Identification of control-oriented models for helicopter flight mechanics

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• Control-oriented physical model identification
• Frequency- and time- domain approaches: advantages and drawbacks
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• Outline of proposed time/frequency domain approach
• Case studies:
  • A small, rpm-controlled quadrotor
  • A not-so-small, pitch-controlled quadrotor
  • A full scale helicopter.
Most helicopters are characterized by an unstable behaviour.

Helicopter control systems design needs accurate models.

Intrinsic limitations in nonlinear physical modelling call for full or partial resort to empirical modelling → increasing attention given to system identification.
Main difficulties in rotorcraft model identification:

- Intrinsically multivariable (MIMO) problem
- High order dynamics
- Most rotorcraft vehicles are open loop unstable
  - need for closed-loop identification techniques
- Community wants continuous-time, physically parameterised models
  - need for continuous-time identification techniques
- Expensive flight experiments
  - need to use all available flight data

Objective:

- Continuous-time identification algorithm able to deal with closed-loop MIMO systems using time- and frequency-domain data
Rotorcraft model identification

Typical Input classes

Time-domain data

Frequency-domain data

DLR 3211

Automatic SWEEP

Time

Lateral input

Time

Lateral input

$10 \leq s \leq 100$

DOUBLET

Manual SWEEP

Time

Lateral input

Time

Lateral input

$\geq 100$
The dynamics of a rotorcraft during steady flight (e.g., hover, forward flight) can be well described using a MIMO LTI continuous-time system:

\[
\dot{x}(t) = A(\theta)x(t) + B(\theta)u(t)
\]

\[
y(t) = C(\theta)x(t) + D(\theta)u(t)
\]

where the system matrices depend on unknown parameters (i.e., physical parameters).

The objective is to estimate the unknown parameters \( \theta \).
Iterative time-domain approaches
(e.g., OE, EE, see Jategaonkar 2006)

\[
\min_{\theta} J(\theta) = \sum_{k=1}^{N} (y(t_k) - \hat{y}(t_k, \theta))^{T} R^{-1}(y(t_k) - \hat{y}(t_k, \theta))
\]

Grey-Box TD Model identification (OE)
Iterative time-domain approaches (e.g., OE, EE, see Jategaonkar 2006)

Outline

\[
\min_{\theta} J(\theta) = \sum_{k=1}^{N} (y(t_k) - \hat{y}(t_k, \theta))^T R^{-1} \hat{y}(t_k) - \hat{y}(t_k, \theta))
\]

- **Advantage**: shorter, cheaper and safer experiments (DLR 3211 sequences)
- **Drawback**: computationally slow (a lot of data-points)
- **Drawback**: simulation of unstable models
- **Drawback**: initial guess needed.
Frequency-domain approaches
\(\text{(e.g., CIFER, see Tischler and Remple 2006)}\)

**Outline**

\[
\min_{\theta} J(\theta) = \sum_{i=1}^{n_{\omega}} W(\omega_i) \left[ \left| G_s(\omega_i, \theta) - |\hat{T}(\omega_i)| \right|^2 + W_p \left( \angle G_s(\omega_i, \theta) - \angle \hat{T}(\omega_i) \right)^2 \right]
\]
Frequency-domain approaches (e.g., CIFER, see Tischler and Remple 2006)

- **Advantage**: computationally **fast** (few data-points)
- **Advantage**: deal with unstable system in a very natural way (phase signs)
- **Drawback**: long and **costly** experiments (sweeps)
- **Drawback**: initial guess needed.

Outline

\[
\min_{\theta} J(\theta) = \sum_{i=1}^{n} W(\omega_i) \left[ \left| G_i(\omega_i, \theta) - \hat{T}(\omega_i) \right|^2 + W_p \left( \angle G_i(\omega_i, \theta) - \angle \hat{T}(\omega_i) \right)^2 \right]
\]
Non-iterative time-domain approach
(e.g., CT subspace model identification methods)

- In the system identification community Subspace Model Identification (SMI) was proposed about 25 years ago to handle black-box MIMO problems in a numerically stable way.
- SMI has proved extremely successful in a number of industrial applications.
- The discrete-time case has been studied extensively.
- The continuous-time case has been investigated in a number of contributions, mainly for the open-loop setting.
- Main downside: impossibility to impose a fixed basis to the state space representation, i.e., the identified models are unstructured.
Non-iterative time-domain approach (e.g., CT subspace methods, see Bergamasco and Lovera)

Advantage: shorter, cheaper, and safer experiments
Advantage: computationally efficient and robust
Advantage: no model structure and initial guess (high order model can be eventually considered)
Drawback: no control on state space basis of identified models, i.e., no physical model.
Continuous-time PBSID algorithm
Model class, assumptions, approach

- Consider the MIMO LTI continuous-time system
  \[ dx(t) = Ax(t)dt + Bu(t)dt + Kde(t), \quad x(0) = x_0 \]
  \[ y(t)dt = Cx(t)dt + Du(t)dt + de(t) \]
  (in innovation form for simplicity) where \( x \in \mathbb{R}^n, u \in \mathbb{R}^m, y \in \mathbb{R}^p \)

  
  Assumptions

  - \( de(t) \) Wiener process
  - \((A,B,C,D,K)\) such that \((A,C)\) observable and \((A,[B\ K])\) controllable
  - system possibly operating in closed-loop

  Approach

  - Convert the model to discrete-time via an **exact** signals-based method
  - Apply the discrete-time PBSID SMI algorithm
  - Retrieve the original continuous-time model, *i.e.*, \((A,B,C,D,K)\)
Continuous-time PBSID algorithm

Definitions

• Consider the first order all-pass transfer function

\[ w(s) = \frac{s - a}{s + a}, \quad a > 0 \]

• \( w(s) \) generates the family of Laguerre filters, defined as

\[ \mathcal{L}_i(s) = w^i(s)\mathcal{L}_0(s), \quad \mathcal{L}_0(s) = \sqrt{2a}\frac{1}{(s + a)} \]

• Denote with \( \ell_i(t) \) the impulse response of the \( i \)-th Laguerre filter. The set

\[ \{\ell_0, \ell_1, \ldots, \ell_i, \ldots\} \]

is an orthonormal basis of \( L_2(0,1) \).
\[ dx(t) = Ax(t)dt + Bu(t)dt + Kde(t) \]
\[ y(t)dt = Cx(t)dt + Du(t)dt + de(t) \]

Matrix transformations

\[ A_o = (A - aI)^{-1}(A + aI) \]
\[ B_o = \sqrt{2a}(A - aI)^{-1}B \]
\[ C_o = -\sqrt{2aC}(A - aI)^{-1} \]
\[ D_o = D - C(A - aI)^{-1}B \]

Signal projections

\[ \tilde{u}(k) = \int_0^\infty \ell_k(t)u(t)dt \]
\[ \tilde{y}(k) = \int_0^\infty \ell_k(t)y(t)dt \]
\[ \tilde{e}(k) = \int_0^\infty \ell_k(t)de(t) \]

\[ \xi(k + 1) = A_o\xi(k) + B_o\tilde{u}(k) + K_o\tilde{e}(k) \]
\[ \tilde{y}(k) = C_o\xi(k) + D_o\tilde{u}(k) + \tilde{e}(k) \]

Discrete index $k$: basis order
Consider the discrete-time system

\[
\xi(k + 1) = A_o \xi(k) + B_o \tilde{u}(k) + K_o \tilde{e}(k)
\]
\[
\tilde{y}(k) = C_o \xi(k) + D_o \tilde{u}(k) + \tilde{e}(k)
\]

Closed-loop predictor matrices

\[
\tilde{A}_o = A_o - K_o C_o
\]
\[
\tilde{B}_o = B_o - K_o D_o
\]

\[
\tilde{z}(k) = \begin{bmatrix} \tilde{u}(k) \\ \tilde{y}(k) \end{bmatrix}, \quad \tilde{B}_o = \begin{bmatrix} \tilde{B}_o & K_o \end{bmatrix}
\]

Prediction Form

\[
\xi(k + 1) = \tilde{A}_o \xi(k) + \tilde{B}_o \tilde{z}(k)
\]
\[
\tilde{y}(k) = C_o \xi(k) + D_o \tilde{u}(k) + \tilde{e}(k)
\]
The discrete-time PBSID algorithm: the data equation

Iterating $p-1$ times the state equation one gets

$$
\xi(k + 2) = A_o^2 \xi(k) + \begin{bmatrix} A_o & B_o \end{bmatrix} \begin{bmatrix} \tilde{z}(k) \\ \tilde{z}(k + 1) \end{bmatrix}
$$

\[ \vdots \]

$$
\xi(k + p) = A_o^p \xi(k) + \mathcal{K}^p Z^{0,p-1}
$$

where

$$
\mathcal{K}^p = \begin{bmatrix} A_o^{p-1} & B_0 & \ldots & B_o \end{bmatrix}
$$

and

$$
Z^{0,p-1} = \begin{bmatrix} \tilde{z}(k) \\ \vdots \\ \tilde{z}(k + p - 1) \end{bmatrix}
$$

Extended controllability matrix

Input-output “past” data
The discrete-time PBSID algorithm: the data equation

- The predictor is AS by assumption, so
  \[ A_0^p \xi(k) \simeq 0 \]
  for sufficiently large values of \( p \) and
  \[ \xi(k + p) \simeq \mathcal{K}^p Z^{0,p-1} \]

- Then, the input-output behaviour of the system is given by the data equation:
  \[
  \tilde{y}(k + p) \simeq C_0 \mathcal{K}^p Z^{0,p-1} + D_0 \tilde{u}(k + p) + \tilde{e}(k + p)
  \]
  \[
  \vdots
  \]
  \[
  \tilde{y}(k + p + f) \simeq C_0 \mathcal{K}^p Z^{f,p+f-1} + D_0 \tilde{u}(k + p + f) + \tilde{e}(k + p + f)
  \]

- Finally, the state space matrices can be recovered from the data equation using Least Squares techniques.

\( p \): past window length
\( f \): future window length
Continuous-time PBSID algorithm

Summary

Data collection

\((u(t_i), y(t_i))\)

Signal projections

\((\tilde{u}(k), \tilde{y}(k))\)

Discrete-time data

\[\begin{align*}
\dot{x}(t) &= \tilde{A}x(t) + \tilde{B}u(t) \\
y(t) &= \tilde{C}x(t) + \tilde{D}u(t)
\end{align*}\]

Black-box continuous-time identified model

PBSID algorithm

\[\begin{align*}
\xi(k+1) &= A_0\xi(k) + B_0\tilde{u}(k) \\
\tilde{y}(k) &= C_0\xi(k) + D_0\tilde{u}(k)
\end{align*}\]

Matrix transformations

Discrete-time identified model
Continuous-time PBSID algorithm: comments

- The computation of the signals transformations

\[ \tilde{u}(k) = \int_0^\infty \ell_k(t)u(t)\,dt \quad \tilde{y}(k) = \int_0^\infty \ell_k(t)y(t)\,dt \]

allows to deal with \textit{non uniform sampling}.

- Data from \textit{different experiments} can be naturally \textit{merged} in the identification procedure.

- The identification algorithm is based on QR and SVD factorisations (very efficient implementations are available in Matlab).
Continuous-time PBSID algorithm
Estimation of model uncertainty

- The asymptotic theory of SMI methods has been studied extensively
- Estimates are asymptotically Gaussian
- Expressions for the asymptotic variance are extremely cumbersome (see Chiuso 2005, Chiuso 2007, van Wingerden 2012).
- Proposed approach: use the bootstrap method to estimate model uncertainty (along the lines of Bittanti, Lovera 2000)
- Analysis of the bootstrap method for CT SMI is ongoing.
Continuous-time PBSID algorithm
Estimation of model uncertainty: a bootstrap-based approach

\[(u_i, y_i) \quad i = 0, \ldots, N\]

\[\begin{align*}
  e_i^{(1)} & \quad e_i^{(2)} & \quad e_i^{(4)} \\
  e_i^{(3)} & \quad e_i^{(\cdots)} & \quad e_i^{(M)} \\
\end{align*}\]

\[\begin{align*}
  \begin{pmatrix}
    u_i, e_i^{(1)}, \tilde{y}_i^{(1)} \\
    u_i, e_i^{(\cdots)}, \tilde{y}_i^{(\cdots)} \\
    u_i, e_i^{(2)}, \tilde{y}_i^{(2)} \\
    u_i, e_i^{(M)}, \tilde{y}_i^{(M)}
  \end{pmatrix}
\end{align*}\]

\[\dot{x} = \hat{A}x + \hat{B}u + \hat{K}e\]
\[y = \hat{C}x + \hat{D}u + e\]
\[e \sim \mathcal{N}(0, \sigma_e^2)\]

\[\begin{pmatrix}
  \hat{A} & \hat{B} \\
  \hat{C} & \hat{D}
\end{pmatrix}^{(1)}\]
\[\begin{pmatrix}
  \hat{A} & \hat{B} \\
  \hat{C} & \hat{D}
\end{pmatrix}^{(M)}\]
\[\begin{pmatrix}
  \hat{A} & \hat{B} \\
  \hat{C} & \hat{D}
\end{pmatrix}^{(\cdots)}\]
From unstructured to structured models

- SMI enabled the possibility of dealing with MIMO state space identification in a simple and natural way.

- Downside: it is hard to impose a fixed basis to the state space representation. Therefore, it is hard to
  - impose a parameterisation to the state space matrices
  - exploit prior knowledge
  - recover numerical values for physical parameters.

- The problem has been recently addressed in, e.g., Xie & Ljung 2002, Parrilo & Ljung 2003, Prot et al. 2012, by solving the bilinear equations resulting from the definition of state space similarity transformations.
Black-box to grey-box model transformation in the frequency-domain

- **Black-box identified model**
  \[ \dot{x}(t) = \hat{A}x(t) + \hat{B}u(t) \]
  \[ y(t) = \hat{C}x(t) + \hat{D}u(t) \]

- **Grey-box model structure**
  \[ \dot{x}(t) = A(\theta)x(t) + B(\theta)u(t) \]
  \[ y(t) = C(\theta)x(t) + D(\theta)u(t) \]

- **$H_\infty$ approach in frequency-domain**
  \[ \theta^* = \arg\min_{\theta} \| \hat{G}_{ns}(s) - G_s(s;\theta) \|_\infty \]

- **Model Error (ME) approach in frequency-domain**
  \[ \min_{\theta} J(\theta) = \sum_{i=1}^{n_\omega} W(\omega_i) \left[ \left| G_s(\omega_i,\theta) - \hat{G}_{ns}(\omega_i) \right|^2 + W_p \left( \angle G_s(\omega_i,\theta) - \angle \hat{G}_{ns}(\omega_i) \right)^2 \right] \]
• The optimization problem can be solved using some recent algorithms available in literature, see Apkarian & Noll 2006 (and in Matlab R2012a, see Gahinet & Apkarian 2011).

• The estimation of the similarity transformation is not necessary (this enables handling of larger problems).

• Frequency-domain data (if available) can be included in the optimization problem.
Black-box to grey-box model transformation in the FD

Outline

1. \( \min_{\hat{A}, \hat{B}, \hat{C}, \hat{D}} J = f(\hat{A}, \hat{B}, \hat{C}, \hat{D}, u(t), y(t)) \)

2. \( \min_{\theta} J(\theta) = \sum_{i=1}^{n_{s}} W(\omega_i) \left[ \left| G_s(\omega_i, \theta) - \hat{G}_{ns}(\omega_i) \right|^2 + W_p \left( \angle G_s(\omega_i, \theta) - \angle \hat{G}_{ns}(\omega_i) \right)^2 \right] \)

- **Advantage:** shorter, **cheaper**, and **safer** experiments (DLR 3211 sequences)
- **Advantage:** computationally **efficient and robust**
Proposed time/frequency domain approach

Outline

1. \[
\min_{\hat{A},\hat{B},\hat{C},\hat{D}} J = f(\hat{A}, \hat{B}, \hat{C}, \hat{D}, u(t), y(t))
\]
   \[
\min_{\theta} J(\theta) = \sum_{i=1}^{n_0} W_i(\omega) \left[ \left| G_s(\omega, \theta) - |\hat{G}_{ns}(\omega)\right|^2 + W_p \left( \angle G_s(\omega, \theta) - \angle \hat{G}_{ns}(\omega) \right)^2 \right]
\]
   \[
   + \sum_{i=1}^{n_0} W_2(\omega) \left[ \left| G_s(\omega, \theta) - |\hat{T}(\omega)\right|^2 + W_p \left( \angle G_s(\omega, \theta) - \angle \hat{T}(\omega) \right)^2 \right]
\]
   \[
   \min_{\theta} J(\theta) = \sum_{k=1}^{N} (y(t_k) - \hat{y}(t_k, \theta))^T R^{-1} (y(t_k) - \hat{y}(t_k, \theta))
\]

- Advantage: FD data (when available) can be eventually included in the optimization problem
- Advantage: all kind of (appropriate) data can be used in the same procedure
Case studies: a small quadrotor

Introduction

Experimental setup

- Mikrokopter platform
- Equipped for outdoor flight
- Sampling onboard at 100Hz
- Automatic excitation
- Attitude control (closed-loop)
Case studies: a small quadrotor
Hover condition

\[ \dot{x}(t) = Ax(t) + Bu(t) + Ke(t) \]
\[ y(t) = Cx(t) + Du(t) + e(t) \]

Stable modes

Unstable modes
Case studies: a small quadrotor
Identification-oriented flight testing

- Manual excitation too slow → automatic excitation
- Input signal:
  3211 piece-wise constant sequence
- Input channels are excited one by one
- Identification phase
  Multiple 3211 datasets (~20s)
- Cross-validation phase (order and parameters selection)
  a 3211 dataset (~7s)
- Validation: response to doublet inputs.
Case studies: a small quadrotor
Collective and yaw models

Collective

Yaw
Case studies: a small quadrotor
Longitudinal and lateral models: TD validation

Longitudinal

Lateral
Case studies: a small quadrotor
Longitudinal and lateral models: Bode plots

**Longitudinal**

\[ \frac{q}{u_{lon}} \]

Output, Input: \( q \) vs. Input: \( u_{lon} \)

Frequency (rad/s)

\[ \frac{a_x}{u_{lon}} \]

Output: \( a_x \) vs. Input: \( u_{lon} \)

Frequency (rad/s)

**Lateral**

\[ \frac{p}{u_{lat}} \]

Output, Input: \( p \) vs. Input: \( u_{lat} \)

Frequency (rad/s)

\[ \frac{a_y}{u_{lat}} \]

Output: \( a_y \) vs. Input: \( u_{lat} \)

Frequency (rad/s)
Problem: identification of the pitch attitude dynamics for the Aermatica Anteos RPA.
Case studies: a not-so-small quadrotor
Experimental data: indoor testing
Pitch response, identified from indoor data
Case studies: a full-scale helicopter

Research project aimed at developing methods and tools for identification of full-scale helicopter flight dynamics.

Results obtained in piloted simulations (flight simulator based based on the FlightLab code).
AW189 model identification
Simulator introduction and data description

• AW189 simulator: nonlinear model with certified FCS, 80kts steady flight condition

• 4 inputs and 8 outputs are considered

\[ u = \begin{bmatrix} \delta_{\text{col}} & \delta_{\text{lat}} & \delta_{\text{lon}} & \delta_{\text{ped}} \end{bmatrix}^T \]

\[ y = \begin{bmatrix} p & q & r & \varphi & a_x & a_y & a_z \end{bmatrix}^T \]

• Each input has been excited separately

• Time-domain data:
  • 8 datasets: 2 for each input channel (DLR3211 for identification, doublet for cross-validation)
  • Manual excitation

• Frequency-domain data:
  • 8 datasets: 2 for each input channel (2xSweep)
  • Automatic excitation
AW189 model identification
Control-oriented physical model

• LTI MIMO model

\[ \dot{x}(t) = A(\theta)x(t) + B(\theta)u(t) \]
\[ y(t) = C(\theta)x(t) + D(\theta)u(t) \]

• where the state vector (6-DOF) is

\[ x = [\varphi \ \vartheta \ u \ v \ w \ p \ q \ r]^T \]

• Physical model has 64 unknowns parameters \( \theta \) to be estimated

• Validation dataset: a manual pseudo-random excitation in closed-loop

• Root Mean Square error of validation dataset as comparison index

\[ RMS_{err} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (y(t_i) - \hat{y}(t_i))^2} \]
• Primary input manually excited using 3211 sequence and frequency sweeps; secondary inputs manually controlled to stay close to trim
• Several repetitions for each manoeuvre are collected
Close to trim?
Case studies: a full-scale helicopter
Model structure

- FLFM: FlightLAB linearised full-model (55 states)
- FLRM: FlightLAB linearised reduced-model (8 states)

\[
M \ddot{x}(t) = F \dot{x}(t) + Gu(t) \\
y(t) = H_0 x(t) + H_1 \dot{x}(t)
\]

- Grey-box model with 64 free parameters is initialized with FLRM

<table>
<thead>
<tr>
<th>Total M Terms</th>
<th>No. Free F Terms</th>
<th>No. Free G Terms</th>
<th>No. Free Tau Terms</th>
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<td>36</td>
<td>24</td>
</tr>
</tbody>
</table>

- State vector  
  \[
x = [\phi \ \dot{\phi} \ u \ v \ w \ p \ q \ r]^T
\]

- Input vector  
  \[
u = [\delta_{col} \ \delta_{lat} \ \delta_{lon} \ \delta_{ped}]^T
\]

- Output vector  
  \[
y = [p \ q \ r \ \phi \ \dot{\phi} \ a_x \ a_y \ a_z]^T
\]
Case studies: a full-scale helicopter
Overall approach

FD data (sweep) → CIFER (FRESPID/ MISOSA/ COMPOSITE) → FRF

TD data (DLR 3211) → CT-PBSID₀ → BB

Model Structure & Initial guess

Grey-Box FD Model identification

GB₁ → GB₂

Grey-Box TD Model identification

GB₃
Case studies: a full-scale helicopter
Validation results

• Root Mean Square Error comparison
  • BB: Black-box identified model
  • GB_1: Grey-box identified model (Step 2a)
  • GB_2: Grey-box identified model (Step 2b)
  • GB_3: Grey-box identified model (Step 3)
  • CIFER
  • Initial Guess
Case studies: a full-scale helicopter
Time-domain comparison
Case studies: a full-scale helicopter
Frequency responses comparison 1/4
Case studies: a full-scale helicopter
Frequency responses comparison 2/4
Case studies: a full-scale helicopter
Frequency responses comparison 3/4

- TET/DCOL
- TET/DLAT
- TET/DLON
- TET/DPED

- PHI/DCOL
- PHI/DLAT
- PHI/DLON
- PHI/DPED

Magnitude (dB) vs. Frequency (rad/sec)
Phase (deg) vs. Frequency (rad/sec)
Case studies: a full-scale helicopter
Frequency responses comparison 4/4
Conclusions

• Rotorcraft model identification has been considered

• An overview of the state-of-the-art of rotorcraft model identification has been provided

• Continuous-time predictor-based subspace identification algorithm and black-box to grey-box transformation have been introduced

• A novel approach combining time and frequency domain data has been presented and discussed

• Preliminary results based on the AW189 simulation example has shown the viability of the proposed approach