ANALYSIS OF LOAD PATTERNS IN RUBBER COMPONENTS FOR VEHICLES

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Outline

Motivation

Software Description

Connectivity

Constitutive Laws

Application: Car Suspension Model

Conclusions & Acknowledgements
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Conclusions & Acknowledgements
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- Blend of exact, arbitrary kinematics & nonlinear finite elements
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- Today: fully developed industrial-grade computational tool
- Blend of exact, arbitrary kinematics & nonlinear finite elements
- Ideal playground for multidisciplinary problems
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  → multibody dynamics
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- Hutchinson traditionally developed specialized finite element analysis capabilities in-house
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Industrial requirement: preserve in-house analysis capabilities for multibody analysis as well; solution:
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- Industrial requirement: preserve in-house analysis capabilities for multibody analysis as well; solution:
  \[\Rightarrow\text{use free software}\]

as an alternative to “reinventing the wheel”
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- mainly applied to rotorcraft dynamics and aeroservoelasticity, but it is currently exploited (at Politecnico di Milano and by 3rd parties) in projects involving:
  - Rotorcraft dynamics (helicopters, tiltrotors)
  - Aircraft landing gear analysis
  - Robotics and mechatronics, including real-time simulation
  - Automotive
  - Wind turbines
  - Biomechanics
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Further information at http://www.aero.polimi.it/~mbdyn/
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Connectivity

Mechanical systems are modeled as **nodes**, that provide *shared* equations and degrees of freedom, connected by **elements**, that contribute to *shared* equations and optionally provide *private* ones.

- Relevant elements can be:
  - **rigid bodies** → contribute inertia properties to nodes
  - **joints** → add private algebraic equations that constrain nodes → contribute constraint reactions to shared equations
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Connectivity

Selected test application: car suspensions model

a wide variety of deformable components model rubber parts
Connectivity

Deformable components:
Connectivity

Deformable components:

- nodes exchange configuration-dependent forces and moments

\[ F = F(u, \dot{u}, \ldots) \]

\[ u = u(u_1, u_2) \]

\[ F_1 = T_1(u_1, u_2) \]

\[ F_2 = T_2(u_1, u_2) \]

Connectivity and constitutive models development is decoupled; provided an adequately expressive API is designed, they mutually benefit from each other.
Connectivity

Deformable components:

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- separation between constitutive model...

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Deformable components implemented in MBDyn:
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Deformable components implemented in MBDyn:

- 1D component: rod
- 3D component: angular spring
- 3D component: linear spring
- 6D component: linear & angular spring
- 6D component: geometrically “exact”, composite ready beam
- Component Mode Synthesis (CMS) element
Connectivity

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1D Component: Rod

- connects two points \( p_1, p_2 \)
- optionally offset from the respective nodes:
  
  \[
  p_1 = x_1 + f_1 \\
  p_2 = x_2 + f_2
  \]

- offsets are rigidly connected to nodes

  \[
  f_1 = R_1 \bar{f}_1 \\
  f_2 = R_2 \bar{f}_2
  \]

- straining related to distance between points \( p_1 \) and \( p_2 \):

  \[
  l = p_2 - p_1 \\
  \varepsilon = \frac{\sqrt{l^T l}}{l_0} - 1
  \]
3D Component: Angular Spring

- connects two nodes $x_1$, $x_2$
- straining related to perturbation $\theta$ of relative orientation $R = R_1^T R_2$
- the joint has no location in space (it is typically paired to a spherical hinge or other relative position constraint)
3D Component: Angular Spring

- relative orientation matrix $\mathbf{R} = \mathbf{R}_1^T \mathbf{R}_2$
3D Component: Angular Spring

- relative orientation matrix \( \mathbf{R} = \mathbf{R}_1^T \mathbf{R}_2 \)
- \( \rightarrow \) implies that the component constitutive properties are intrinsically referred to node 1
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- relative orientation vector $\mathbf{\theta} = a \times (\exp^{-1} (\mathbf{R}))$
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- relative orientation vector $\mathbf{\theta} = \mathbf{ax} \left( \exp^{-1} (\mathbf{R}) \right)$
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- $\rightarrow$ implies that the component constitutive properties are intrinsically referred to node 1
- relative orientation vector $\theta = \alpha \left( \exp^{-1}(\mathbf{R}) \right)$
- relative angular velocity $\mathbf{\omega} = \mathbf{R}_1^T (\mathbf{\omega}_2 - \mathbf{\omega}_1)$
- internal moment $\overline{\mathbf{M}} = \overline{\mathbf{M}}(\theta, \mathbf{\omega})$, referred to node 1
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- internal moment \( \mathbf{\bar{M}} = \mathbf{\bar{M}}(\mathbf{\theta}, \mathbf{\omega}) \), referred to node 1
- contribution to virtual work \( \delta \mathcal{L} = -\mathbf{\theta}_\delta^T \mathbf{\bar{M}} \)
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- contribution to virtual work $\delta \mathcal{L} = -\theta^T \delta \mathbf{M}$
- relative orientation virtual perturbation $\theta_\delta = \mathbf{R}_1^T (\theta_{2\delta} - \theta_{1\delta})$
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- relative angular velocity \( \mathbf{\omega} = \mathbf{R}_1^T (\mathbf{\omega}_2 - \mathbf{\omega}_1) \)
- internal moment \( \mathbf{\overline{M}} = \mathbf{\overline{M}} (\mathbf{\theta}, \mathbf{\omega}) \), referred to node 1
- contribution to virtual work \( \delta \mathcal{L} = -\mathbf{\theta}_\delta^T \mathbf{\overline{M}} \)
- relative orientation virtual perturbation \( \mathbf{\theta}_\delta = \mathbf{R}_1^T (\mathbf{\theta}_2^\delta - \mathbf{\theta}_1^\delta) \)
- contributions to node equilibrium

\[
\mathbf{M}_1 = \mathbf{M} \\
\mathbf{M}_2 = -\mathbf{M}
\]

with \( \mathbf{M} = \mathbf{R}_1 \mathbf{\overline{M}} \)
3D Component: Angular Spring

Pros:

▶ the formulation is straightforward

Cons:

▶ the model is biased towards one node
▶ as a consequence, formulating any constitutive law but isotropic may not be straightforward

With linear anisotropic constitutive law, if connectivity is reversed:
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► reversing the order of the nodes always solved the problem

Rationale:

► the behavior of the component is intrinsically independent from the ordering of the connectivity
► if the connectivity formulation depends on its ordering, the constitutive properties need to take care of invariance
► otherwise, connectivity must take care of invariance itself
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- relative orientation $\mathbf{R} = \mathbf{R}_1^T \mathbf{R}_2$ remains the same
3D Component: “Invariant” Angular Spring

- relative orientation \( R = R_1^T R_2 \) remains the same
- constitutive properties referred to mid-rotation \( \tilde{\theta} = \frac{1}{2} \theta \),
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- perturbation of intermediate orientation $\tilde{\theta}_\delta = \left( \mathbf{I} + \tilde{\mathbf{R}} \right)^{-1} \theta_\delta$
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- perturbation of intermediate orientation $\tilde{\theta}_\delta = \left( \mathbf{I} + \tilde{\mathbf{R}} \right)^{-1} \theta_\delta$
- absolute mid-rotation orientation $\hat{\mathbf{R}} = \mathbf{R}_1 \tilde{\mathbf{R}} = \mathbf{R}_2 \tilde{\mathbf{R}}^T$
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- absolute mid-rotation orientation $\hat{\mathbf{R}} = \mathbf{R}_1 \tilde{\mathbf{R}} = \mathbf{R}_2 \tilde{\mathbf{R}}^T$
- relative angular velocity $\bar{\boldsymbol{\omega}} = \hat{\mathbf{R}}^T (\omega_2 - \omega_1)$
- internal moment is now $\overline{\mathbf{M}} = \overline{\mathbf{M}} (\theta, \bar{\boldsymbol{\omega}})$
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- relative angular velocity $\overline{\omega} = \hat{\mathbf{R}}^T (\omega_2 - \omega_1)$
- internal moment is now $\overline{\mathbf{M}} = \overline{\mathbf{M}}(\theta, \overline{\omega})$
- internal moment in the absolute frame $\mathbf{M} = \hat{\mathbf{R}} \overline{\mathbf{M}}$
3D Component: “Invariant” Angular Spring

Pros:

- the model is no longer biased towards one node
- simpler constitutive laws may be formulated
3D Component: “Invariant” Angular Spring

Pros:
- the model is no longer biased towards one node
- simpler constitutive laws may be formulated

Cons:
- the formulation is less straightforward
- little bit more computationally expensive
3D Component: Linear Spring

- Same as rod, but...
- Straining related to distance between points $\varepsilon = p_2 - p_1$
- The joint does not react pure relative rotation (pin; usually paired to relative orientation constraint)
3D Component: Linear Spring

- same issue about “attached” vs. “invariant” formulation
3D Component: Linear Spring

- same issue about “attached” vs. “invariant” formulation
- formulation of invariant case even less straightforward
  (not presented here, but fully developed and implemented in the software)
3D Component: Linear Spring

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Same linear anisotropic constitutive law, transverse shear case
6D Components

- linear & angular spring
6D Components

- linear & angular spring
  - models fully coupled bushings
6D Components

- linear & angular spring
  - models fully coupled bushings
  - formulation fully developed, but...
6D Components

- linear & angular spring
  - models fully coupled bushings
  - formulation fully developed, but...
  - ...only partially implemented, essentially because of limited usefulness so far

see Ghiringhelli et al., AIAA Journal, 2000
6D Components

- linear & angular spring
  - models fully coupled bushings
  - formulation fully developed, but...
  - ...only partially implemented, essentially because of limited usefulness so far

- kinematically “exact”, composite ready beam
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Outline

Motivation

Software Description

Connectivity

**Constitutive Laws**

Application: Car Suspension Model

Conclusions & Acknowledgements
Constitutive Laws

- define the input/output relationship required by deformable components

\[ F = F(u, \dot{u}) \]
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▸ this aspect is emphasized in the implementation exploiting C++ templates for different dimensionalities (1D, 3D, 6D)

\[ \delta F_n = \frac{\partial F_n}{\partial u_n} \delta u_n + \frac{\partial F_n}{\partial \dot{u}_n} \delta \dot{u}_n \]
Constitutive Laws

Use:

- call \texttt{Update()} at each iteration
- \texttt{subsequent calls to $F()$, $FDE()$, $FDEPrime()$} allow to access the force and its partial derivatives (e.g. to compute the residual or the contribution to the Jacobian matrix)
- as soon as \texttt{AfterConvergence()} is called, the solution converged, and the final state can be consolidated, if needed
- the paper contains examples of C++ meta-code for constitutive laws, including isotropic and orthotropic templates
- or, refer to \texttt{mbdyn/base/constltp*} files in the source code
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- The deformable components have been applied to the analysis of a car suspension model.
Application: Car Suspension Model

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- the model consists in the full front and rear suspension system of a generic car (not representative of a specific vehicle)
Application: Car Suspension Model

- the chassis is modeled as a rigid body
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  (the use of a CMS model is foreseen for further validation)
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- the model consists in about 1300 equations, related to
  - about 100 structural nodes
  - about 60 nonlinear beam elements
  - more than 80 joints
Typical analysis:

- consists in evaluating the loads in rubber components when the model is subjected to test rig excitation
Application: Car Suspension Model

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▶ excitation pattern: acceleration imposed to front right axle
Application: Car Suspension Model

Force and moment in front right shock absorber top bushing
Application: Car Suspension Model

Force in front right shock absorber

CG acceleration
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Conclusions & Acknowledgements
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- The work illustrates versatile modeling of structural components for mechanics analysis of rubber components.

- Component behavior dependence on connectivity has been eliminated by invariant deformable components, without formulating connectivity-dependent constitutive properties.

- The features illustrated in this work will be distributed shortly, with the next release of the software (1.3.0).

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ANALYSIS OF LOAD PATTERNS IN RUBBER COMPONENTS FOR VEHICLES

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Questions?