POLITECNICO DI MILANO





Identification of control-oriented models for helicopter flight mechanics

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- Rotorcraft model identification: objectives and constraints
- Control-oriented physical model identification
- Frequency- and time- domain approaches: advantages and drawbacks
- Non-iterative time-domain approach: continuous-time predictor-based subspace identification algorithm
- Outline of proposed time/frequency domain approach
- Case studies:
  - A small, rpm-controlled quadrotor
  - A not-so-small, pitch-controlled quadrotor
  - A full scale helicopter.

- Most helicopters are characterized by an unstable behaviour
- Helicopter control systems design needs accurate models
- Intrinsic limitations in nonlinear physical modelling call for full or partial resort to empirical modelling → increasing attention given to system identification

Rotorcraft model identification
Main issues and objectives

Main difficulties in rotorcraft model identification:

- Intrinsically multivariable (MIMO) problem
- High order dynamics
- Most rotorcraft vehicles are open loop unstable
  - need for closed-loop identification techniques
- Community wants continuous-time, physically parameterised models
  - need for continuous-time identification techniques
- Expensive flight experiments
  - need to use all available flight data

Objective:

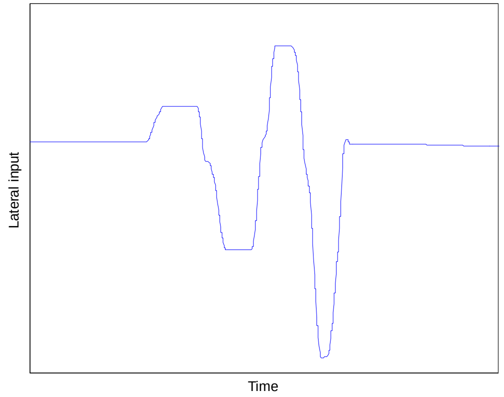
- Continuous-time identification algorithm able to deal with closed-loop MIMO systems using time- and frequency-domain data

Rotorcraft model identification
Typical Input classes

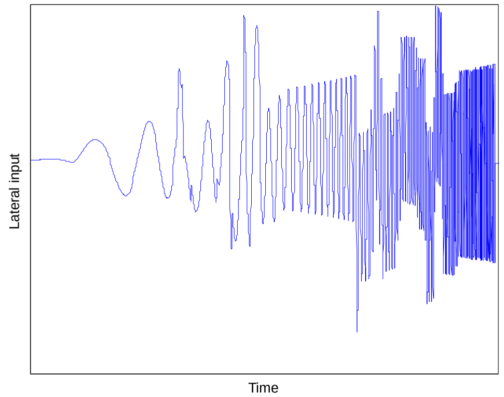
Time-domain data

Frequency-domain data

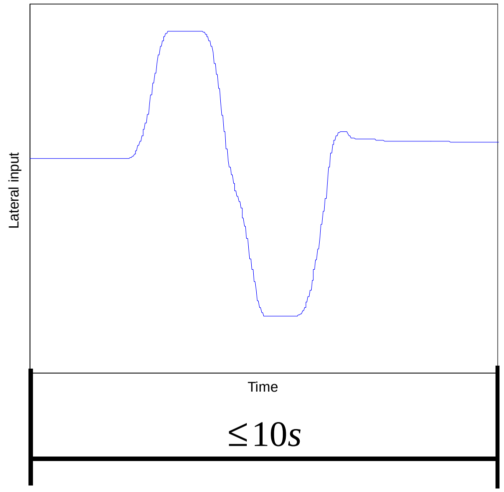
DLR 3211



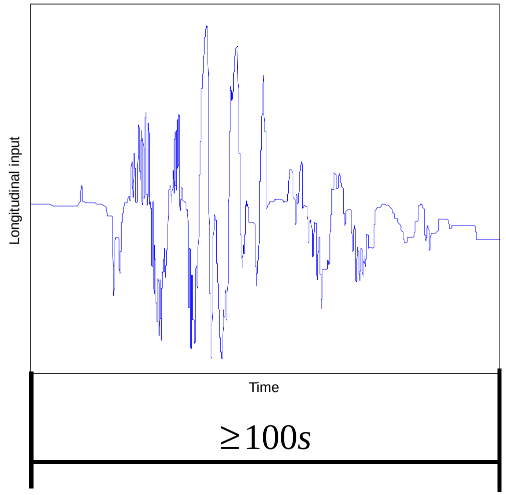
Automatic SWEEP



DOUBLET



Manual SWEEP



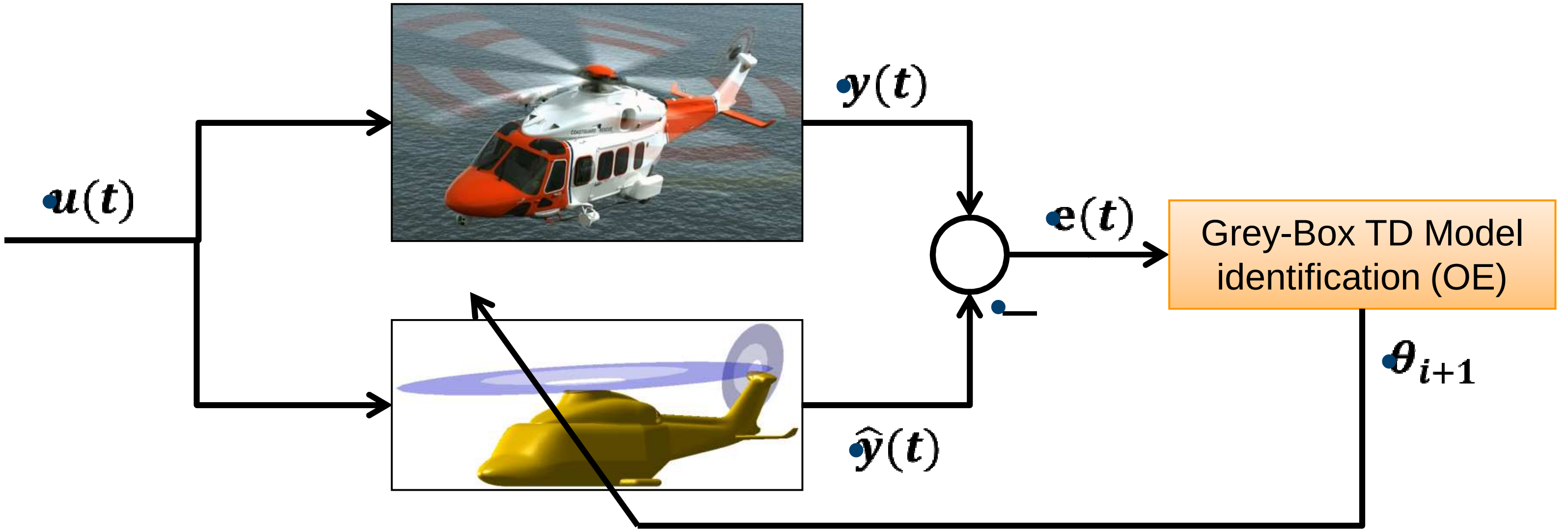
- The dynamics of a rotorcraft during steady flight (*e.g.*, hover, forward flight)

can be well described using a MIMO LTI continuous-time system

$$\dot{x}(t) = A(\theta)x(t) + B(\theta)u(t)$$
$$y(t) = C(\theta)x(t) + D(\theta)u(t)$$

where the system matrices depend on unknown parameters (*i.e.*, physical parameters)

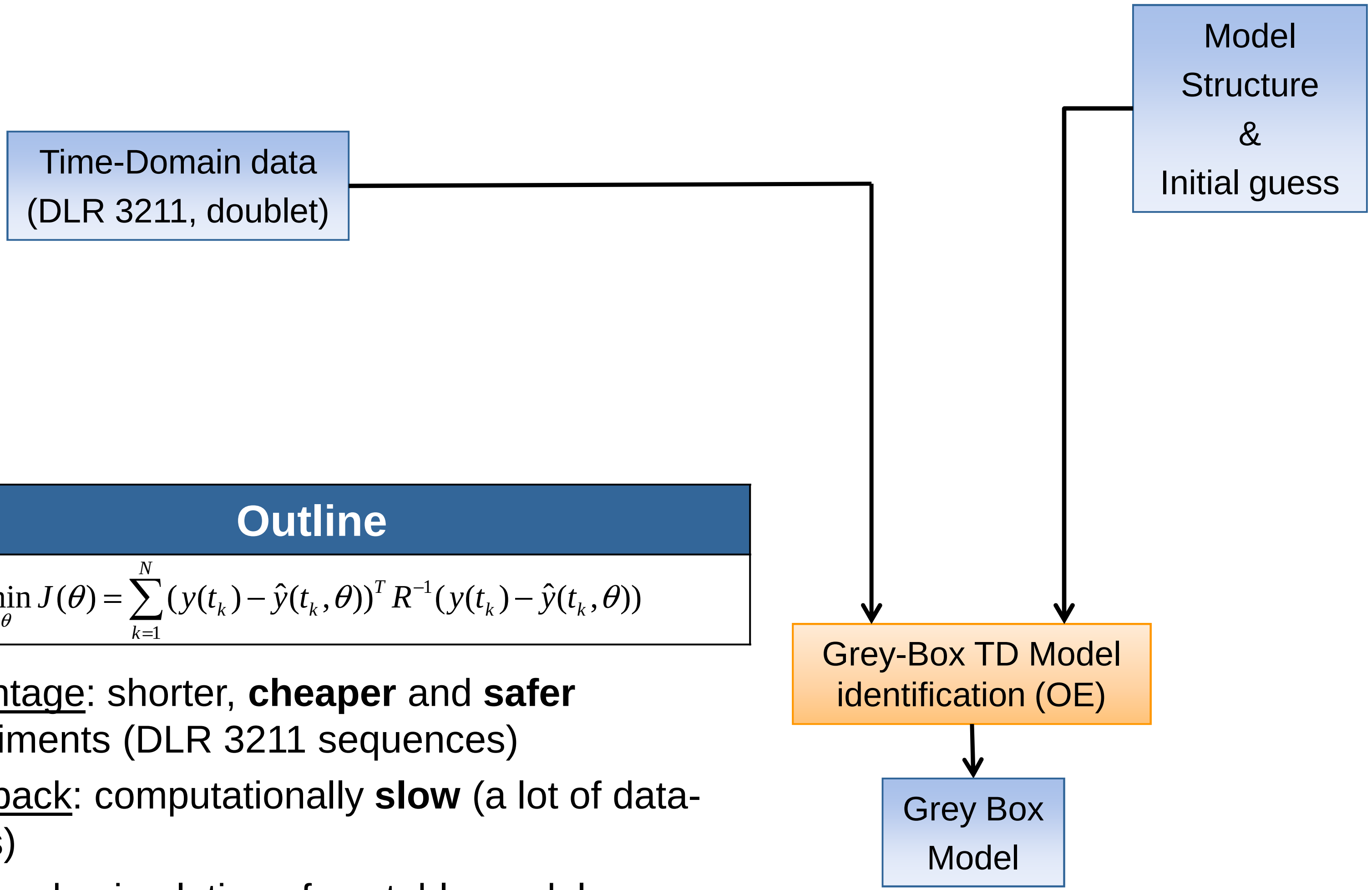
- The objective is to estimate the unknown parameters θ



Outline

$$\min_{\theta} J(\theta) = \sum_{k=1}^{N} (y(t_k) - \hat{y}(t_k,\theta))^T R^{-1} (y(t_k) - \hat{y}(t_k,\theta))$$

Iterative time-domain approaches
(*e.g.*, OE, EE, see Jategaonkar 2006)

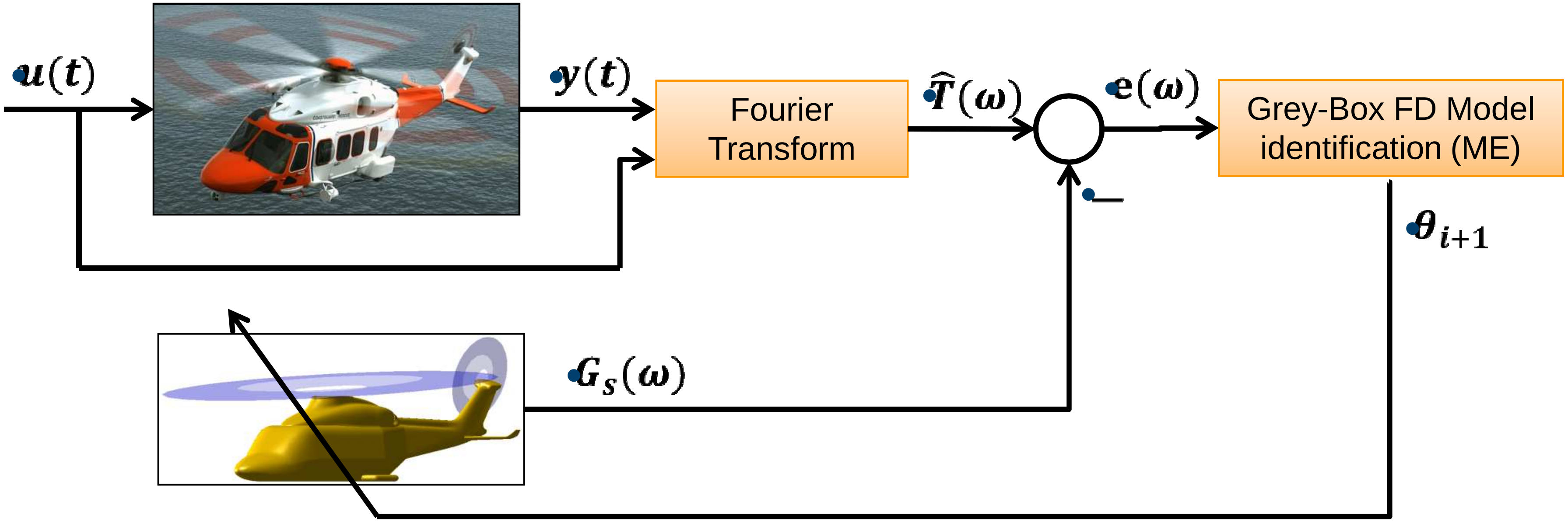


Outline

$$\min_{\theta} J(\theta) = \sum_{k=1}^{N} (y(t_k) - \hat{y}(t_k,\theta))^T R^{-1} (y(t_k) - \hat{y}(t_k,\theta))$$

- Advantage: shorter, **cheaper** and **safer** experiments (DLR 3211 sequences)
- Drawback: computationally **slow** (a lot of data-points)
- Drawback: simulation of unstable models
- Drawback: initial guess needed.

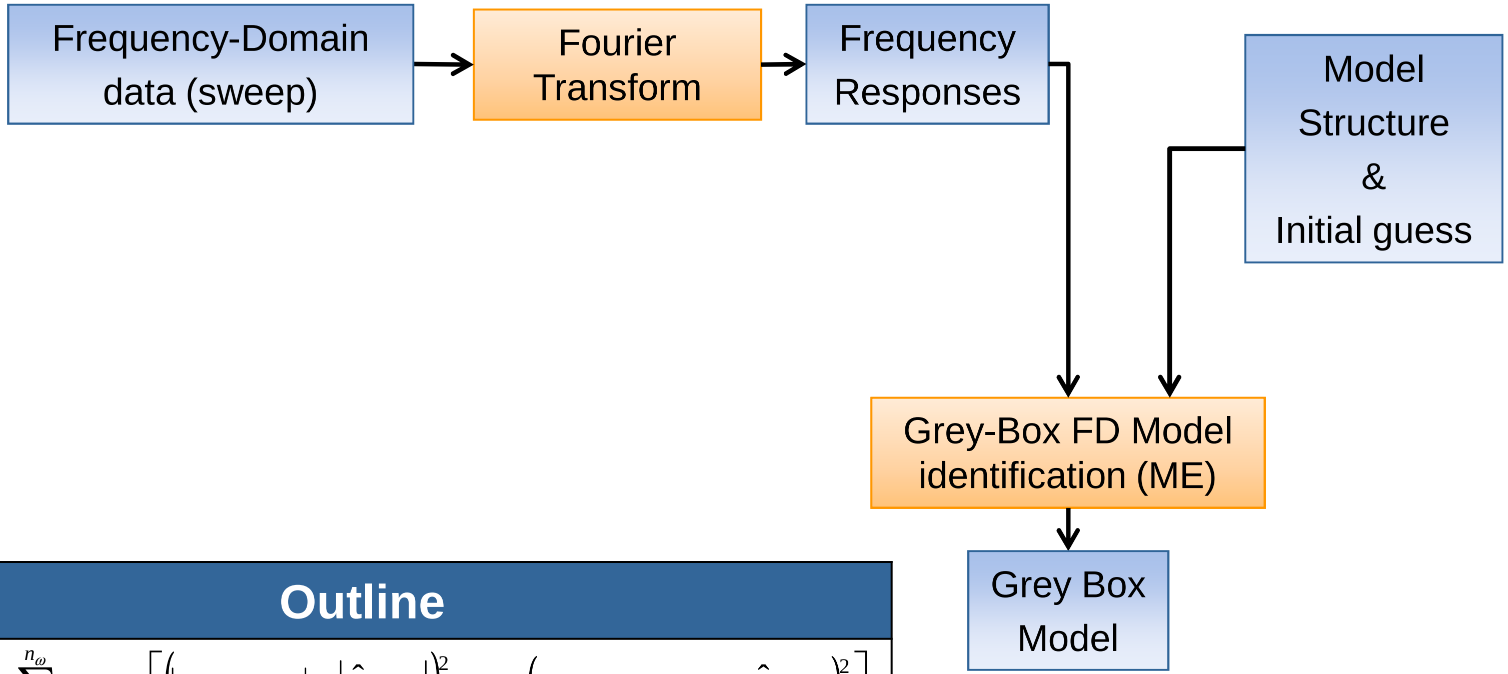
Frequency-domain approaches
(*e.g.*, CIFER, see Tischler and Remple 2006)



Outline

$$\min_{\theta} J(\theta) = \sum_{i=1}^{n_\omega} W(\omega_i)\left[\left(\left|G_s(\omega_i,\theta)\right| - \left|\hat{T}(\omega_i)\right|\right)^2 + W_p\left(\angle G_s(\omega_i,\theta) - \angle \hat{T}(\omega_i)\right)^2\right]$$

Frequency-domain approaches
(*e.g.*, CIFER, see Tischler and Remple 2006)



Outline

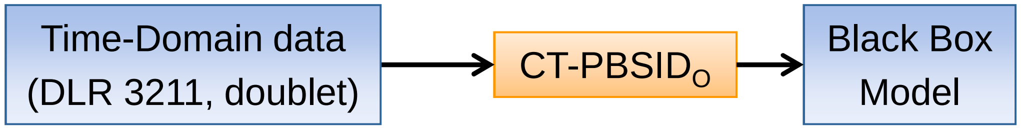
$$\min_{\theta} J(\theta) = \sum_{i=1}^{n_\omega} W(\omega_i)\left[\left(\left|G_s(\omega_i,\theta)\right| - \left|\hat{T}(\omega_i)\right|\right)^2 + W_p\left(\angle G_s(\omega_i,\theta) - \angle \hat{T}(\omega_i)\right)^2\right]$$

- Advantage: computationally **fast** (few data-points)
- Advantage: deal with unstable system in a very natural way (phase signs)
- Drawback: long and **costly** experiments (sweeps)
- Drawback: initial guess needed.

Non-iterative time-domain approach
(*e.g.*, CT subspace model identification methods)

- In the system identification community Subspace Model Identification (SMI) was proposed about 25 years ago to handle black-box MIMO problems in a numerically stable way
- SMI has proved extremely successful in a number of industrial applications
- The discrete-time case has been studied extensively
- The continuous-time case has been investigated in a number of contributions, mainly for the open-loop setting
- Main downside: impossibility to impose a fixed basis to the state space representation, *i.e.*, the identified models are unstructured

Non-iterative time-domain approach
(e.g., CT subspace methods, see Bergamasco and Lovera)



Outline
$\min_{\hat{A},\hat{B},\hat{C},\hat{D}} J = f(\hat{A},\hat{B},\hat{C},\hat{D},u(t),y(t))$

- Advantage: shorter, **cheaper**, and **safer** experiments
- Advantage: computationally **efficient and robust**
- Advantage: no model structure and initial guess (high order model can be eventually considered)
- Drawback: no control on state space basis of identified models, i.e., no physical model.

- Consider the MIMO LTI continuous-time system

$$dx(t) = \boxed{A}x(t)dt + \boxed{B}u(t)dt + \boxed{K}de(t),\ x(0) = x_0$$
$$y(t)dt = \boxed{C}x(t)dt + \boxed{D}u(t)dt + de(t)$$

(in innovation form for simplicity) where $x \in \mathbb{R}^n,\ u \in \mathbb{R}^m,\ y \in \mathbb{R}^p$

Assumptions

- *de*(*t*) Wiener process
- (*A*,*B*,*C*,*D*,*K*) such that (*A*,*C*) observable and (*A*,[*B K*]) controllable
- system possibly operating in closed-loop

Approach

- Convert the model to discrete-time via an <u>exact</u> signals-based method
- Apply the discrete-time PBSID SMI algorithm
- Retrieve the original continuous-time model, *i.e.*, *(A,B,C,D,K)*

Definitions

- Consider the first order all-pass transfer function

$$w(s) = \frac{s-a}{s+a}, \qquad a > 0$$

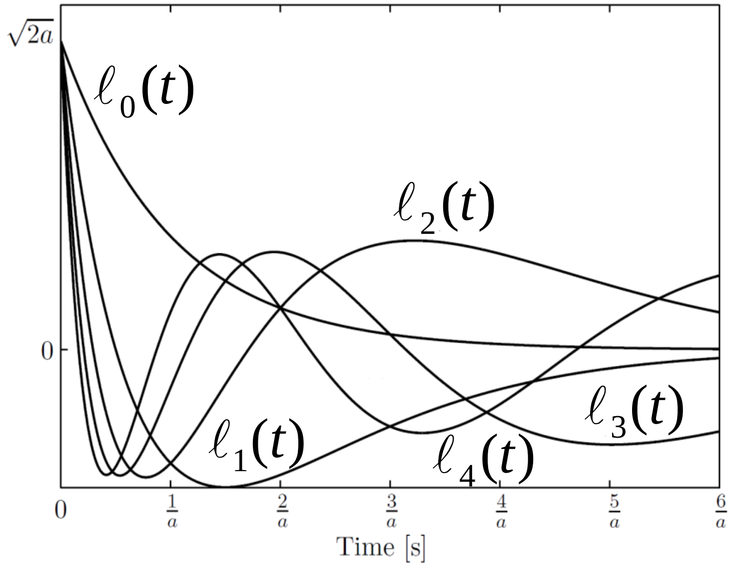
- w(s) generates the family of Laguerre filters, defined as

$$\mathcal{L}_i(s) = w^i(s)\mathcal{L}_0(s), \qquad \mathcal{L}_0(s) = \sqrt{2a}\frac{1}{(s+a)}$$

- Denote with $\ell_i(t)$ the impulse response of the *i*-th Laguerre filter. The set

$$\{\ell_0, \ell_1, \ldots, \ell_i, \ldots\}$$

is an orthonormal basis of $L_2(0,1)$.



$$dx(t) = Ax(t)dt + Bu(t)dt + Kde(t)$$
$$y(t)dt = Cx(t)dt + Du(t)dt + de(t)$$

Matrix transformations

$$A_o = (A - aI)^{-1}(A + aI)$$
$$B_o = \sqrt{2a}(A - aI)^{-1}B$$
$$C_o = -\sqrt{2a}C(A - aI)^{-1}$$
$$D_o = D - C(A - aI)^{-1}B$$

Signal projections

$$\tilde{u}(k) = \int_0^\infty \ell_k(t)u(t)dt$$
$$\tilde{y}(k) = \int_0^\infty \ell_k(t)y(t)dt$$
$$\tilde{e}(k) = \int_0^\infty \ell_k(t)de(t)$$

$$\xi(k+1) = A_o\xi(k) + B_o\tilde{u}(k) + K_o\tilde{e}(k)$$
$$\tilde{y}(k) = C_o\xi(k) + D_o\tilde{u}(k) + \tilde{e}(k)$$

Discrete index *k*: basis order

The discrete-time PBSID algorithm: model in predictor form

Consider the discrete-time system

$$\xi(k+1) = A_O\xi(k) + B_O\tilde{u}(k) + K_O\tilde{e}(k)$$
$$\tilde{y}(k) = C_O\xi(k) + D_O\tilde{u}(k) + \tilde{e}(k)$$

Innovation Form

Closed-loop predictor matrices

$$\bar{A}_O = A_O - K_O C_O$$
$$\bar{B}_O = B_O - K_O D_O$$

$$\tilde{z}(k) = \begin{bmatrix} \tilde{u}(k) \\ \tilde{y}(k) \end{bmatrix}, \quad \tilde{B}_O = \begin{bmatrix} \bar{B}_O & K_O \end{bmatrix}$$

$$\xi(k+1) = \bar{A}_O\xi(k) + \tilde{B}_O\tilde{z}(k)$$
$$\tilde{y}(k) = C_O\xi(k) + D_O\tilde{u}(k) + \tilde{e}(k)$$

Prediction Form

The discrete-time PBSID algorithm: the data equation

Iterating *p-1* times the state equation one gets

$$\xi(k+2) = \bar{A}_o^2 \xi(k) + \begin{bmatrix} \bar{A}_o \tilde{B}_o & \tilde{B}_o \end{bmatrix} \begin{bmatrix} \tilde{z}(k) \\ \tilde{z}(k+1) \end{bmatrix}$$

$$\vdots$$

$$\xi(k+p) = \bar{A}_o^p \xi(k) + \mathcal{K}^p Z^{0,p-1}$$

where

$$\mathcal{K}^p = \begin{bmatrix} \bar{A}_o^{p-1} \tilde{B}_0 & \dots & \tilde{B}_o \end{bmatrix}$$

Extended controllability matrix

and

$$Z^{0,p-1} = \begin{bmatrix} \tilde{z}(k) \\ \vdots \\ \tilde{z}(k+p-1) \end{bmatrix}$$

Input-output "past" data

The discrete-time PBSID algorithm: the data equation

- The predictor is AS by assumption, so

$$\bar{A}_O^p \xi(k) \simeq 0$$

for sufficiently large values of *p* and

$$\xi(k+p) \simeq \mathcal{K}^p Z^{0,p-1}$$

p: past window length
f: future window length

- Then, the input-output behaviour of the system is given by the data equation:

$$\tilde{y}(k+p) \simeq C_O \mathcal{K}^p Z^{0,p-1} + D_O \tilde{u}(k+p) + \tilde{e}(k+p)$$
$$\vdots$$
$$\tilde{y}(k+p+f) \simeq C_O \mathcal{K}^p Z^{f,p+f-1} + D_O \tilde{u}(k+p+f) + \tilde{e}(k+p+f)$$

- Finally, the state space matrices can be recovered from the data equation using Least Squares techniques.

Data collection

$$(u(t_i), y(t_i))$$

Signal projections

Discrete-time data

$$(\tilde{u}(k), \tilde{y}(k))$$

PBSID algorithm

$$\xi(k+1) = A_o \xi(k) + B_o \tilde{u}(k)$$
$$\tilde{y}(k) = C_o \xi(k) + D_o \tilde{u}(k)$$

Discrete-time identified model

-1

Matrix transformations

$$\dot{x}(t) = \hat{A}x(t) + \hat{B}u(t)$$
$$y(t) = \hat{C}x(t) + \hat{D}u(t)$$

Black-box continuous-time identified model

Continuous-time PBSID algorithm: comments

- The computation of the signals transformations

$$\tilde{u}(k) = \int_0^\infty \ell_k(t)u(t)dt \quad \tilde{y}(k) = \int_0^\infty \ell_k(t)y(t)dt$$

allows to deal with **non uniform sampling**.

- Data from **different experiments** can be naturally **merged** in the identification procedure.

- The identification algorithm is based on QR and SVD factorisations (very efficient implementations are available in Matlab).

Continuous-time PBSID algorithm
Estimation of model uncertainty

- The asymptotic theory of SMI methods has been studied extensively
- Estimates are asymptotically Gaussian
- Expressions for the asymptotic variance are extremely cumbersome (see Chiuso 2005, Chiuso 2007, van Wingerden 2012).
- Proposed approach: use the bootstrap method to estimate model uncertainty (along the lines of Bittanti, Lovera 2000)
- Analysis of the bootstrap method for CT SMI is ongoing.

$$(u_i, y_i) \quad i = 0, \cdots, N$$

$$\dot{x} = \hat{A}x + \hat{B}u + \hat{K}e$$
$$y = \hat{C}x + \hat{D}u + e$$

$$e \sim \mathcal{N}(0, \sigma_e^2)$$

$$e_i^{(1)} \quad e_i^{(2)} \quad e_i^{(4)}$$
$$e_i^{(3)} \quad e_i^{(\cdots)} \quad e_i^{(M)}$$

$$(u_i, e_i^{(1)}, \tilde{y}_i^{(1)}) \quad (u_i, e_i^{(\cdots)}, \tilde{y}_i^{(\cdots)})$$
$$(u_i, e_i^{(2)}, \tilde{y}_i^{(2)}) \quad (u_i, e_i^{(M)}, \tilde{y}_i^{(M)})$$

$$\begin{bmatrix} \widehat{A} & \widehat{B} \\ \widehat{C} & \widehat{D} \end{bmatrix}^{(1)} \quad \begin{bmatrix} \widehat{A} & \widehat{B} \\ \widehat{C} & \widehat{D} \end{bmatrix}^{(\cdots)} \quad \begin{bmatrix} \widehat{A} & \widehat{B} \\ \widehat{C} & \widehat{D} \end{bmatrix}^{(M)}$$

From unstructured to structured models

- SMI enabled the possibility of dealing with MIMO state space identification in a simple and natural way

- Downside: it is hard to impose a fixed basis to the state space representation. Therefore, it is hard to
  - impose a parameterisation to the state space matrices
  - exploit prior knowledge
  - recover numerical values for physical parameters.

- The problem has been recently addressed in, e.g., Xie & Ljung 2002, Parrilo & Ljung 2003, Prot *et al.* 2012, by solving the bilinear equations resulting from the definition of state space similarity transformations.

Black-box to grey-box model transformation in the frequency-domain

- Black-box identified model

$$\dot{x}(t) = \hat{A}x(t) + \hat{B}u(t)$$
$$y(t) = \hat{C}x(t) + \hat{D}u(t)$$
$$\Rightarrow \quad \hat{G}_{ns}(s)$$

- Grey-box model structure

$$\dot{x}(t) = A(\theta)x(t) + B(\theta)u(t)$$
$$y(t) = C(\theta)x(t) + D(\theta)u(t)$$
$$\Rightarrow \quad G_s(s;\theta)$$

- H_∞ approach in frequency-domain

$$\theta^\star = \arg\min_\theta \|\hat{G}_{ns}(s) - G_s(s;\theta)\|_\infty$$

- Model Error (ME) approach in frequency-domain

$$\min_\theta J(\theta) = \sum_{i=1}^{n_\omega} W(\omega_i)\left[\left(|G_s(\omega_i,\theta)| - |\hat{G}_{ns}(\omega_i)|\right)^2 + W_p\left(\angle G_s(\omega_i,\theta) - \angle \hat{G}_{ns}(\omega_i)\right)^2\right]$$

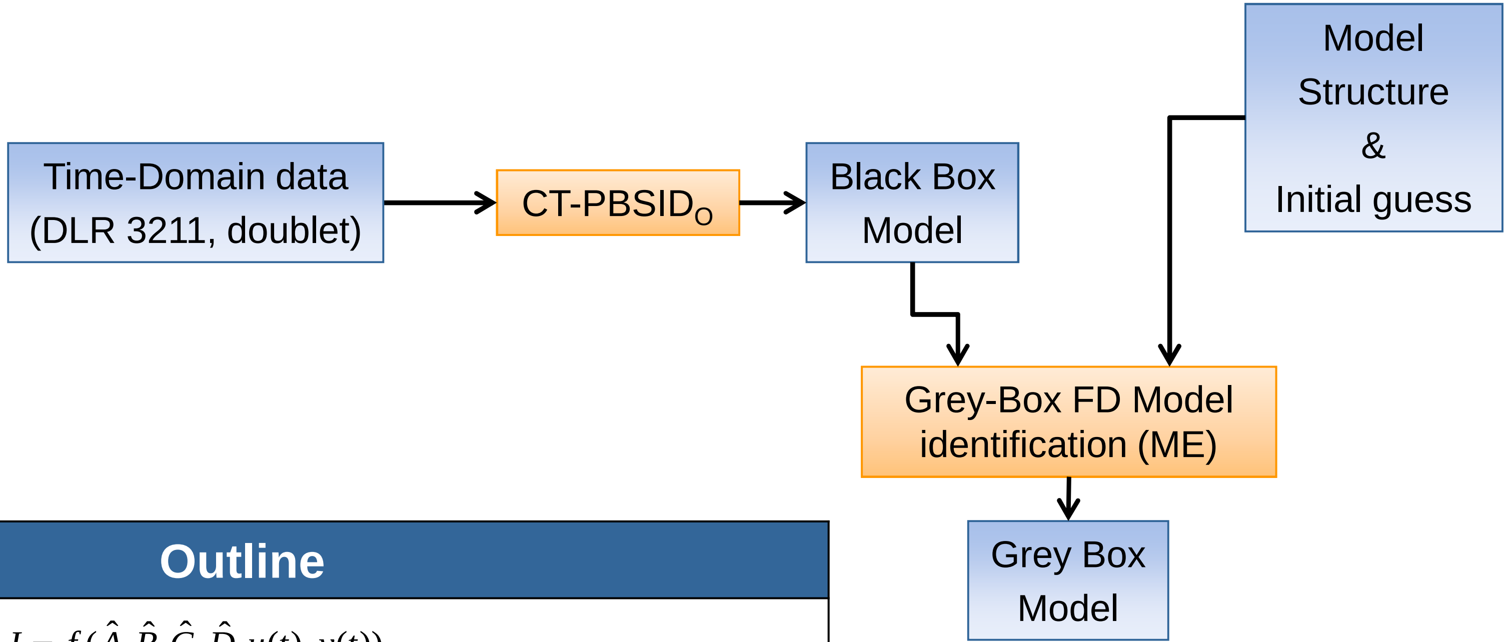
H_∞ approach: comments

- The optimization problem can be solved using some recent algorithms available in literature, see Apkarian & Noll 2006 (and in Matlab R2012a, see Gahinet & Apkarian 2011).

- The estimation of the similarity transformation is not necessary (this enables handling of larger problems).

- Frequency-domain data (if available) can be included in the optimization problem.

Black-box to grey-box model transformation in the FD



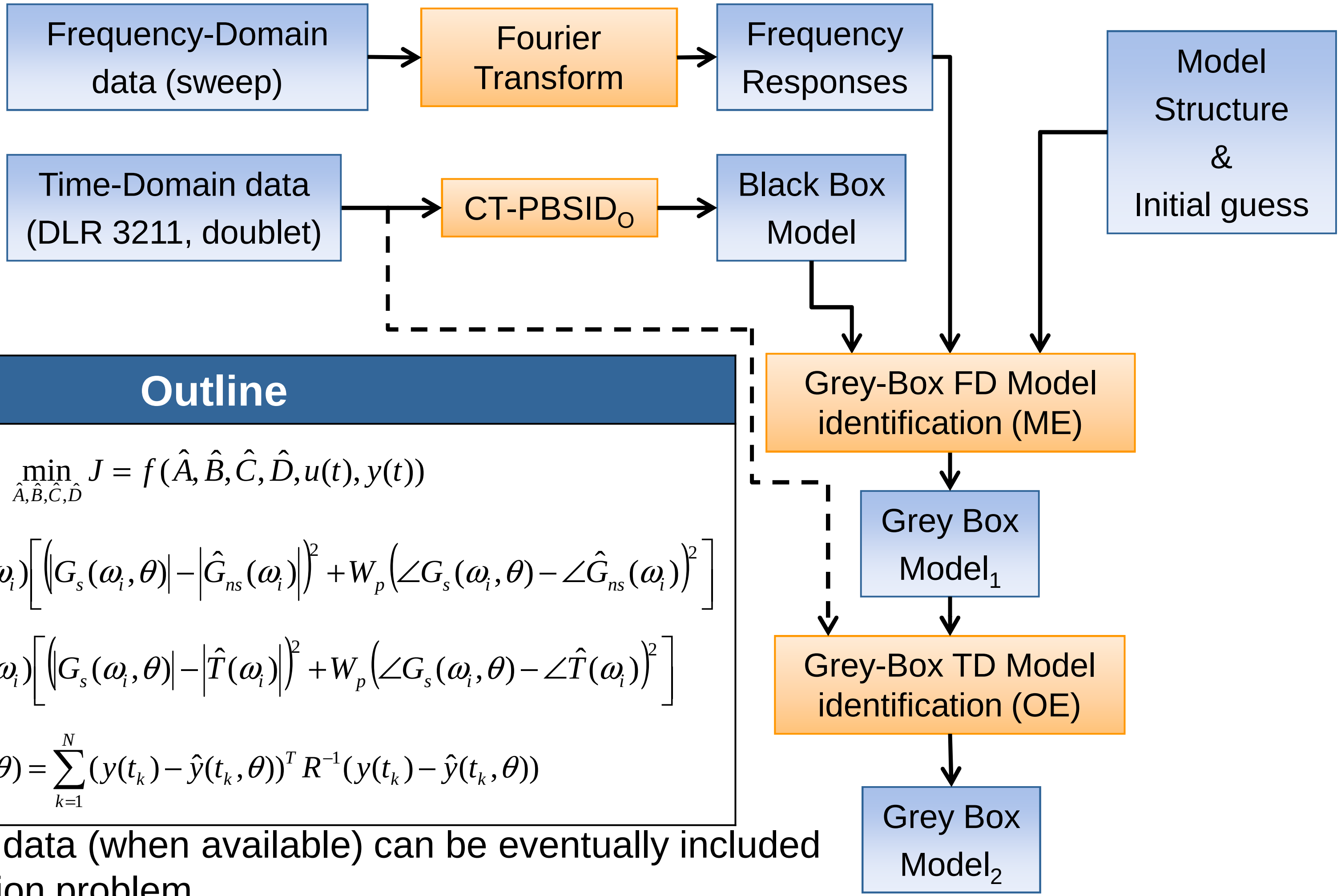
Outline

1. $$\min_{\hat{A},\hat{B},\hat{C},\hat{D}} J = f(\hat{A},\hat{B},\hat{C},\hat{D},u(t),y(t))$$

2. $$\min_{\theta} J(\theta) = \sum_{i=1}^{n_\omega} W(\omega_i)\left[\left(\left|G_s(\omega_i,\theta)\right| - \left|\hat{G}_{ns}(\omega_i)\right|\right)^2 + W_p\left(\angle G_s(\omega_i,\theta) - \angle \hat{G}_{ns}(\omega_i)\right)^2\right]$$

- Advantage: shorter, **cheaper**, and **safer** experiments (DLR 3211 sequences)
- Advantage: computationally **efficient and robust**

Proposed time/frequency domain approach



Outline

1. $$\min_{\hat{A},\hat{B},\hat{C},\hat{D}} J = f(\hat{A},\hat{B},\hat{C},\hat{D},u(t),y(t))$$

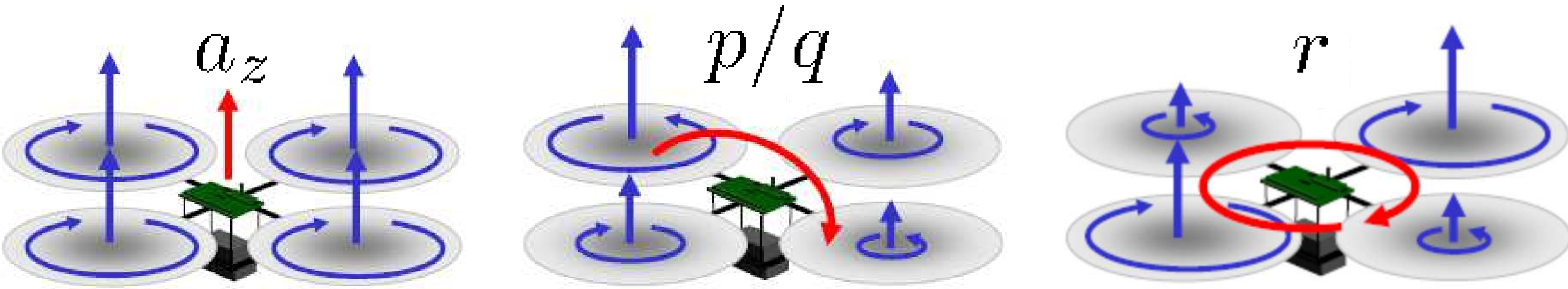
2. $$\min_{\theta} J(\theta) = \sum_{i=1}^{n_\omega} W_1(\omega_i)\left[\left(|G_s(\omega_i,\theta)| - |\hat{G}_{ns}(\omega_i)|\right)^2 + W_p\left(\angle G_s(\omega_i,\theta) - \angle \hat{G}_{ns}(\omega_i)\right)^2\right] + \sum_{i=1}^{n_\omega} W_2(\omega_i)\left[\left(|G_s(\omega_i,\theta)| - |\hat{T}(\omega_i)|\right)^2 + W_p\left(\angle G_s(\omega_i,\theta) - \angle \hat{T}(\omega_i)\right)^2\right]$$

3. $$\min_{\theta} J(\theta) = \sum_{k=1}^{N} (y(t_k) - \hat{y}(t_k,\theta))^T R^{-1} (y(t_k) - \hat{y}(t_k,\theta))$$

- Advantage: FD data (when available) can be eventually included in the optimization problem
- Advantage: all kind of (appropriate) data can be used in the same procedure

Experimental setup

- Mikrokopter platform
- Equipped for outdoor flight
- Sampling onboard at 100Hz
- Automatic excitation
- Attitude control (closed-loop)



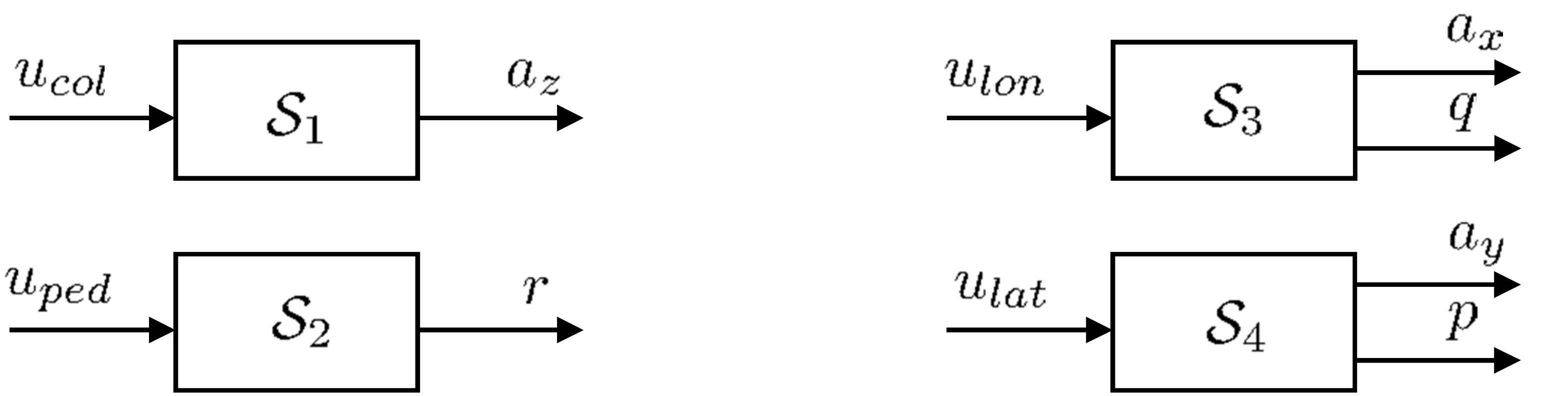
Collective (u_{col}) Lon/Lat (u_{lon}/u_{lat}) Yaw (u_{ped})

Inputs: u_{col}, u_{lon}, u_{lat}, u_{ped}

$$\dot{x}(t) = Ax(t) + Bu(t) + Ke(t)$$
$$y(t) = Cx(t) + Du(t) + e(t)$$

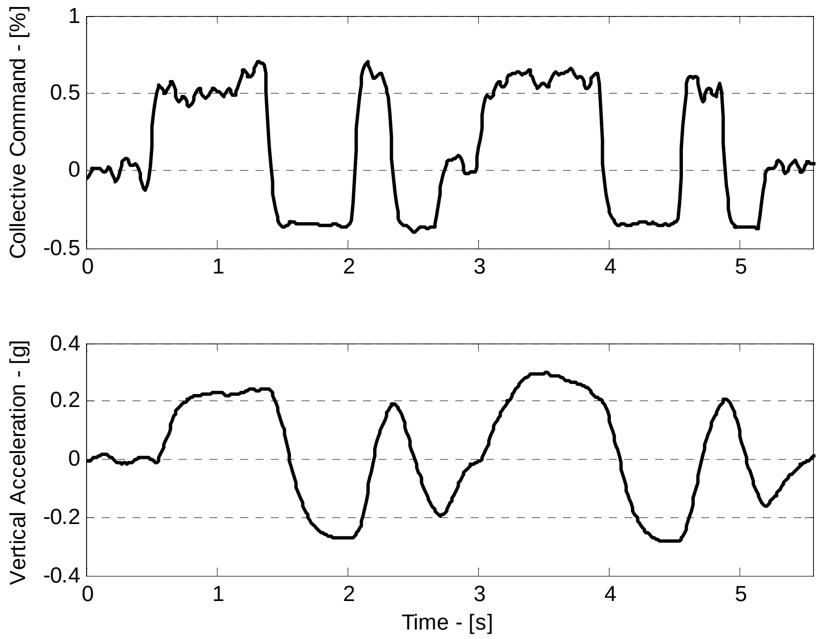
Outputs: a_x, a_y, a_z, p, q, r

Stable modes | **Unstable modes**



Case studies: a small quadrotor
Identification-oriented flight testing

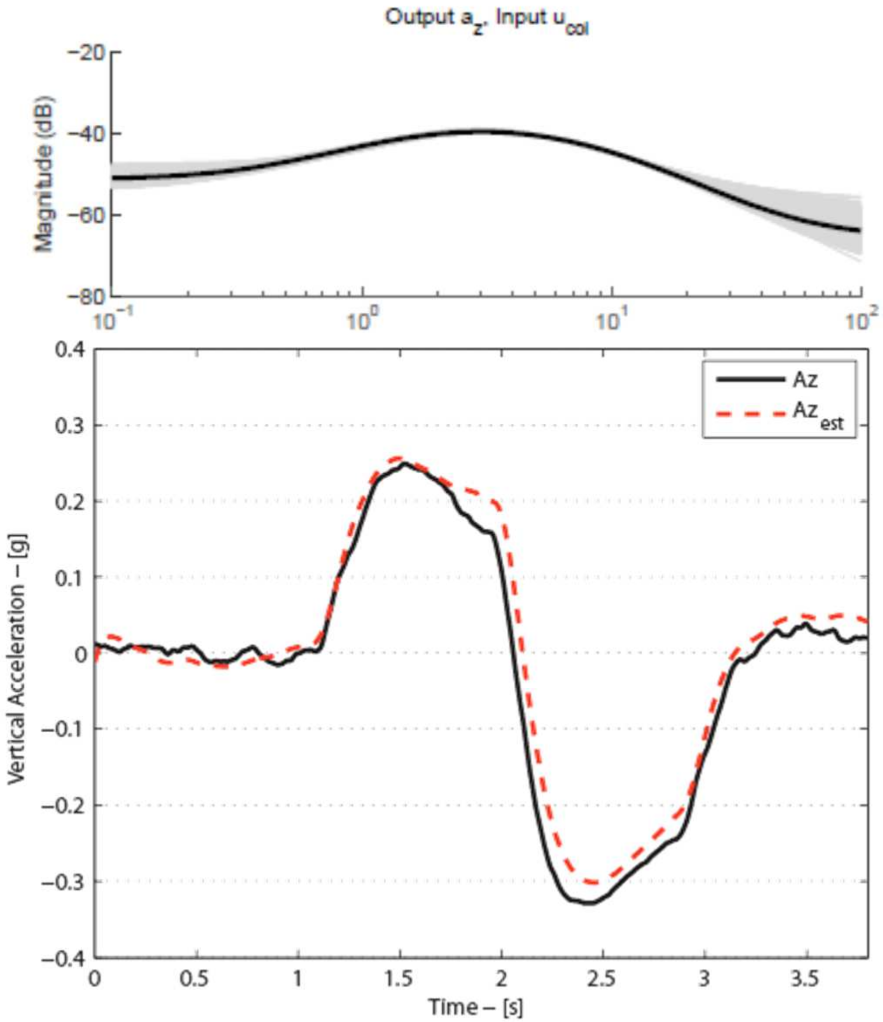
- Manual excitation too slow → automatic excitation
- Input signal:
  3211 piece-wise constant sequence
- Input channels are excited one by one
- Identification phase
  Multiple 3211 datasets (~20s)



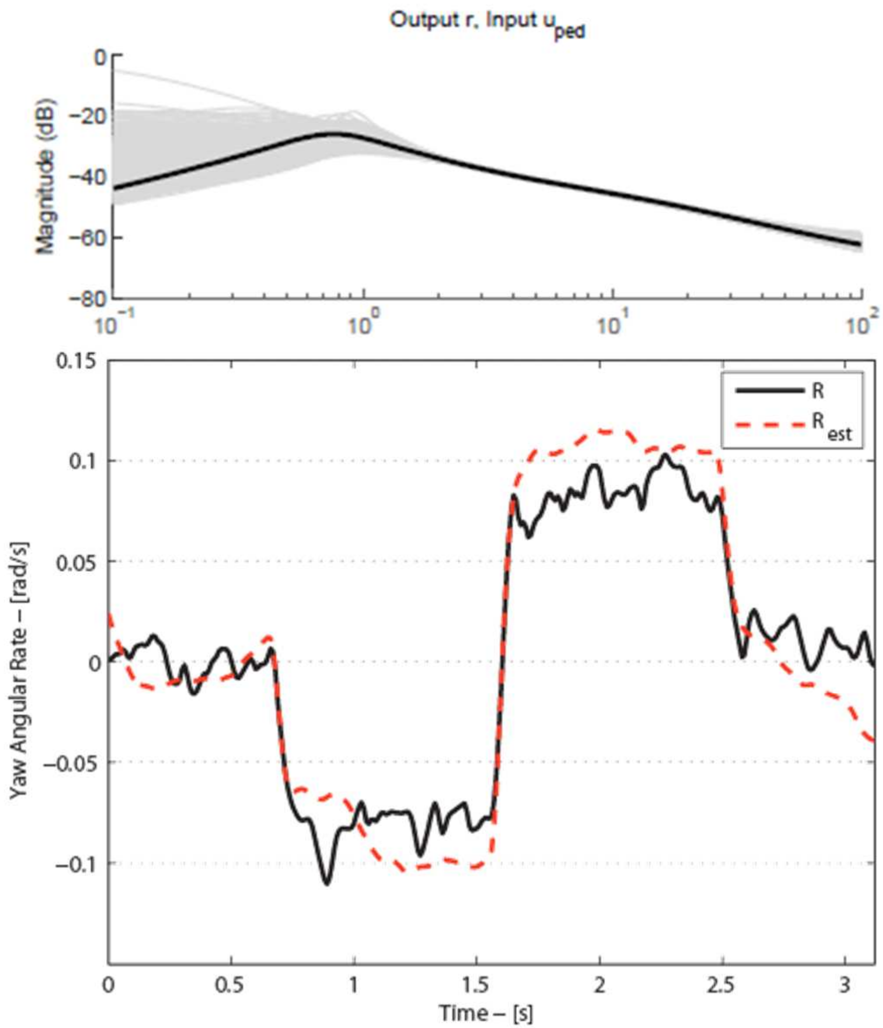
- Cross-validation phase (order and parameters selection)
  a 3211 dataset (~7s)
- Validation: response to doublet inputs.

Case studies: a small quadrotor
Collective and yaw models

Collective



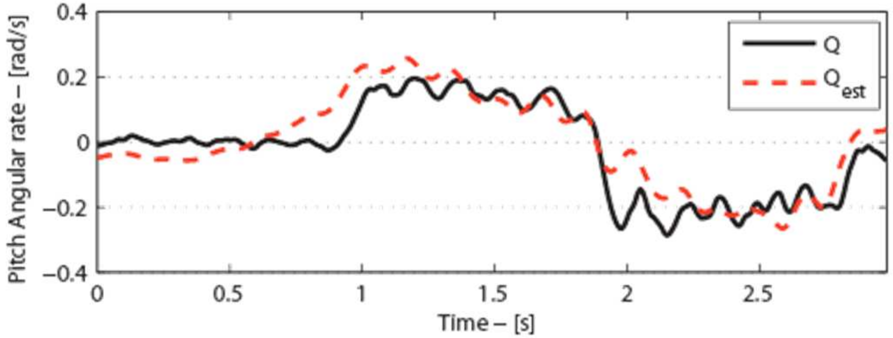
Yaw

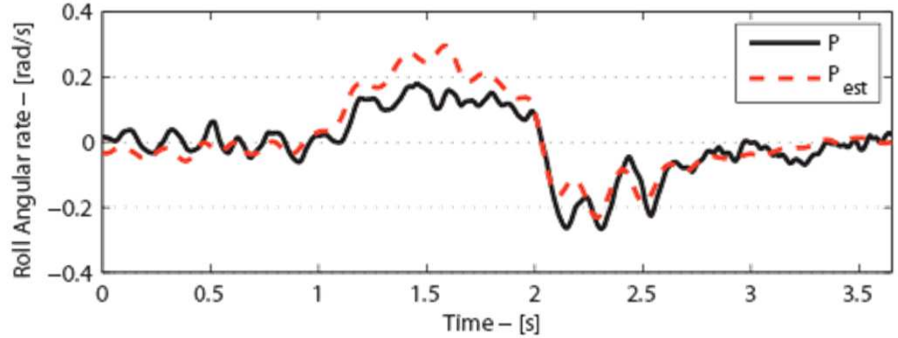


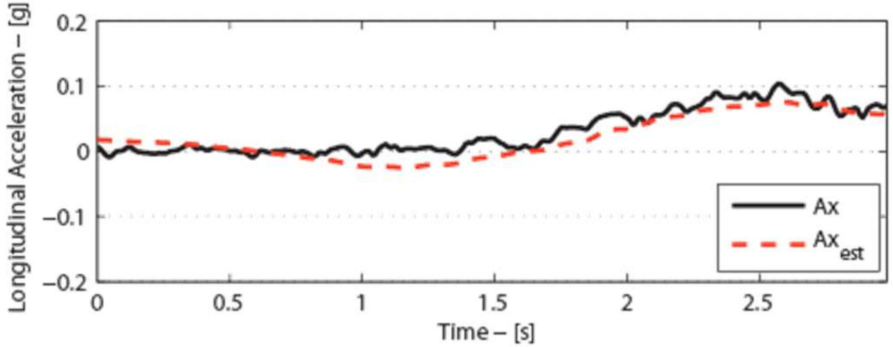
Case studies: a small quadrotor
Longitudinal and lateral models: TD validation

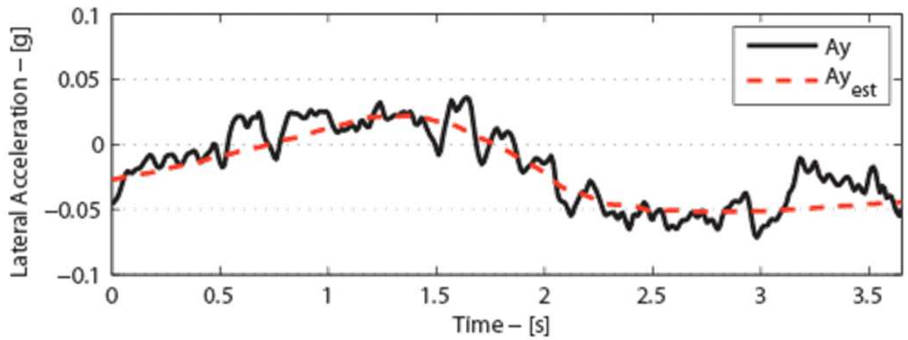
Longitudinal

Lateral

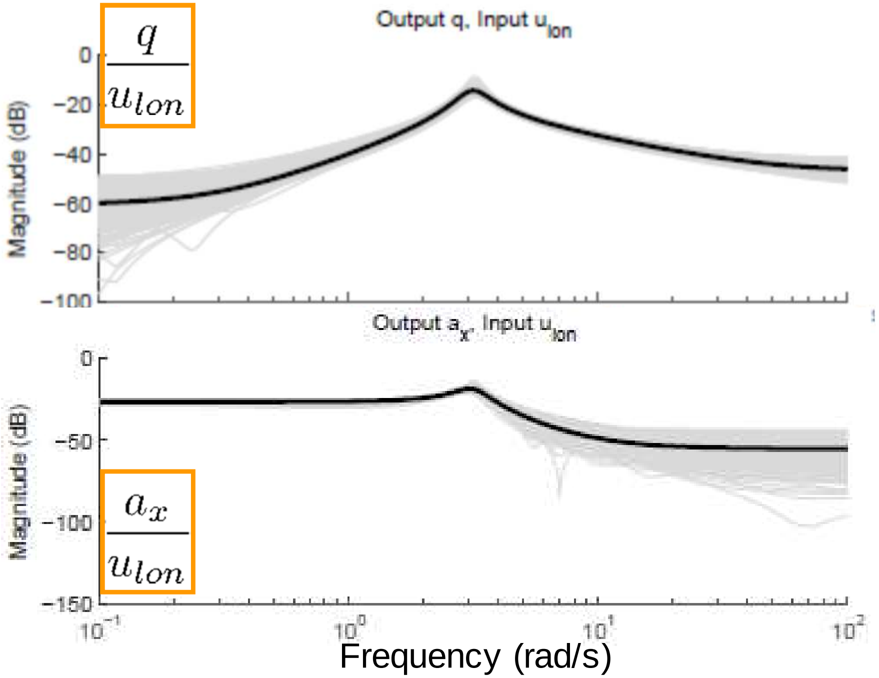




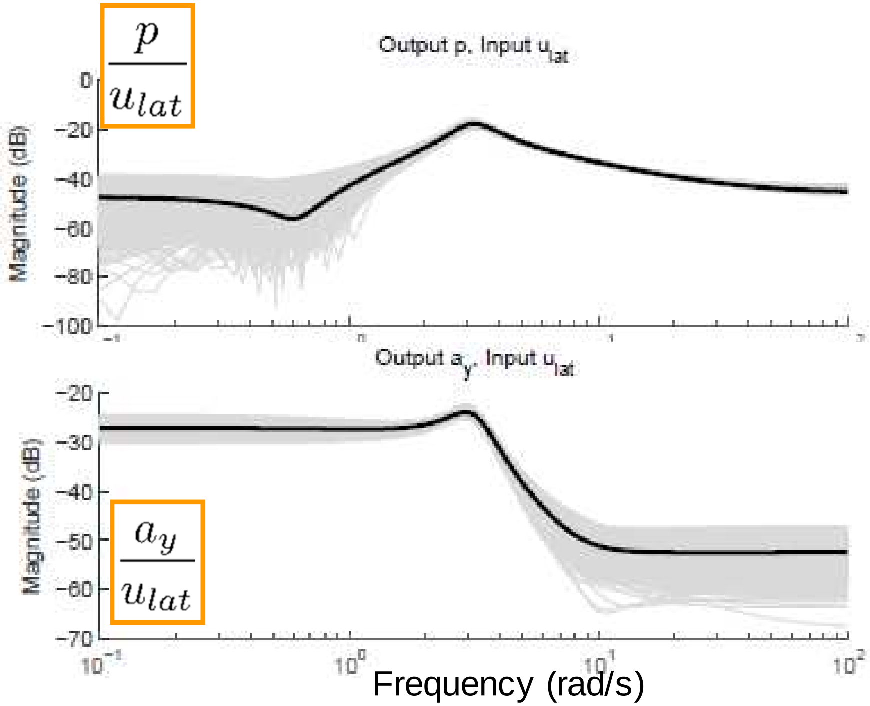




Longitudinal



Lateral



Aermatica

Problem: identification of the pitch attitude dynamics for the Aermatica Anteos RPA.

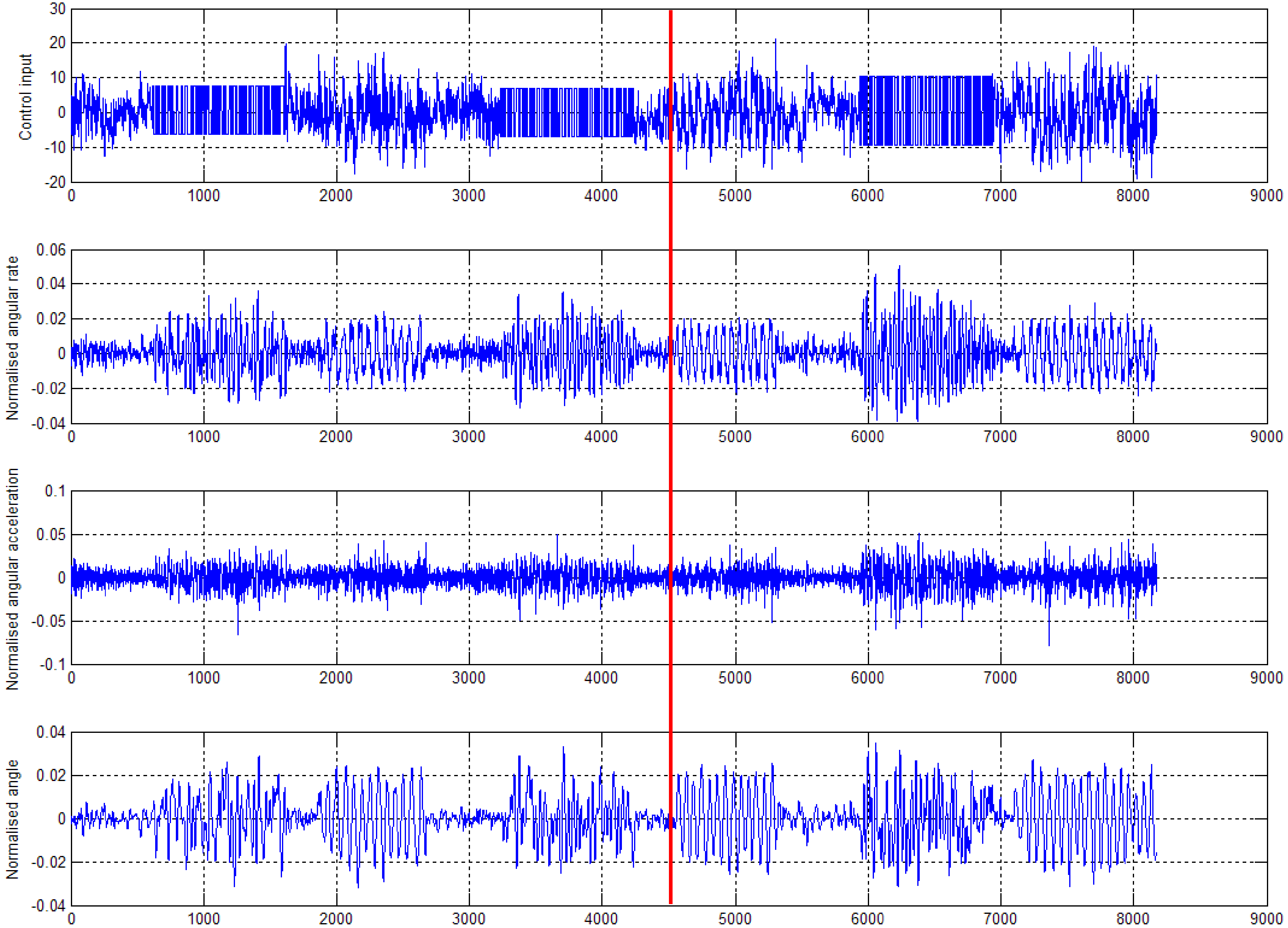
Longitudinal control

Pitch angle

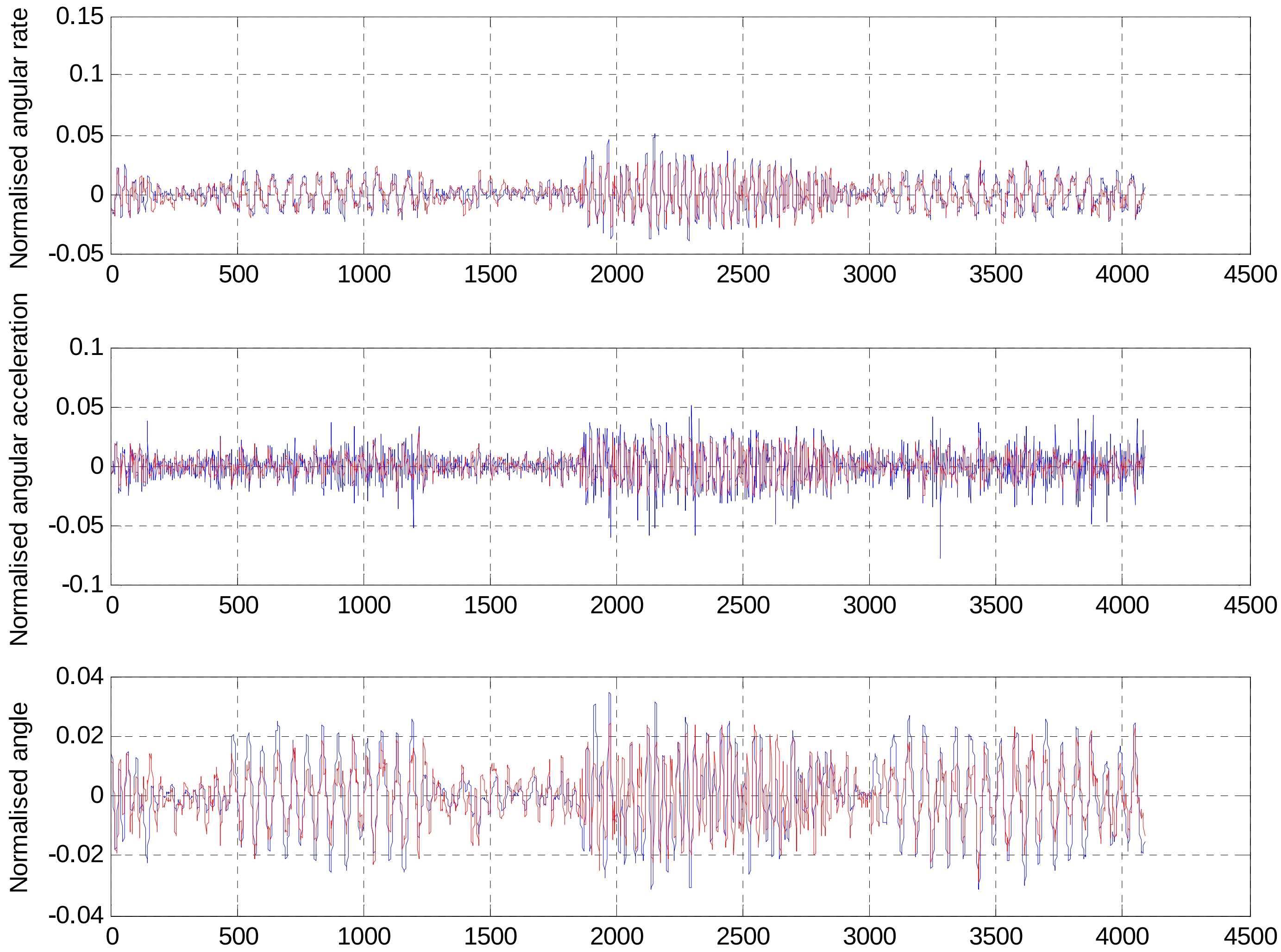
Pitch rate

Pitch acceleration

Case studies: a not-so-small quadrotor
Experimental data: indoor testing



Pitch response, identified from indoor data



Case studies: a full-scale helicopter

Research project aimed at developing methods and tools for identification of full-scale helicopter flight dynamics.

AgustaWestland
A Finmeccanica Company
Advanced Rotorcraft Center - AWPARC
POLITECNICO DI MILANO

Results obtained in piloted simulations (flight simulator based based on the FlightLab code).

AW189 model identification

Simulator introduction and data description

http://www.flightlab.com/

- AW189 simulator: nonlinear model with certified FCS, 80kts steady flight condition
- 4 inputs and 8 outputs are considered

$$u = \begin{bmatrix} \delta_{col} & \delta_{lat} & \delta_{lon} & \delta_{ped} \end{bmatrix}^T \qquad y = \begin{bmatrix} p & q & r & \vartheta & \varphi & a_x & a_y & a_z \end{bmatrix}^T$$

- Each input has been excited separately
- Time-domain data:
  - 8 datasets: 2 for each input channel (DLR3211 for identification, doublet for cross-validation)
  - Manual excitation
- Frequency-domain data:
  - 8 datasets: 2 for each input channel (2xSweep)
  - Automatic excitation

AW189 model identification
Control-oriented physical model

- LTI MIMO model

$$\dot{x}(t) = A(\theta)x(t) + B(\theta)u(t)$$
$$y(t) = C(\theta)x(t) + D(\theta)u(t)$$

- where the state vector (6-DOF) is

$$x = [\varphi \quad \vartheta \quad u \quad v \quad w \quad p \quad q \quad r]^T$$

- Physical model has 64 unknowns parameters θ to be estimated

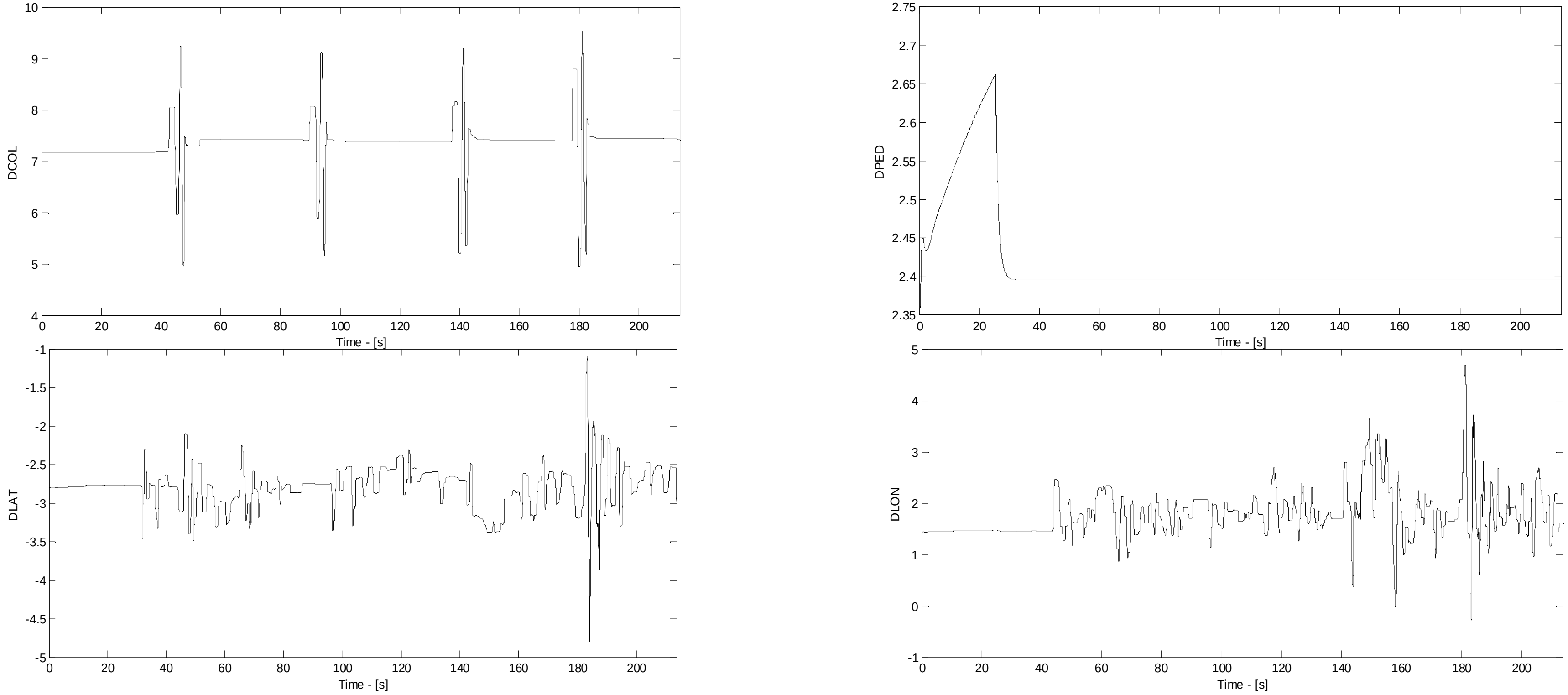
- Validation dataset: a manual pseudo-random excitation in closed-loop

- Root Mean Square error of validation dataset as comparison index

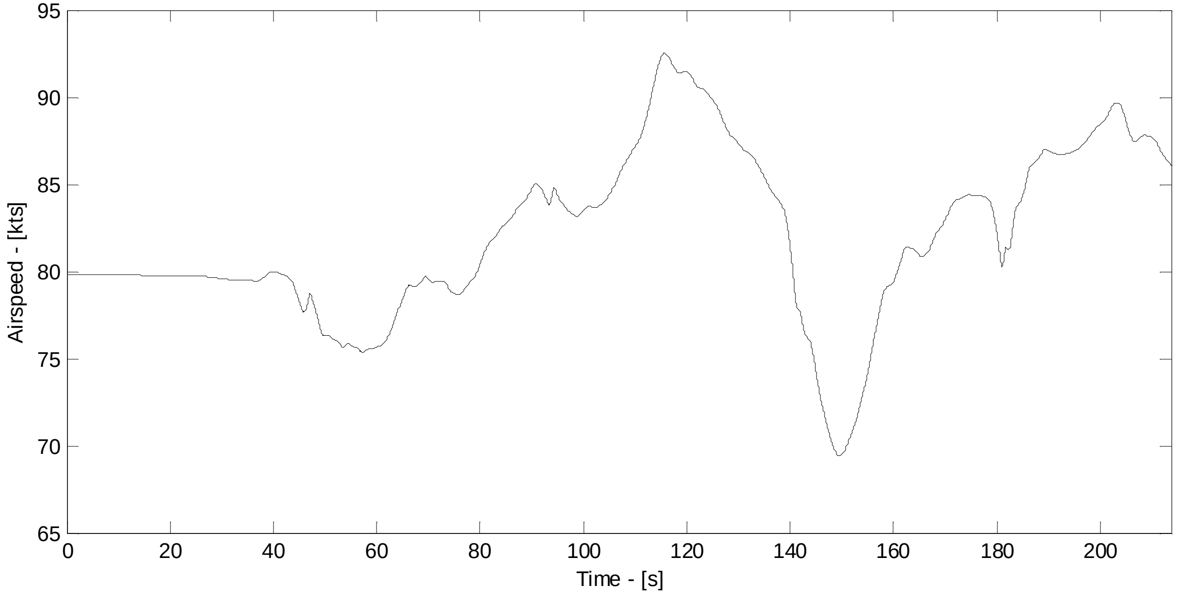
$$RMS_{err} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(y(t_i) - \hat{y}(t_i)\right)^2}$$

Case studies: a full-scale helicopter
Input sequences 1/2

- Primary input manually excited using 3211 sequence and frequency sweeps; secondary inputs manually controlled to stay close to trim
- Several repetitions for each manoeuvre are collected



Close to trim?



Case studies: a full-scale helicopter
Model structure

- FLFM: FlightLAB linearised full-model (55 states)
- FLRM: FlightLAB linearised reduced-model (8 states)

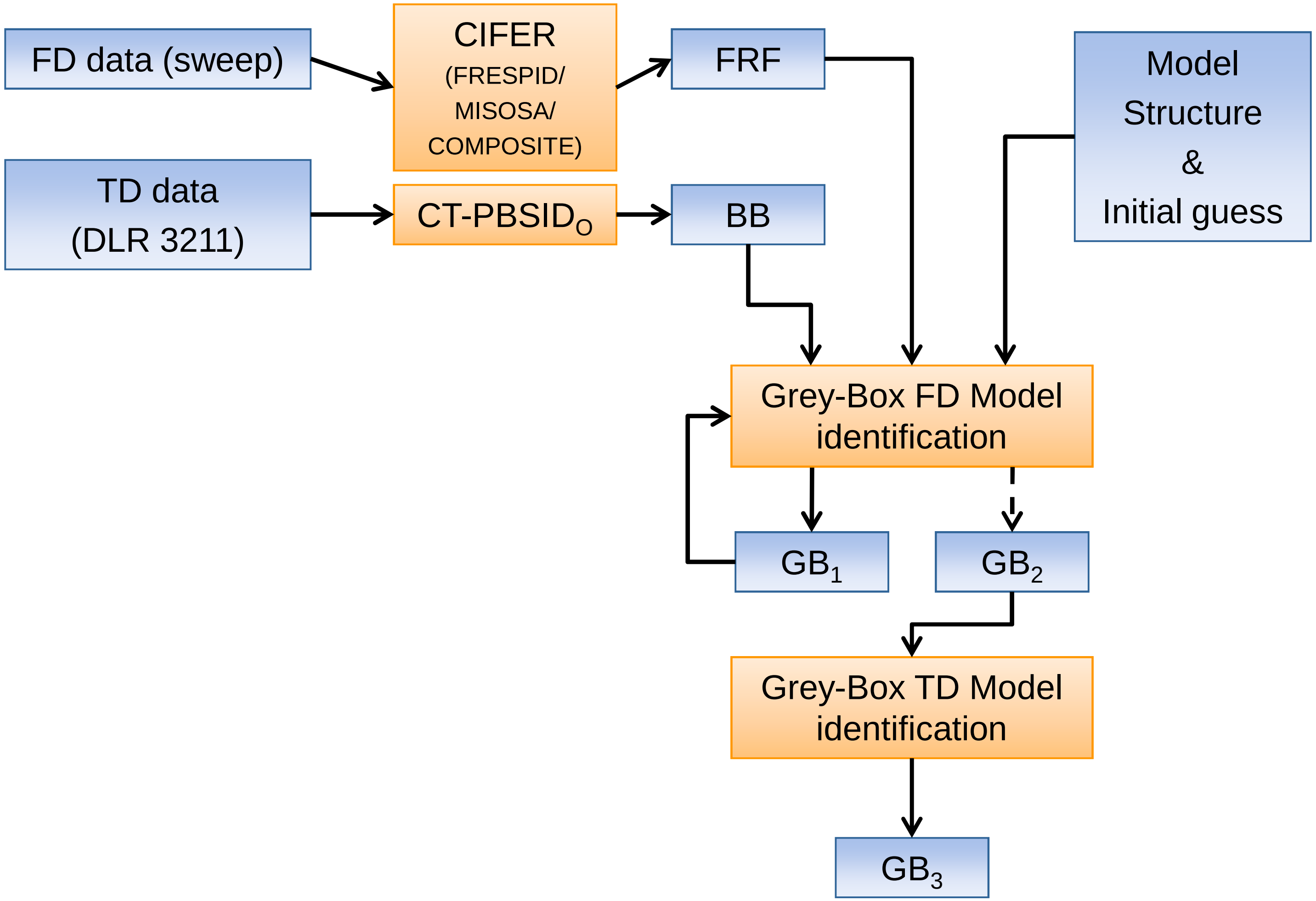
$$M\dot{x}(t) = Fx(t) + Gu(t)$$
$$y(t) = H_0 x(t) + H_1 \dot{x}(t)$$

- Grey-box model with 64 free parameters is initialized with FLRM

Total	No. Free M Terms	No. Free F Terms	No. Free G Terms	No. Free Tau Terms
64	0	36	24	4

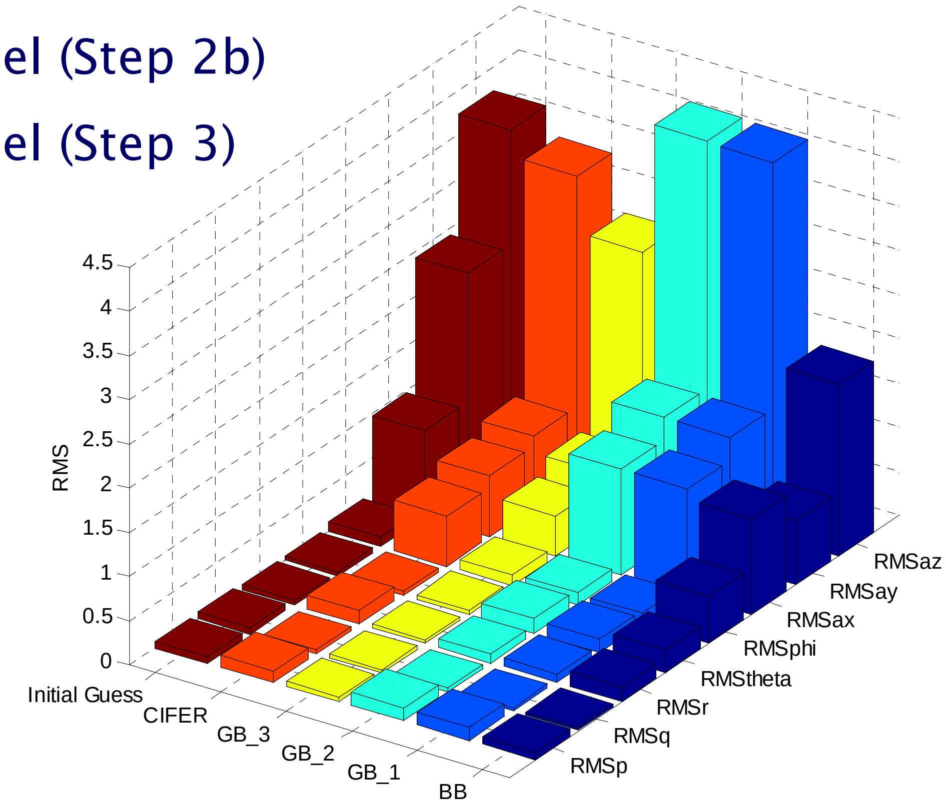
- State vector $x = [\varphi \quad \vartheta \quad u \quad v \quad w \quad p \quad q \quad r]^T$
- Input vector $u = [\delta_{col} \quad \delta_{lat} \quad \delta_{lon} \quad \delta_{ped}]^T$
- Output vector $y = [p \quad q \quad r \quad \vartheta \quad \varphi \quad a_x \quad a_y \quad a_z]^T$

Case studies: a full-scale helicopter
Overall approach

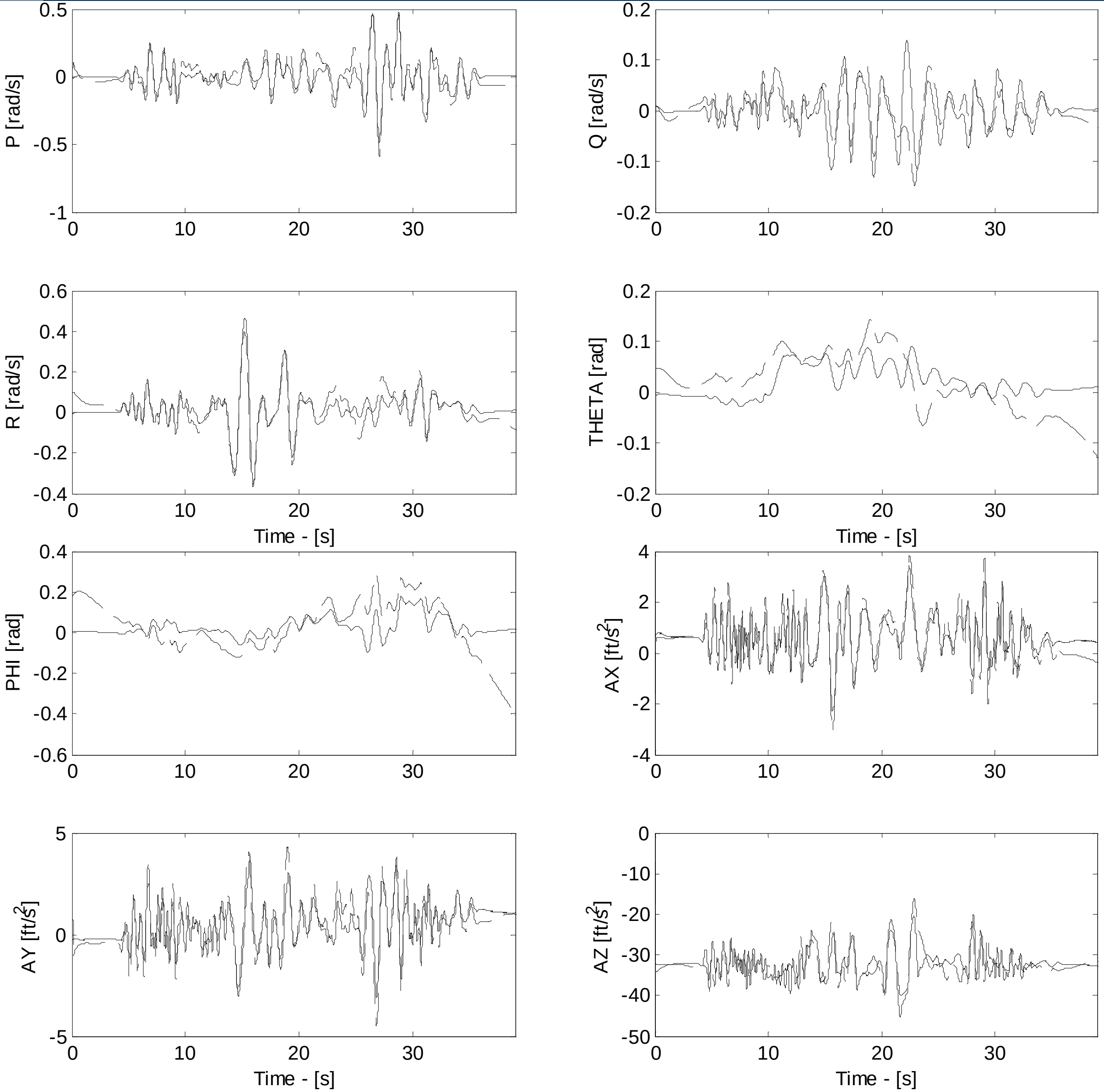


•Root Mean Square Error comparison

- BB: Black-box identified model
- GB_1: Grey-box identified model (Step 2a)
- GB_2: Grey-box identified model (Step 2b)
- GB_3: Grey-box identified model (Step 3)
- CIFER
- Initial Guess

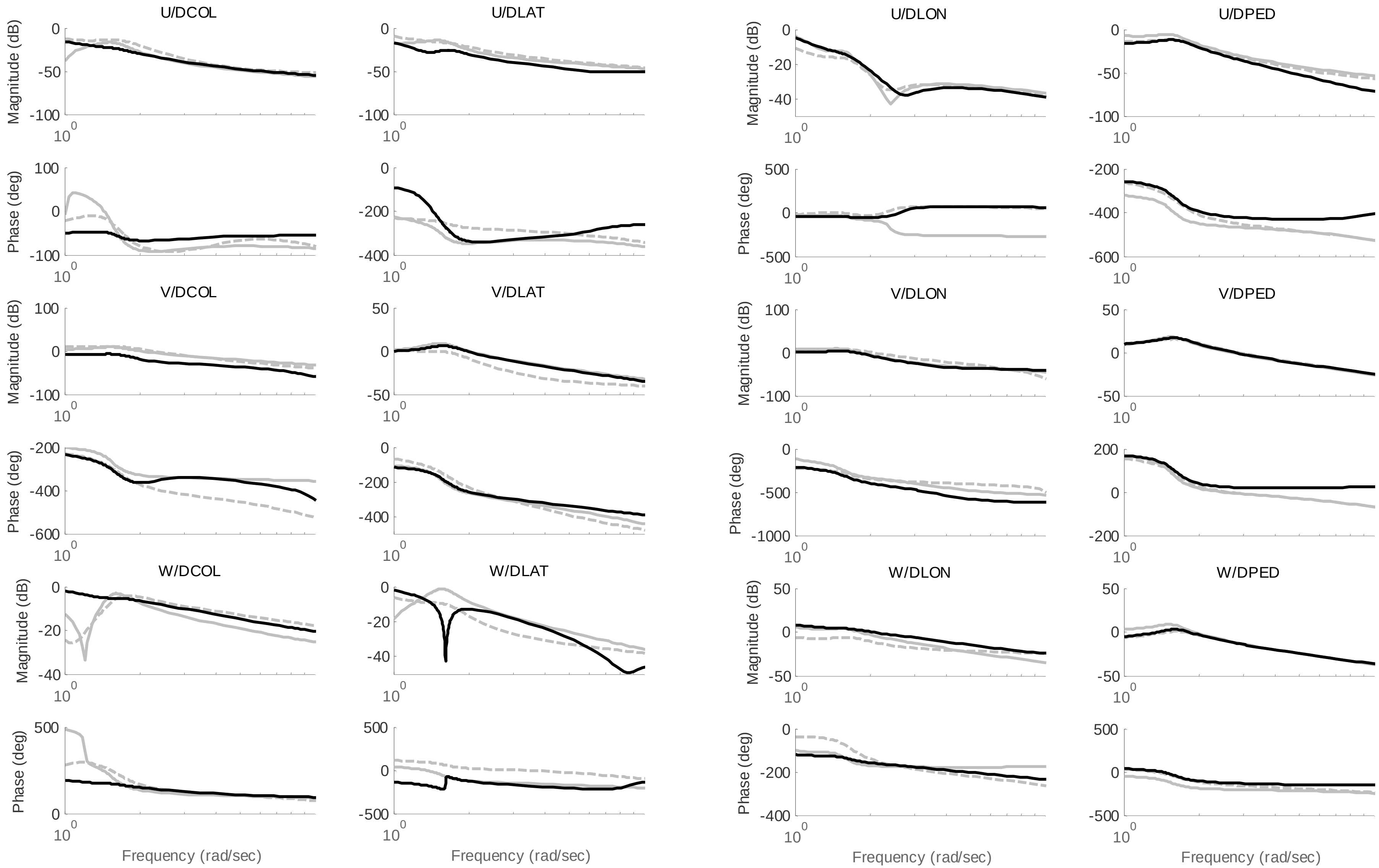


Case studies: a full-scale helicopter
Time-domain comparison

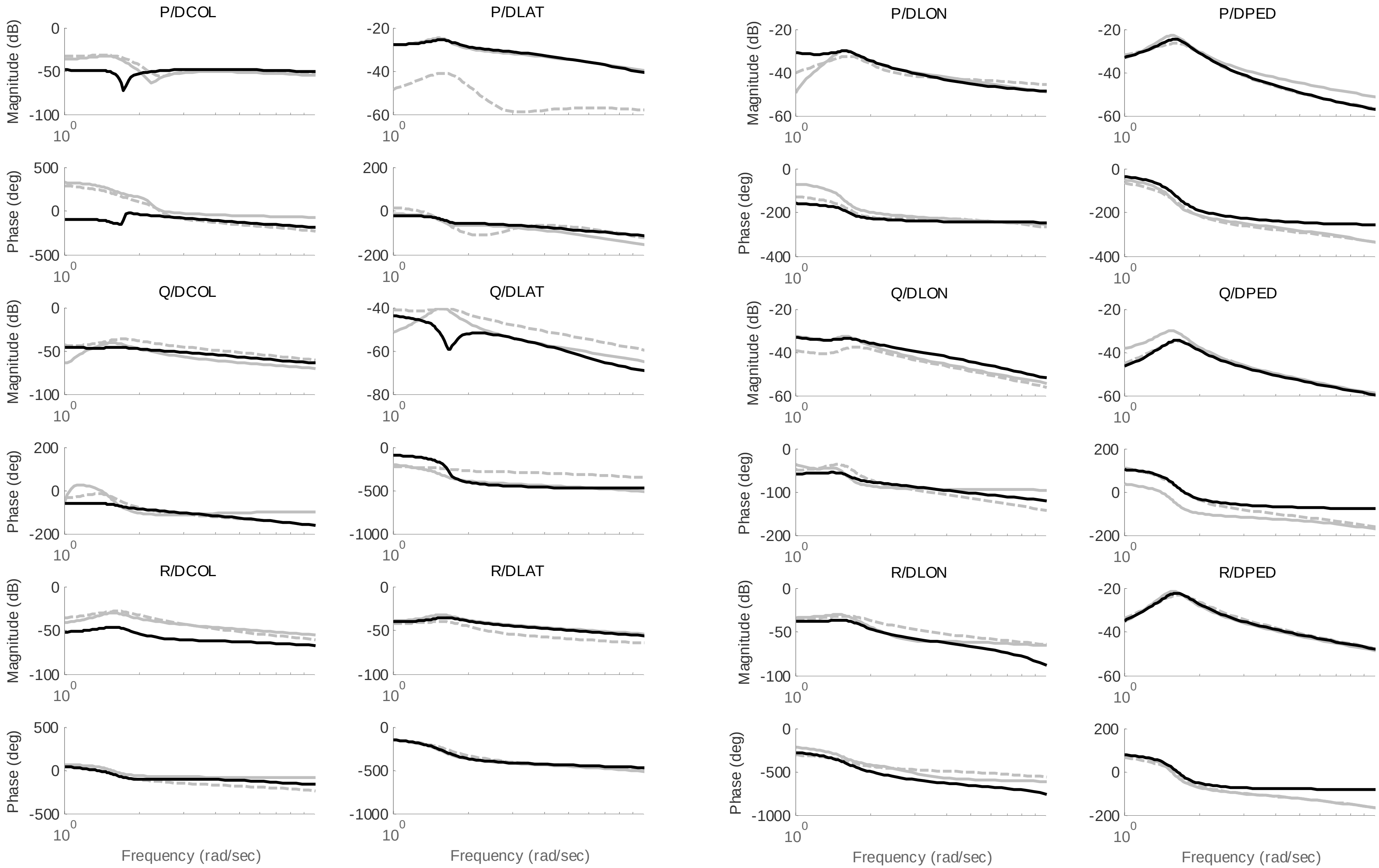


Case studies: a full-scale helicopter

Frequency responses comparison 1/4

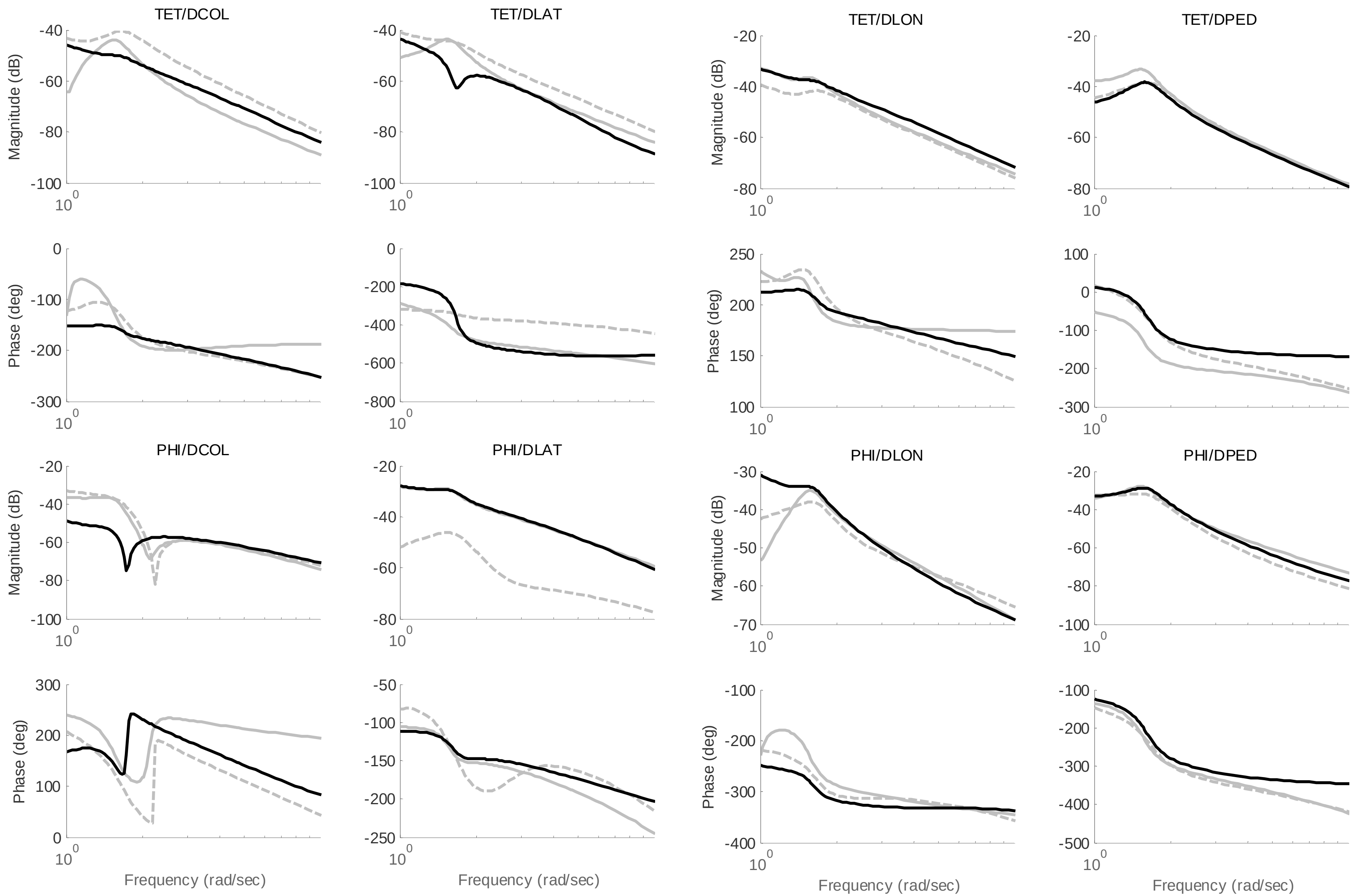


Case studies: a full-scale helicopter
Frequency responses comparison 2/4

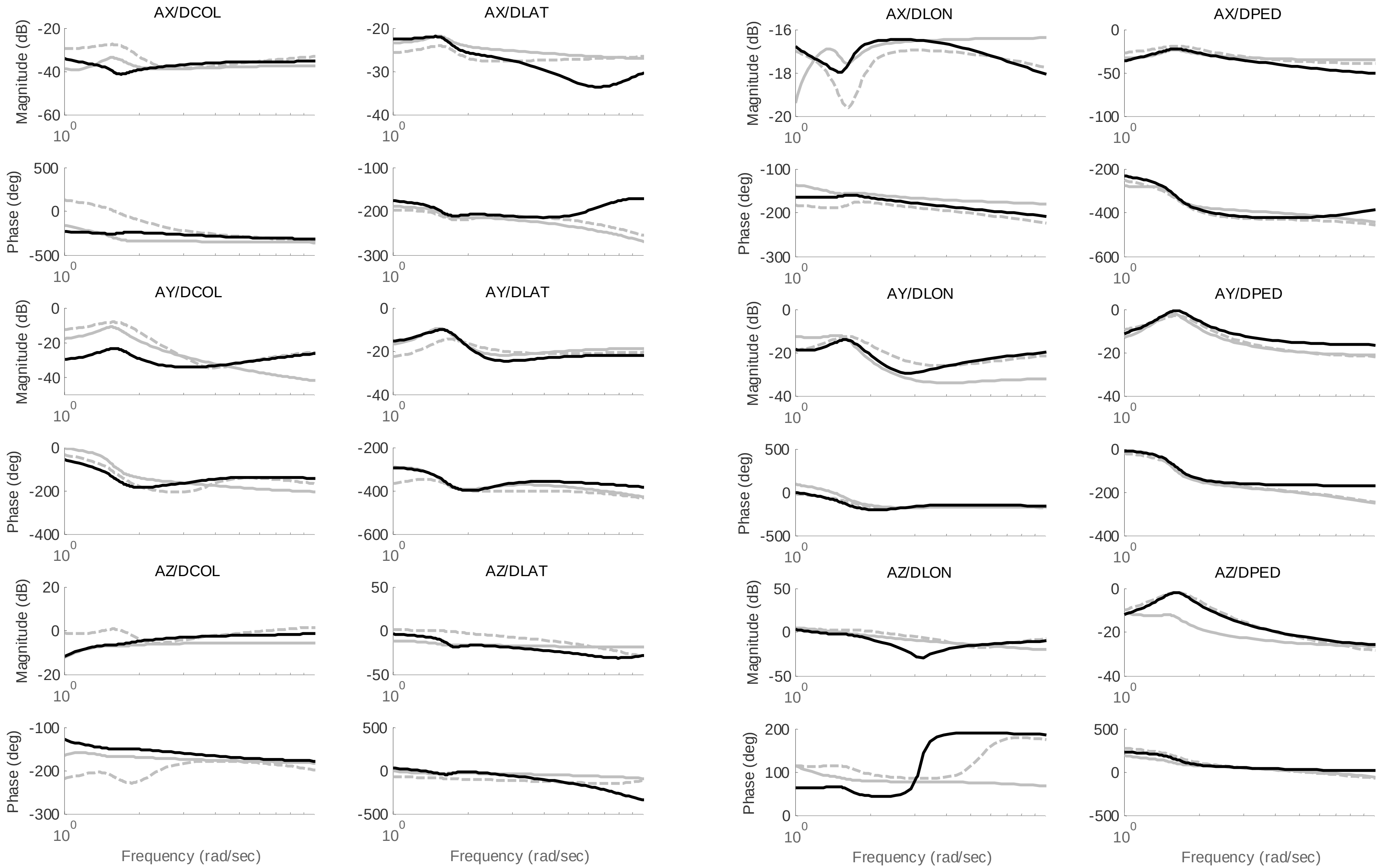


CIFER Id Model

Case studies: a full-scale helicopter
Frequency responses comparison 3/4



Case studies: a full-scale helicopter
Frequency responses comparison 4/4



Conclusions

- Rotorcraft model identification has been considered
- An overview of the state-of-the-art of rotorcraft model identification has been provided
- Continuous-time predictor-based subspace identification algorithm and black-box to grey-box transformation have been introduced
- A novel approach combining time and frequency domain data has been presented and discussed
- Preliminary results based on the AW189 simulation example has shown the viability of the proposed approach