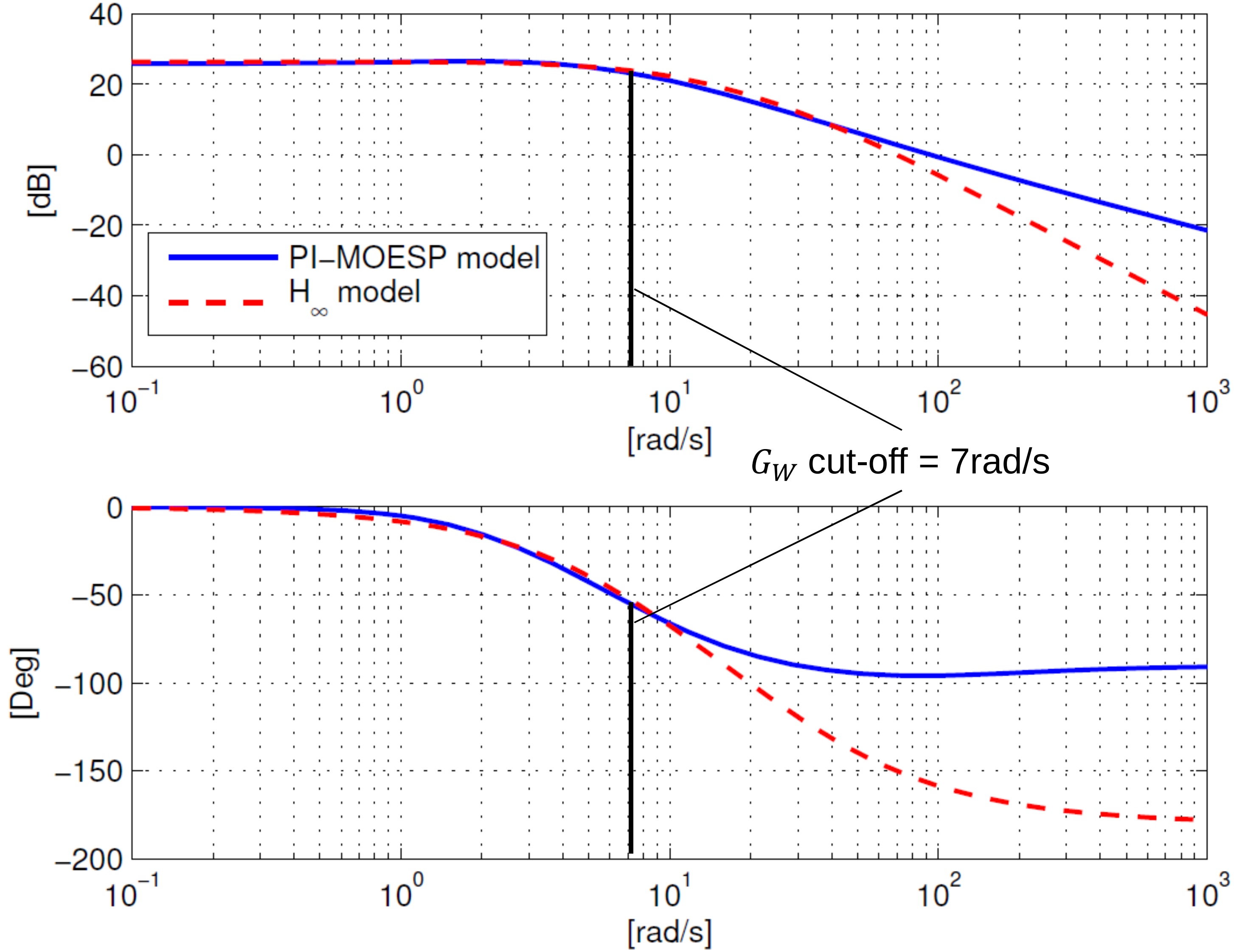


Results and validation: excitation cross-validation dataset

blue lines: measured pitch rate ; red dashed lines: models simulation



Results and validation: structured vs unstructured model



Results and validation: discussion

- Black-box models
  - LMS provides the best performance on excitation cross-validation that degrades significantly on normal operation dataset (worst result): algorithm deeply tied with identification signal, poor generalization capability
  - PI-MOESP guarantees the best VAF on normal operation dataset and almost the best on excitation: benchmark performance

- Grey-box models
  - well known in literature that leads to inferior performance respect to black-box, also for full-scale rotorcraft
  - both OE ML and H_∞ approach perform only slightly less satisfactorily than PI-MOESP, and outperform LMS on normal operation dataset
  - from Bode diagram comparison, the H_∞ model match accurately the PI-MOESP frequency response before the filter cut-off imposed
  - in time domain, with cross-validation dataset as input, the VAF between H_∞ model and PI-MOESP is 96.9%

Concluding remarks

- The problem of characterizing the pitch attitude dynamics of a variable-pitch quadrotor has been considered, applying a number of approaches to its identification from data gathered in laboratory condition

- SMI approach, in view of both its non-iterative nature and capability to replicate accurately experimental data, appears to be the best choice

- H_∞ approach as “structuring” step of SMI black-box model allow to retaining a physically-motivated state space and guarantees acceptable performance in validation