POLITECNICO DI MILANO

    

Advanced Aerospace Control
Exam project

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Outline

- Overview of multirotor UAVs
- Dynamics of a quadrotor helicopter
- Overview of the considered platform
  - Conceptual design and preliminary sizing
  - Characterisation of sensors and actuators
    - Inertial Measurement Unit calibration
    - Actuator analysis
    - Identification of roll and pitch dynamics
- Attitude estimation
- Attitude control law design
- Project task
- Inputs

Overview of multirotor UAVs

- Multirotor helicopters

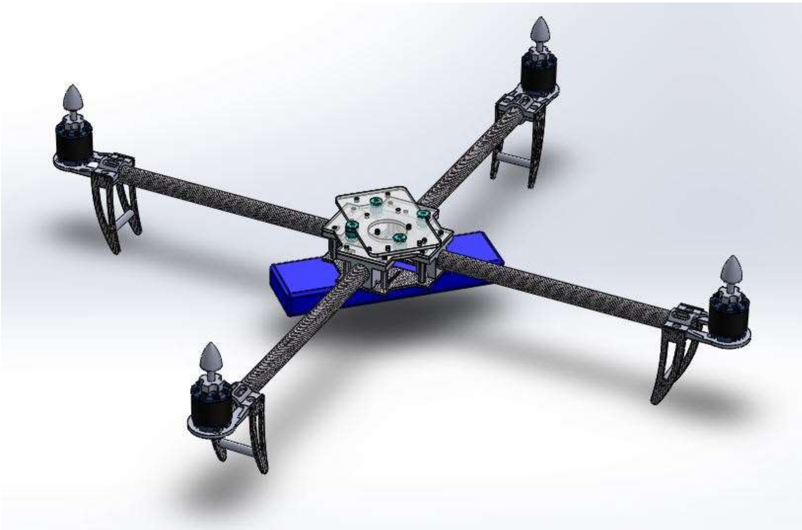
Application areas:

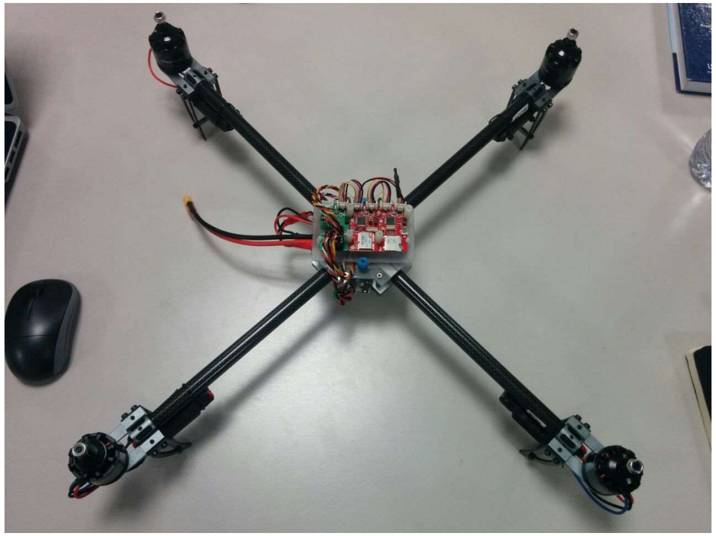
- Inspection
- Surveillance
- Security
- Mapping
- Search & Rescue
- Video and Photography
- …

Depending on the application, different designs might be suitable

Case study: a medium size quadrotor

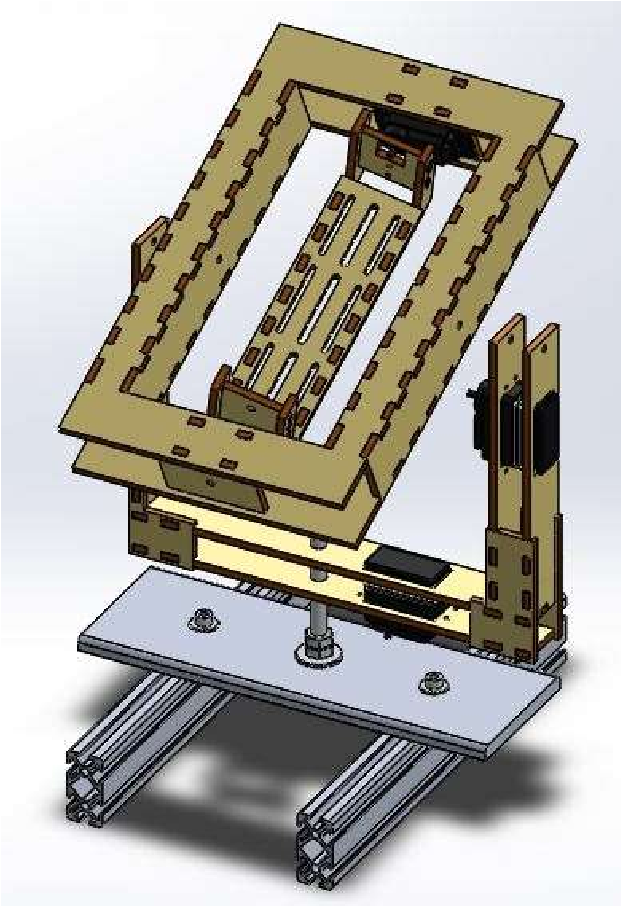
- Design requirements:
  - Take-off weight: < 2 kg
  - Flight time: about 10‘
  - Payload: at least 500 g
  - Suitable as platform for control research and education.
  - Frame configuration: X-quadrotor
  - Frame dimensions: medium size

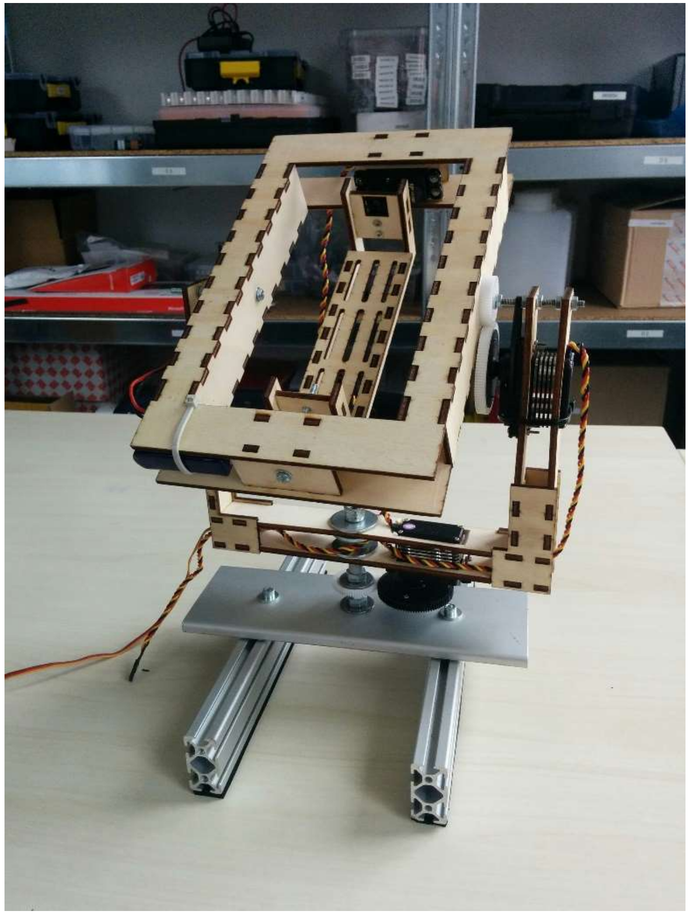




Characterisation of sensors and actuators: IMU calibration

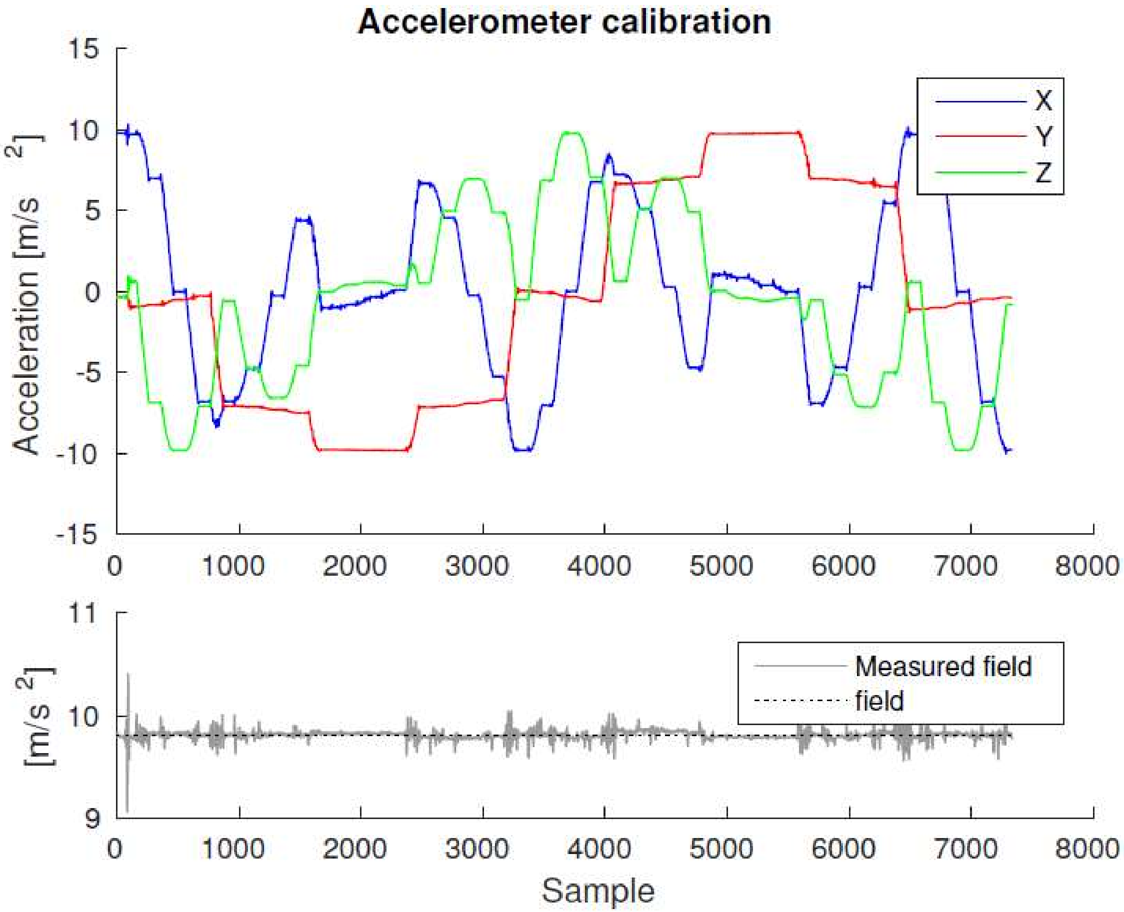
- A calibration platform for the on-board IMU has been designed and built





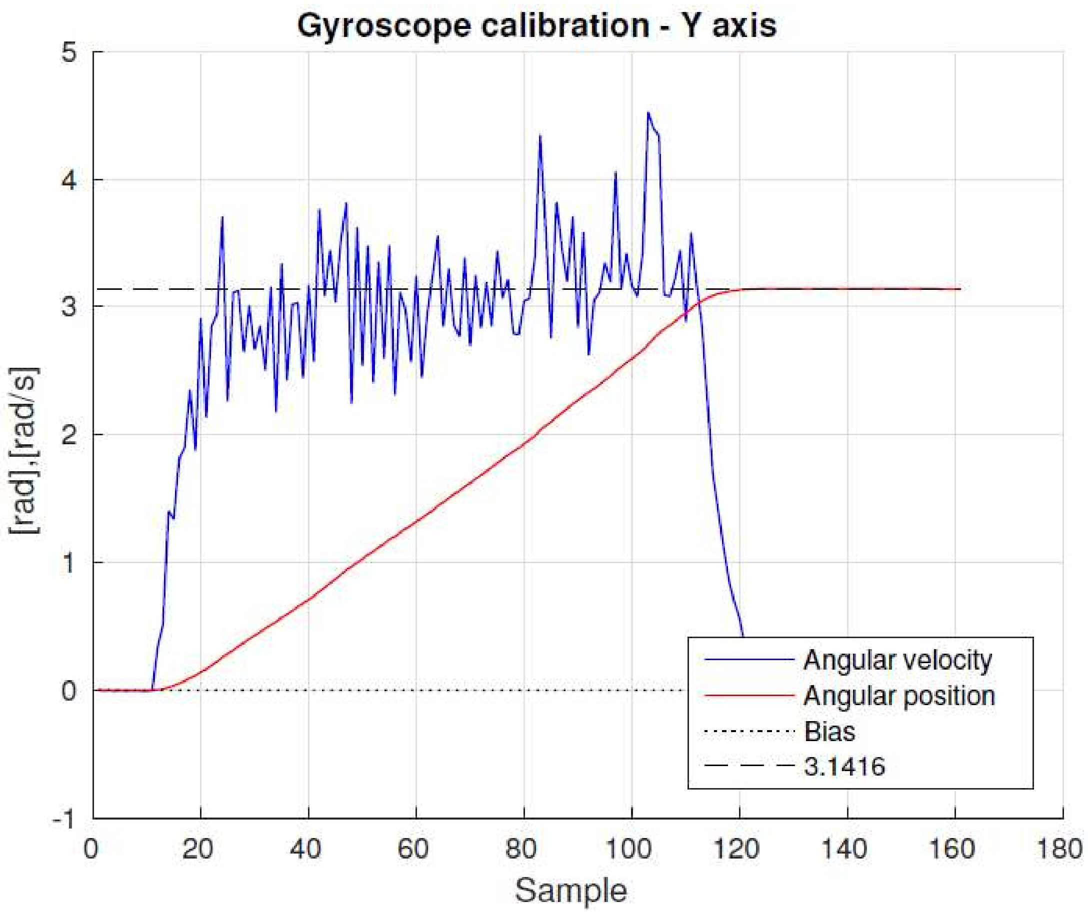
Characterisation of sensors and actuators: IMU calibration

- Accelerometer and magnetometer calibration results



Characterisation of sensors and actuators: IMU calibration

- Gyroscope calibration results

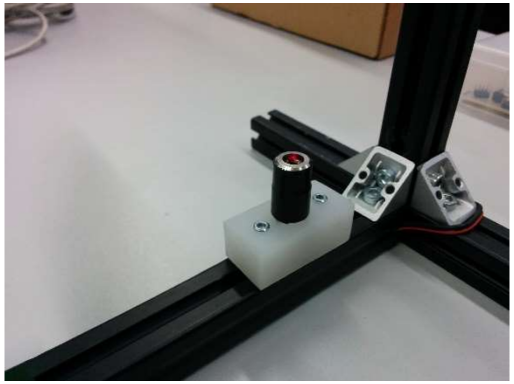


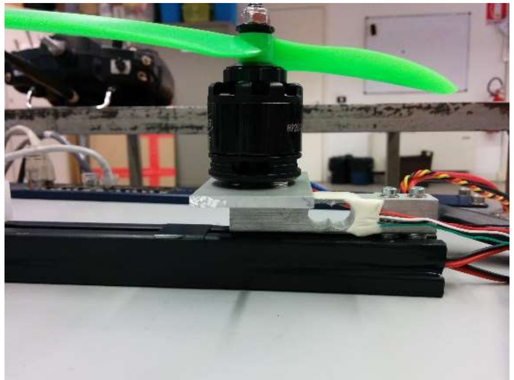
Characterisation of sensors and actuators: motor and propeller

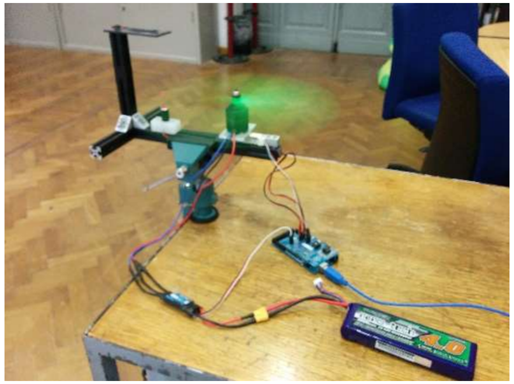
- TESTING PLATFORM
- To measure the propeller's thrust against rotational speed

➢ Sensors
  - Load cell
- Thrust
  - Optical tachometer
- Angular rate

➢ Electronic board
  - Arduino MEGA







STATIC RESPONSE ESTIMATION

- According to rotor momentum theory the following relations between thrust and rotational speed hold

$$T = K_T \Omega^2$$

$$K_T = C_T \rho A R^2$$

$$\longrightarrow$$

$$\widehat{C}_T = \frac{\widehat{K}_T}{\rho A R^2}$$

$$\widehat{C}_P = \frac{\widehat{C}_T^{3/2}}{\sqrt{2}}$$

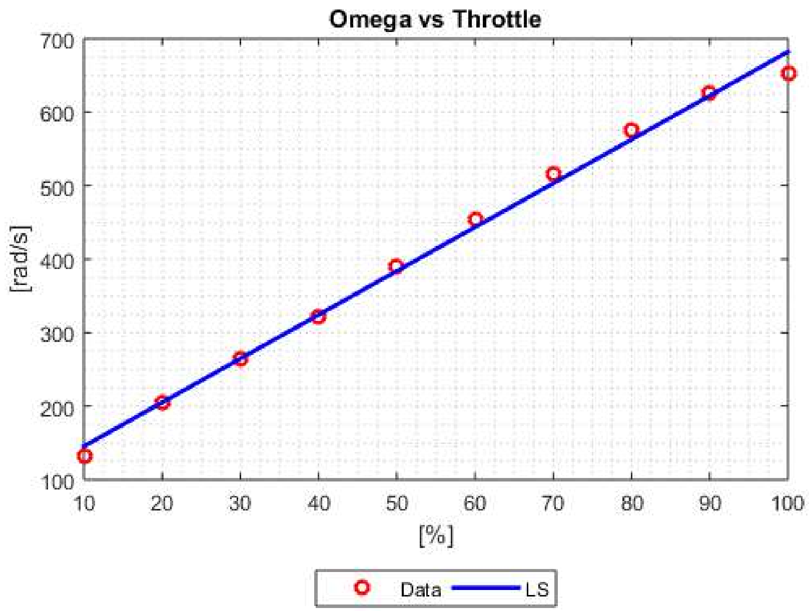
$$\widehat{C}_Q = \widehat{C}_P$$

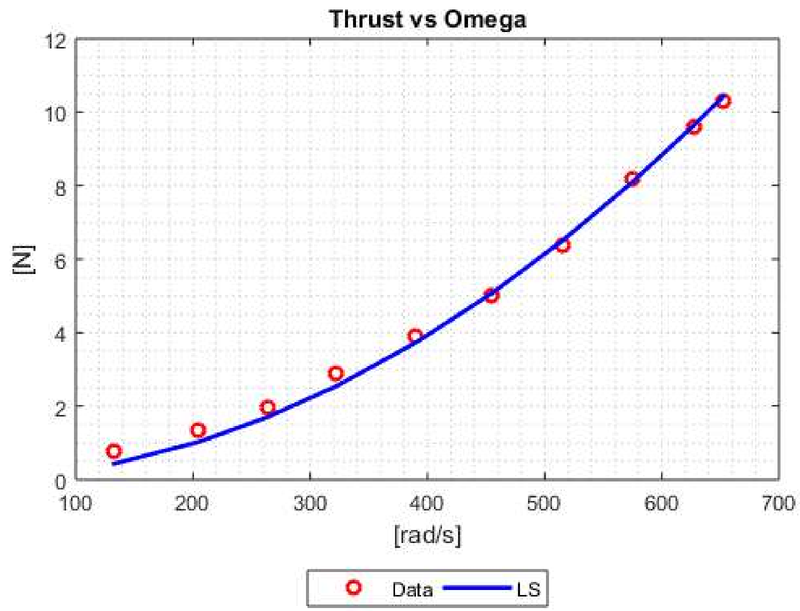
- Relationship between the percentage of throttle and propeller's rotational speed

$$\Omega = \widehat{m} Th_{\%} + \widehat{q}$$

Characterisation of sensors and actuators: motor and propeller

Results of least-squares fit for static motor and propeller model

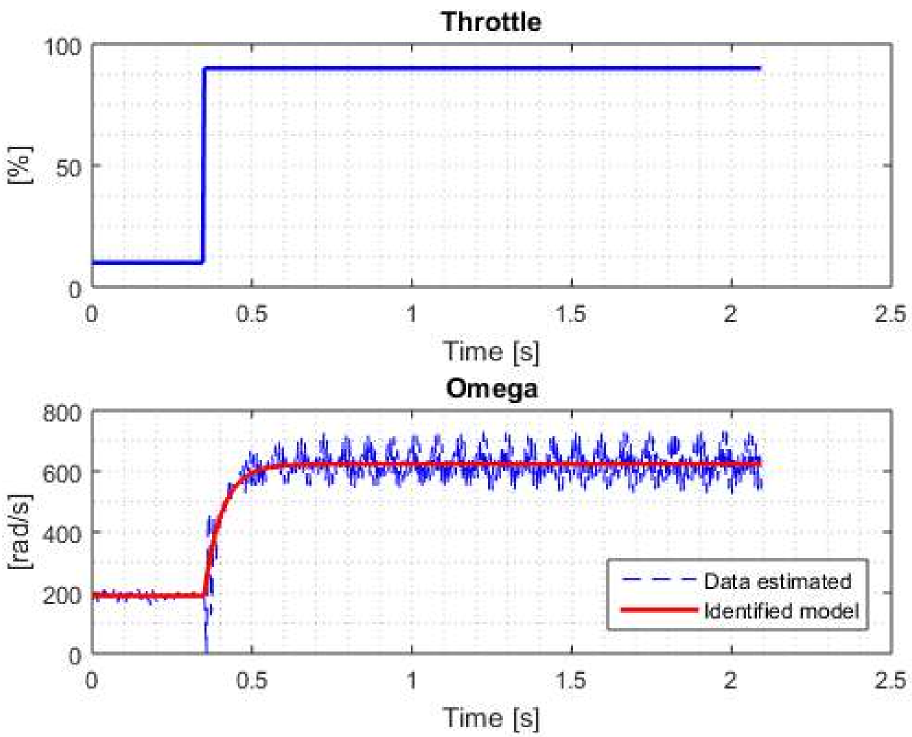




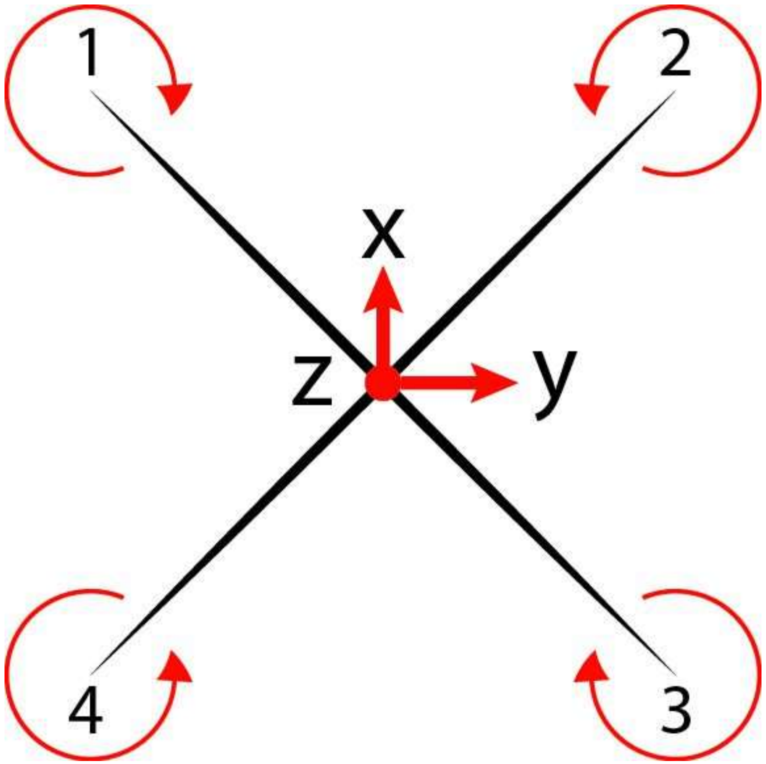
- Linear characteristic
- Quadratic characteristic

Characterisation of sensors and actuators: motor and propeller

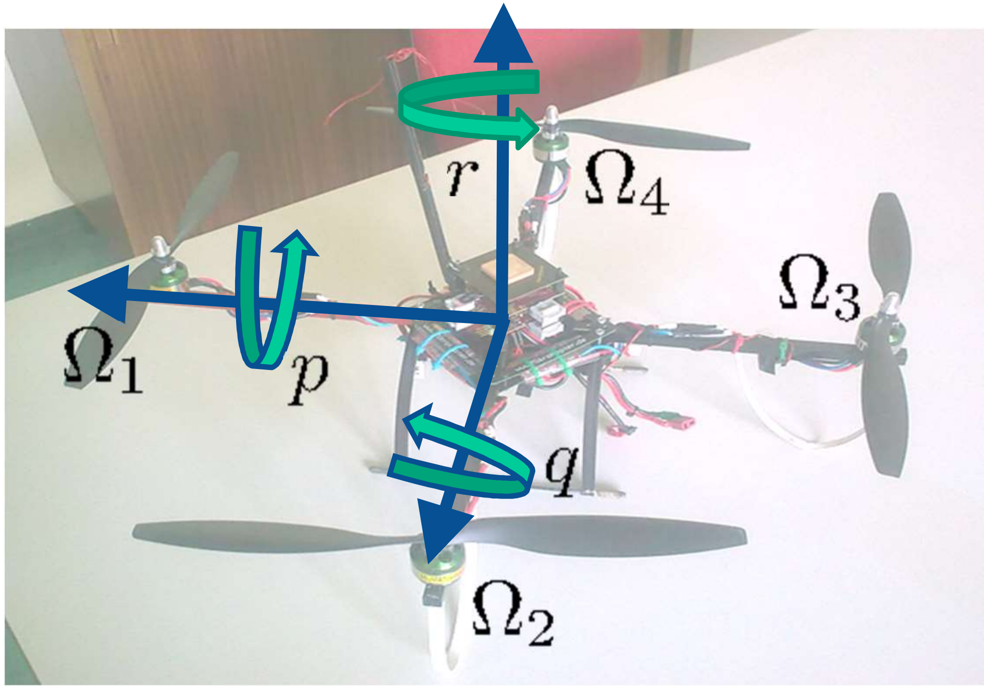
- DINAMIC RESPONSE ESTIMATION
- Assuming a first order dynamical relationship between the percentage of throttle as input and the rotational speed of the propeller as output

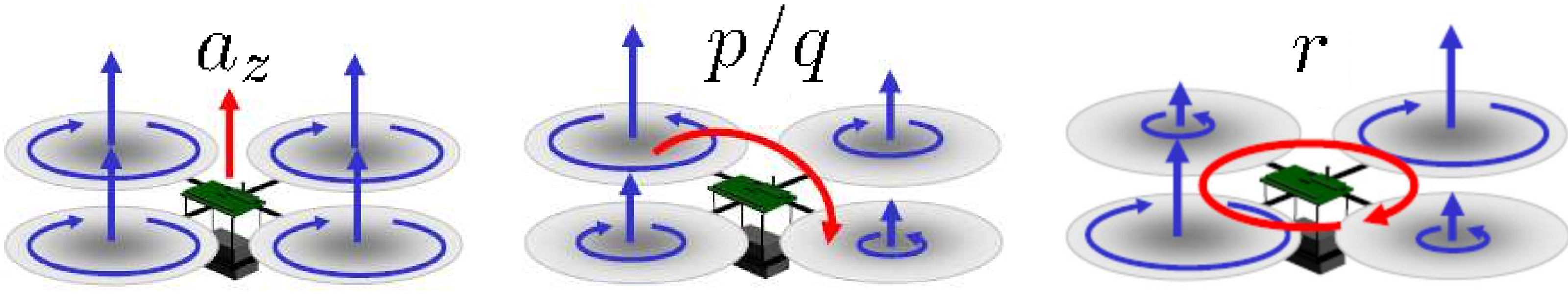


- Quadrotor helicopter configuration



Quadrotor flight control: basic principles





Collective (U_{col}) Pitch/Roll (U_{lon}/U_{lat}) Yaw (U_{ped})

Dynamics of a quadrotor helicopter

- Overall model for linear and angular quadrotor dynamics

$$m\,\dot{V}_b + \omega_b \times (mV_b) = F_g + F_{props}$$
$$I_n\,\dot{\omega}_b + \omega_b \times (I_n\omega_b) = M_{damp} + M_{props}$$

- Gravity component

$$F_g = T_{BE}(\Phi,\Theta,\Psi)\begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} = \begin{bmatrix} -S_\Theta \\ S_\Phi C_\Theta \\ C_\Phi C_\Theta \end{bmatrix} mg.$$

- Damping component

$$M_{damp} = \begin{bmatrix} \frac{\partial L}{\partial p} & 0 & 0 \\ 0 & \frac{\partial M}{\partial q} & 0 \\ 0 & 0 & \frac{\partial N}{\partial r} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

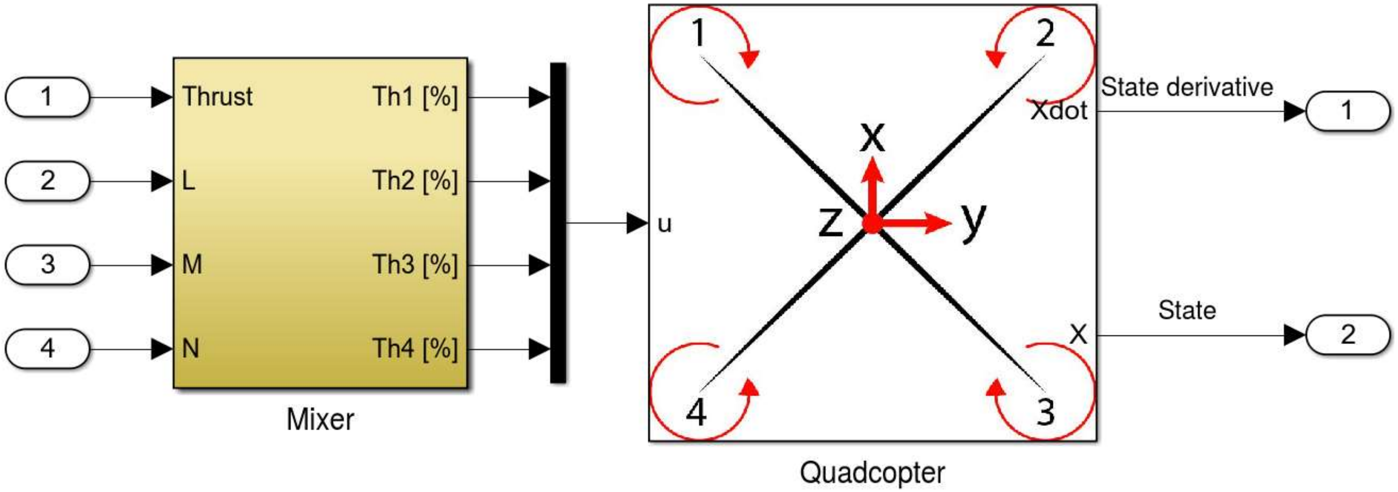
- Propellers thrust and torque

$$F_{props} = -\begin{bmatrix} 0 \\ 0 \\ K_T\left(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2\right) \end{bmatrix} \qquad M_{props} = \begin{bmatrix} K_T \frac{b}{\sqrt{2}}\left(\Omega_1^2 - \Omega_2^2 - \Omega_3^2 + \Omega_4^2\right) \\ K_T \frac{b}{\sqrt{2}}\left(\Omega_1^2 + \Omega_2^2 - \Omega_3^2 - \Omega_4^2\right) \\ K_Q\left(-\Omega_1^2 + \Omega_2^2 - \Omega_3^2 + \Omega_4^2\right) \end{bmatrix}$$

Dynamics of a quadrotor helicopter

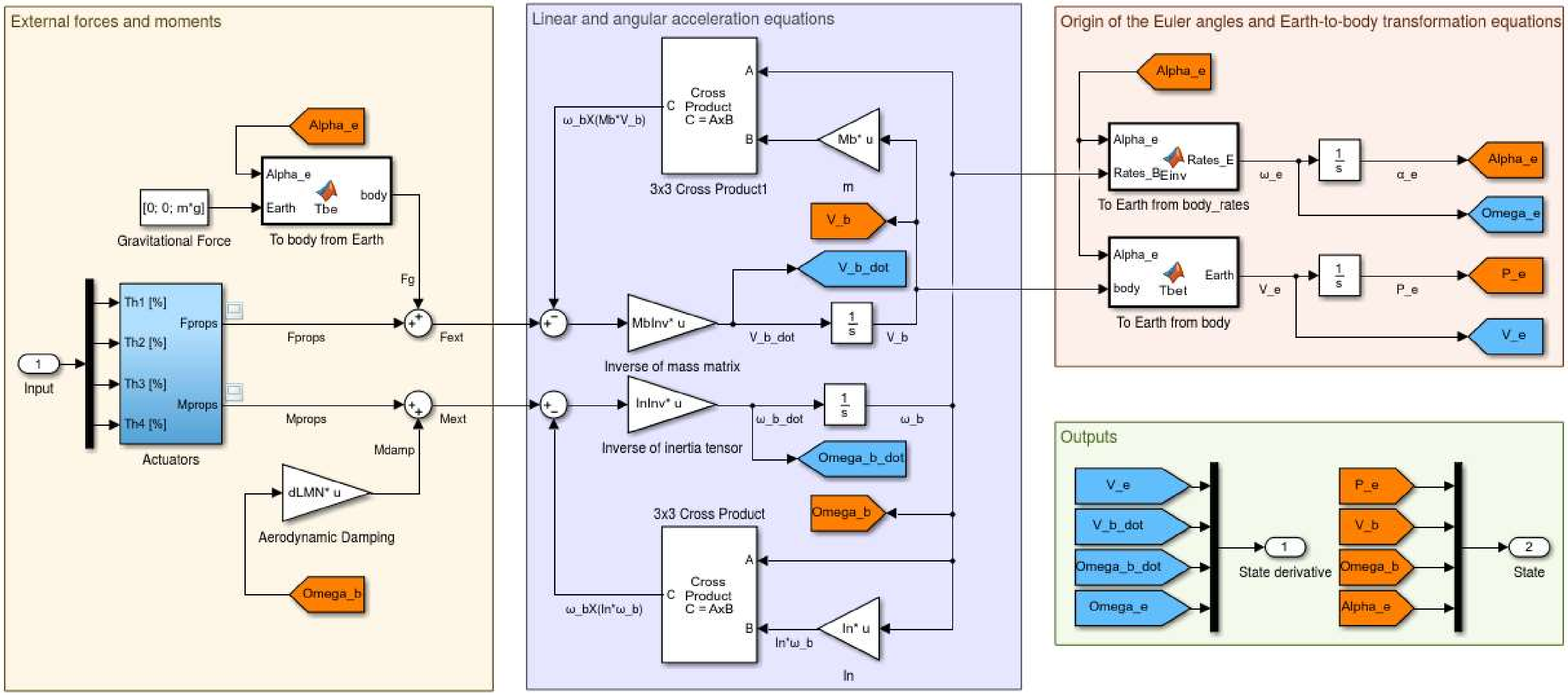
- Mixer matrix

$$\begin{bmatrix} T \\ L \\ M \\ N \end{bmatrix} = \begin{bmatrix} K_T & K_T & K_T & K_T \\ K_T\frac{b}{\sqrt{2}} & -K_T\frac{b}{\sqrt{2}} & -K_T\frac{b}{\sqrt{2}} & K_T\frac{b}{\sqrt{2}} \\ K_T\frac{b}{\sqrt{2}} & K_T\frac{b}{\sqrt{2}} & -K_T\frac{b}{\sqrt{2}} & -K_T\frac{b}{\sqrt{2}} \\ -K_Q & K_Q & -K_Q & K_Q \end{bmatrix} \begin{bmatrix} \Omega_1^2 \\ \Omega_2^2 \\ \Omega_3^2 \\ \Omega_4^2 \end{bmatrix}$$



Dynamics of a quadrotor helicopter

- Simulink multirotor simulation environment



Attitude kinematics and dynamics

$$\dot{\phi} = p + \sin(\phi)\tan(\theta)\, q + \cos(\phi)\tan(\theta)\, r$$

$$\dot{\theta} = \cos(\phi)\, q - \sin(\phi)\, r$$

$$\dot{\psi} = \frac{\sin(\phi)}{\cos(\theta)}\, q + \frac{\cos(\phi)}{\cos(\theta)}\, r$$

$$\dot{p} = \frac{I_y - I_z}{I_x}\, q\, r + \frac{1}{I_x}\, M_p$$

$$\dot{q} = \frac{I_z - I_x}{I_y}\, p\, r + \frac{1}{I_y}\, M_q$$

$$\dot{r} = \frac{I_x - I_y}{I_z}\, p\, q + \frac{1}{I_z}\, M_r$$

On-board attitude estimation

The complementary filter of Mahony et al.

The filter of Mahony *et al.*, is a nonlinear attitude observer formulated on the special orthogonal group SO(3).

Algorithm:

1. Drift estimation:

$$\omega_{mes} = vex\left(\sum_{i=1}^{n} \frac{K_i}{2}\left(b_i \hat{b}_i^T - \hat{b}_i b_i^T\right)\right), \hat{b}_i = \hat{A}_{k-1} r_i,$$

$$vex\,(\Omega \times) = \Omega, \quad \Omega \in \mathcal{R}^3.$$

2. Bias estimation and gyroscope depolarization

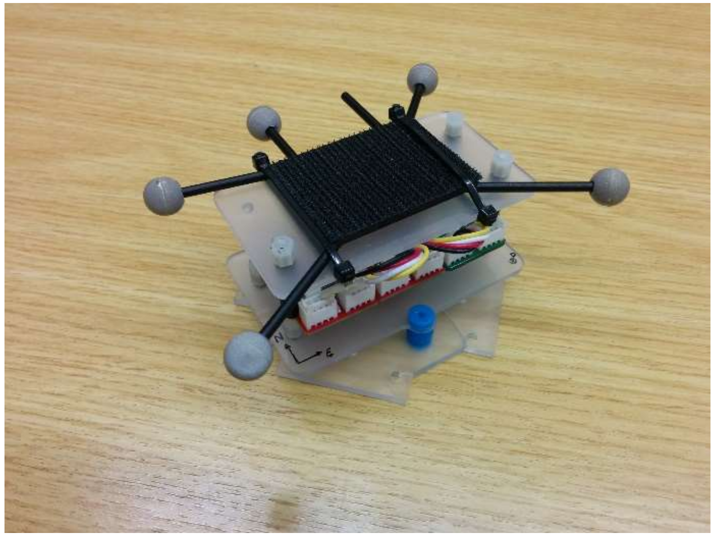
$$\hat{\omega}_k = \tilde{\omega}_k - \hat{\beta}_k - K_p \omega_{mes}, \hat{\beta}_k = \hat{\beta}_{k-1} + K_i \omega_{mes} \delta t.$$

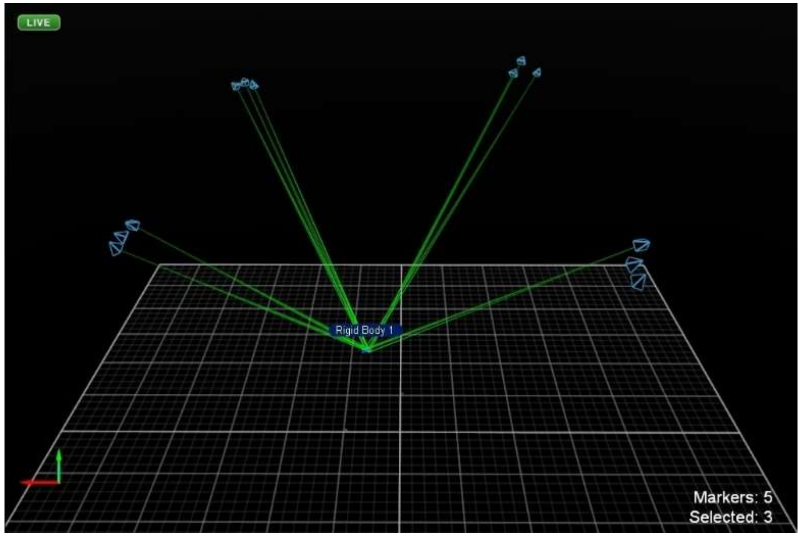
3. Quaternion integration

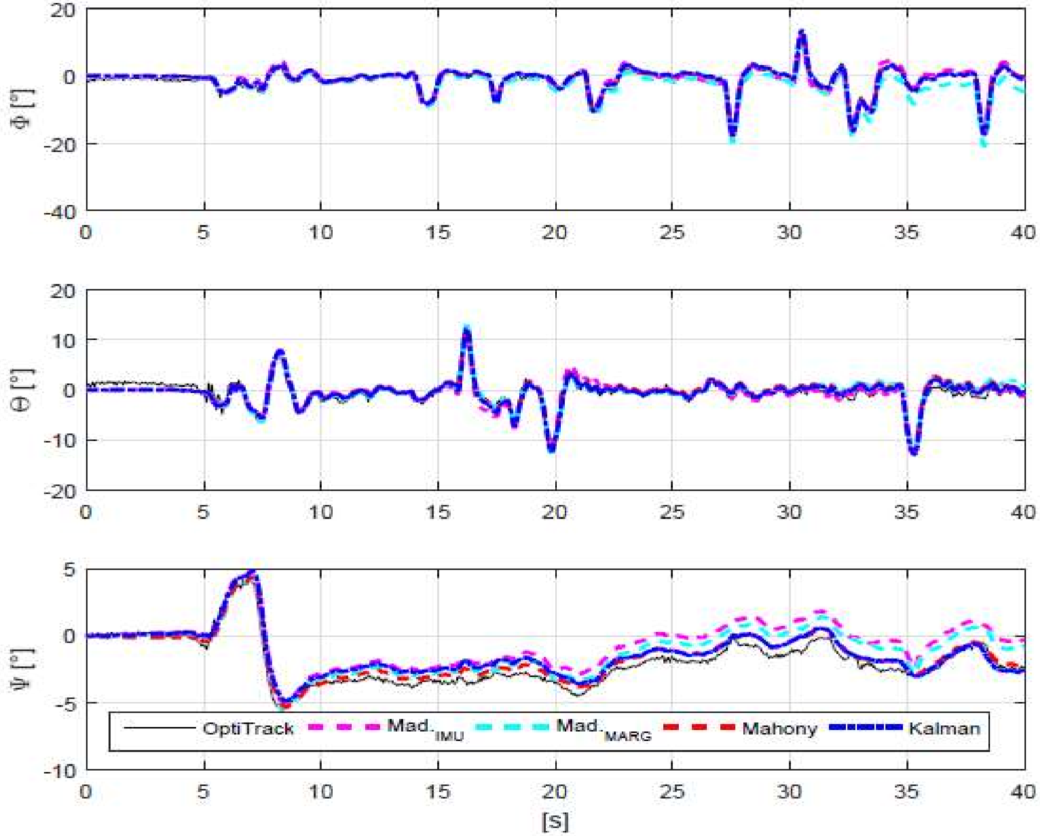
$$\hat{q}_k = \left(I_4 + \frac{1}{2}\Omega\left(\hat{\omega}_k\right)\delta t\right)\hat{q}_{k-1}.$$

On-board attitude estimation

- As an absolute reference of the attitude measure a motion capture system (OptiTrack) has been used.







	Φ_{RMS} [^{o}]	Θ_{RMS} [^{o}]	Ψ_{RMS} [^{o}]	$Mean_{RMS}$ [^{o}]
MEKF	0.7959	0.9331	0.7195	0.8161
Mad.$_{IMU}$	1.1771	1.0543	1.4801	1.2372
Mad.$_{MARG}$	1.8169	1.1816	1.0529	1.3505
Mahony *et al.*	0.7794	1.0728	0.5183	0.7901

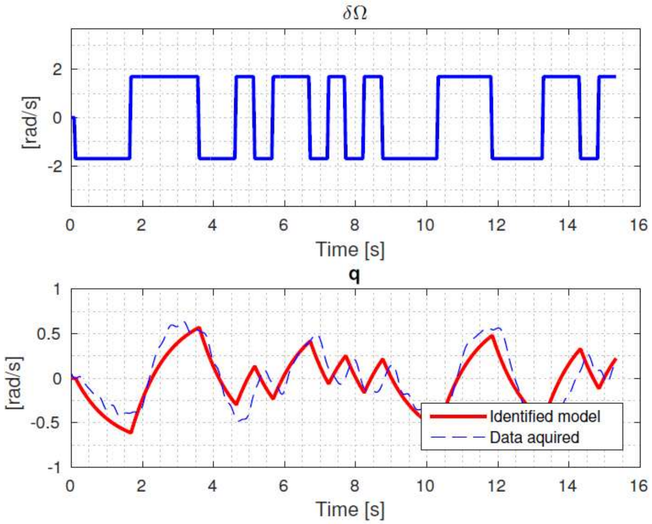
Table 1: RMS errors for the considered filters.

Identification of roll and pitch dynamics

- Linearized pitch model

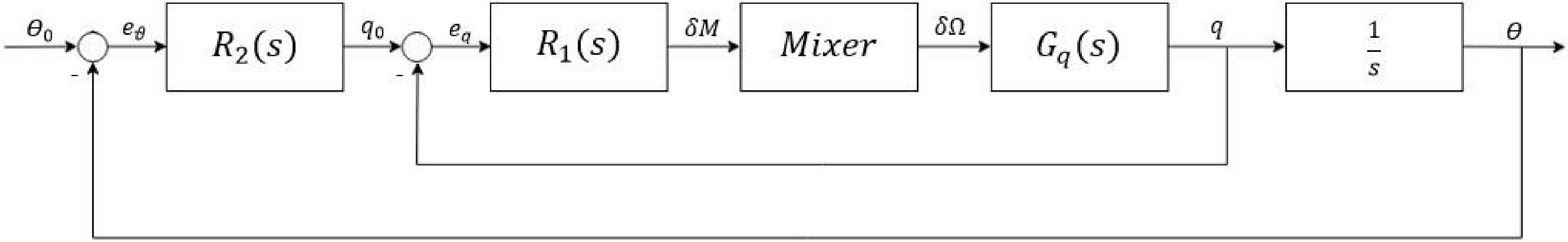
$$I_{yy}\,\dot{q} = \frac{\partial M}{\partial q} q + \frac{\partial M}{\partial u}\delta\Omega$$
$$\dot{\Theta} = q$$

$$\longrightarrow \quad \Theta = \left[I_{yy}, \frac{\partial M}{\partial q} \right]$$

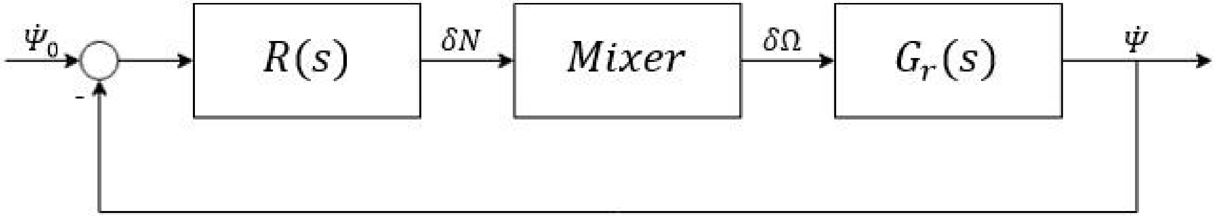


Attitude control law design

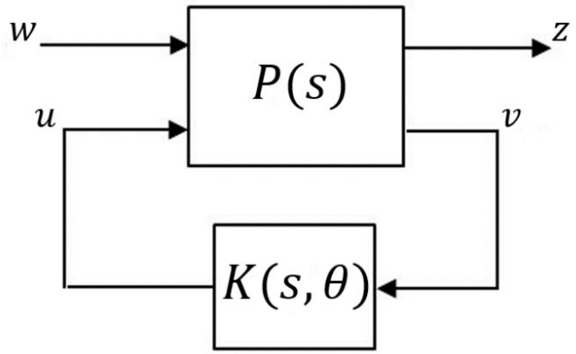
- Pitch and roll control architecture



- Yaw-rate control architecture



- Structured H_∞ synthesis



$$\left\| T_{w \to z}\left(P(s), K(s,\theta)\right) \right\|_\infty$$

- Requirements:
  - Set-point tracking

$$J(\theta) = \left\| \frac{1}{MaxError}\left(F(s,\theta) - 1\right) \right\|_\infty$$

$$MaxError = \frac{(PeakError)s + \omega_c(DCError)}{s + \omega_c}$$

  - Maximum loop gain

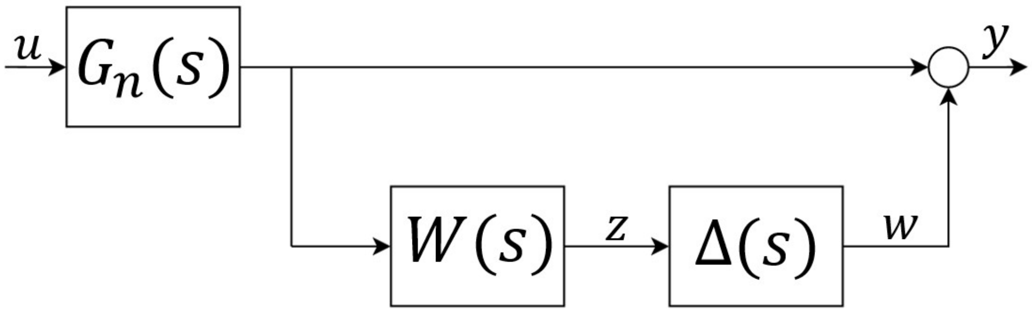
$$J(\theta) = \left\| W_T F(s,\theta) \right\|_\infty$$

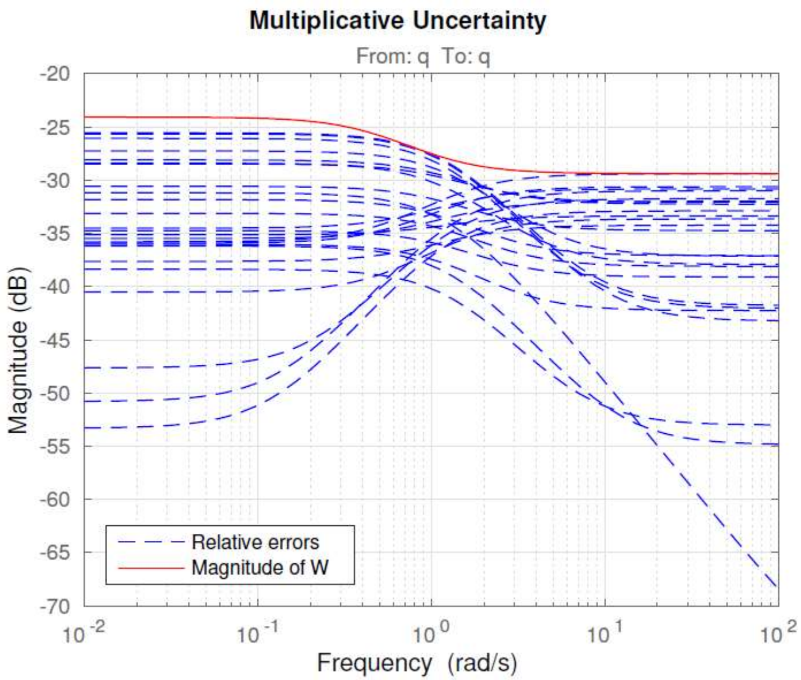
  - Disturbance rejection

$$J(\theta) = \max_{\omega \in \Omega} \left\| W(j\omega) S(j\omega,\theta) \right\|$$

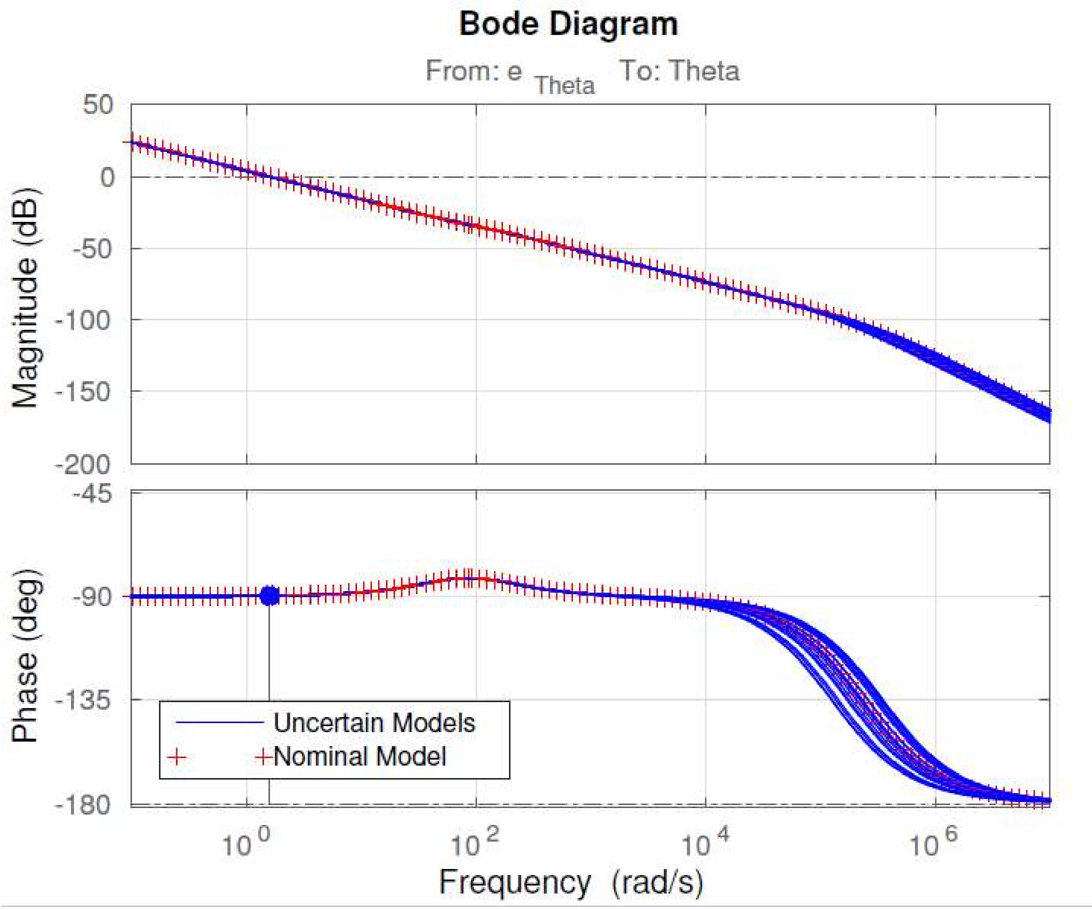
- Robustness analysis

$$G_m(s) = G_n(s)(1 + W(s)\Delta(s)),\ \|\Delta\|_\infty < 1$$



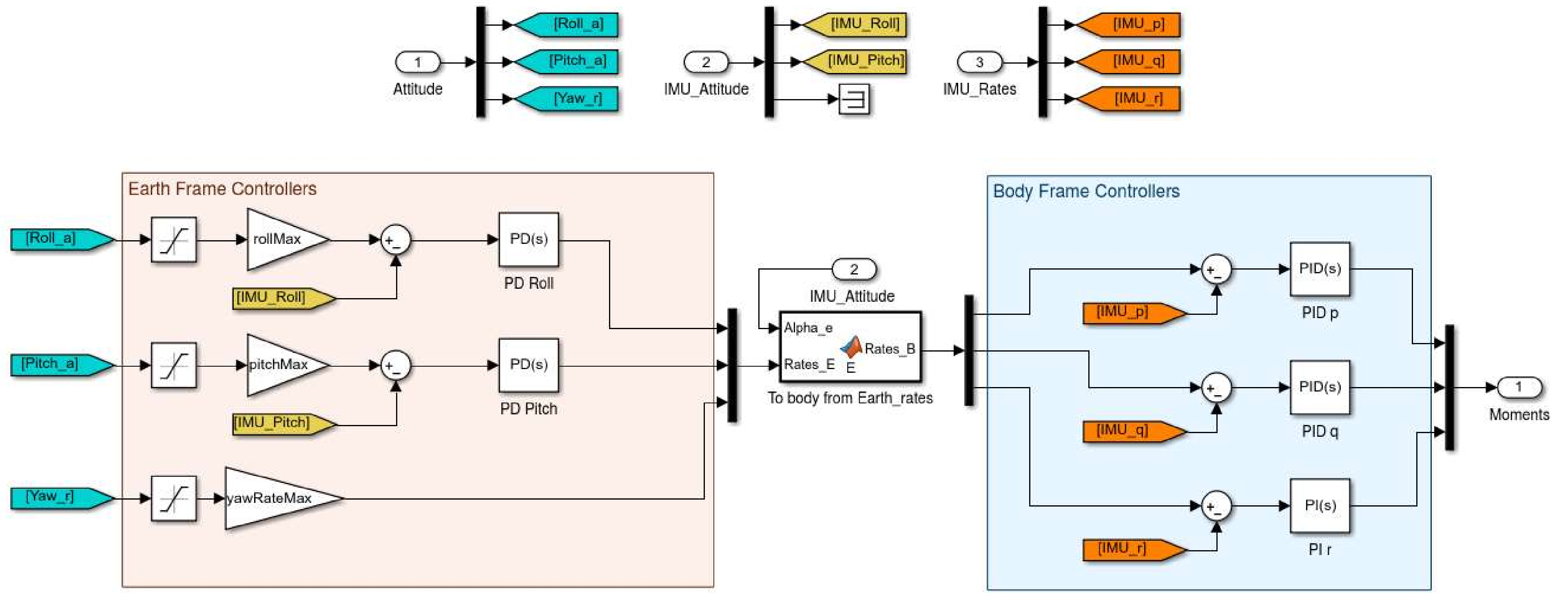


- Robustness analysis -> Loop function



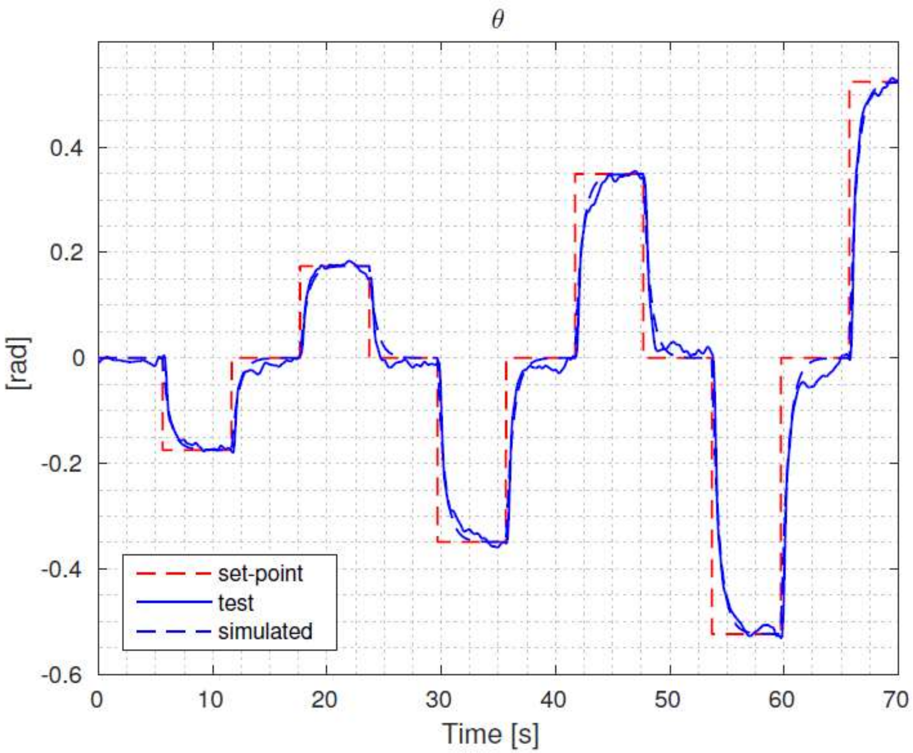
Attitude control law design

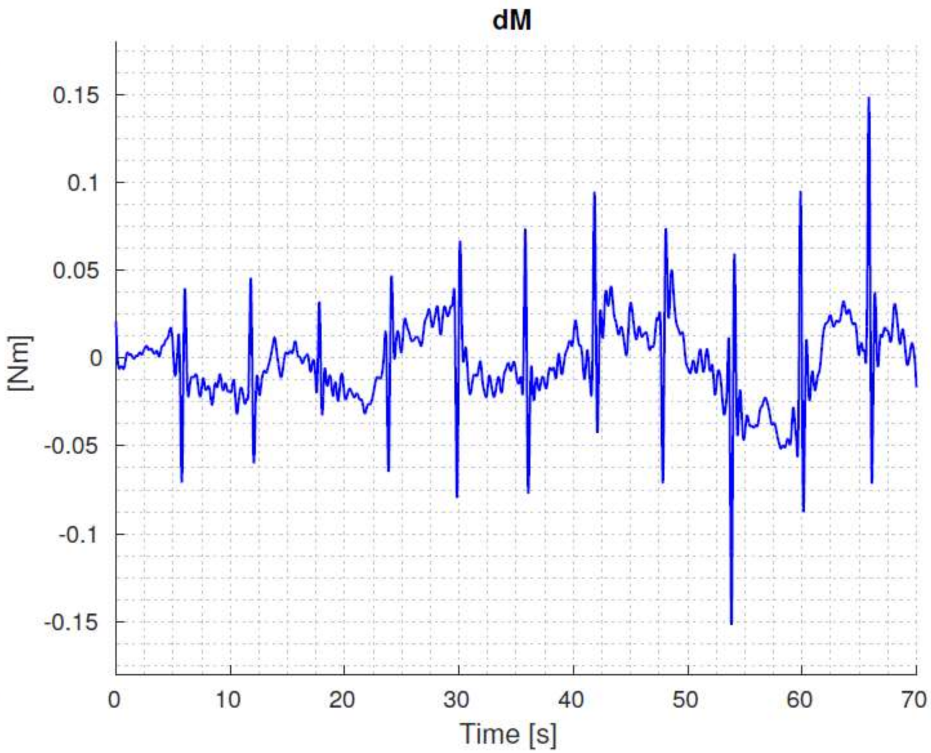
- Simulink implementation



Attitude control law design

- Step response results





Exam project

Inputs:

- Quadrotor simulation model (Simulink)
- Uncertain design model, cascade of
  - identified attitude dynamics
  - Actuator dynamics
  - Dynamics of attitude estimation filter
- Description of current controller structure
- Performance requirements:
  - Tracking
  - Disturbance attenuation
- Benchmark mission profile: position and heading setpoint to be tracked

Exam project

Tasks:

- Construction of uncertain design model
- Definition of performance weights for mixed sensitivity synthesis
- Design of unstructured controller
- Design of structured controller matching current implementation
- Verification in simulation on benchmark mission profile
- In-flight validation on real quadrotor (with one possibility for re-tuning)

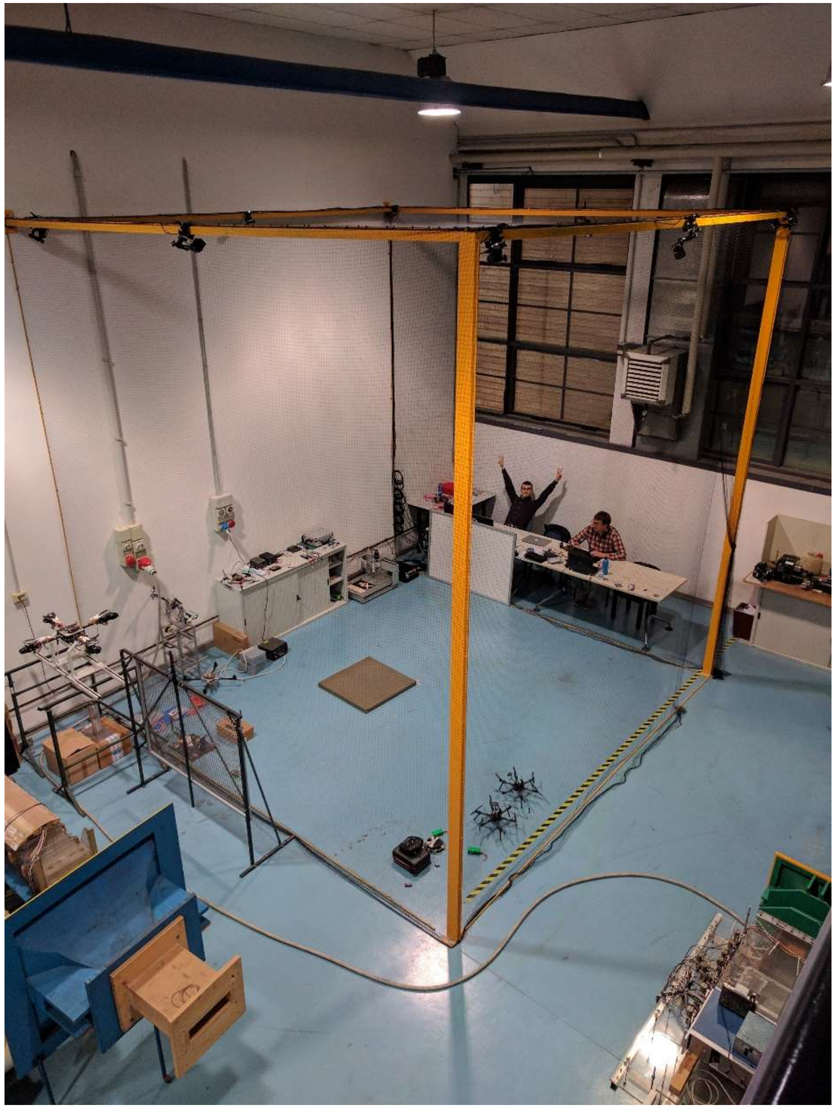
Exam project

Expected outputs:

- Presentation of adopted design approach and design results
- In-flight validation.

Indoor test facility

- Located in building B14
- 5 x 5 x 5 m cage
- Optitrack 3D motion capture system
- Optitrack data (position and attitude) used on-board
- Available for project validation



Practical issues: safety course

- To do even minimal experimental work in the DAER labs an extra safety course must be taken.
- The course takes about half a day and takes place approximately once every two month.
- Next edition: May 5.
- So: if you are interested in the exam project AND plan to do the exam in June/July, let me know asap so I can register you for the course.