ANALYSIS OF LOAD PATTERNS IN RUBBER COMPONENTS FOR VEHICLES

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Politecnico di Milano

Multibody Dynamics 2007

Milano, June 25–28 2007

POLITECNICO DI MILANO

Outline

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  $\longrightarrow$ use free software

  as an alternative to "reinventing the wheel"

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Further information at

http://www.aero.polimi.it/~mbdyn/

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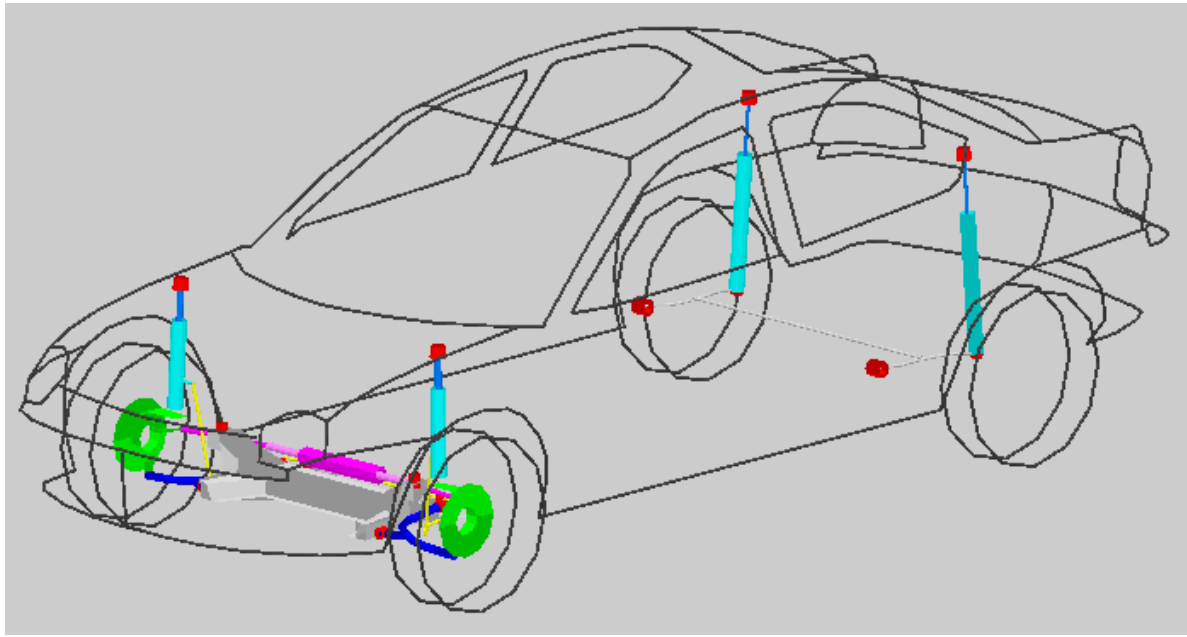
Relevant elements can be

- rigid bodies
  $\longrightarrow$ contribute inertia properties to nodes
- joints
  $\longrightarrow$ add private algebraic equations that constrain nodes
  $\longrightarrow$ contribute constraint reactions to shared equations

Connectivity

Selected test application: car suspensions model

a wide variety of deformable components model rubber parts



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$$\begin{aligned}
\mathbf{u} &= \mathbf{u}\left(\mathbf{u}_1, \mathbf{u}_2\right) \\
\mathbf{F}_1 &= \mathbf{T}_1\left(\mathbf{u}_1, \mathbf{u}_2\right)\mathbf{F} \\
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\end{aligned}$$

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- Connectivity and constitutive models development is decoupled; provided an adequately expressive API is designed, they mutually benefit from each other.

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- 1D component: rod
- 3D component: angular spring
- 3D component: linear spring
- 6D component: linear & angular spring
- 6D component: geometrically "exact", composite ready beam
- Component Mode Synthesis (CMS) element

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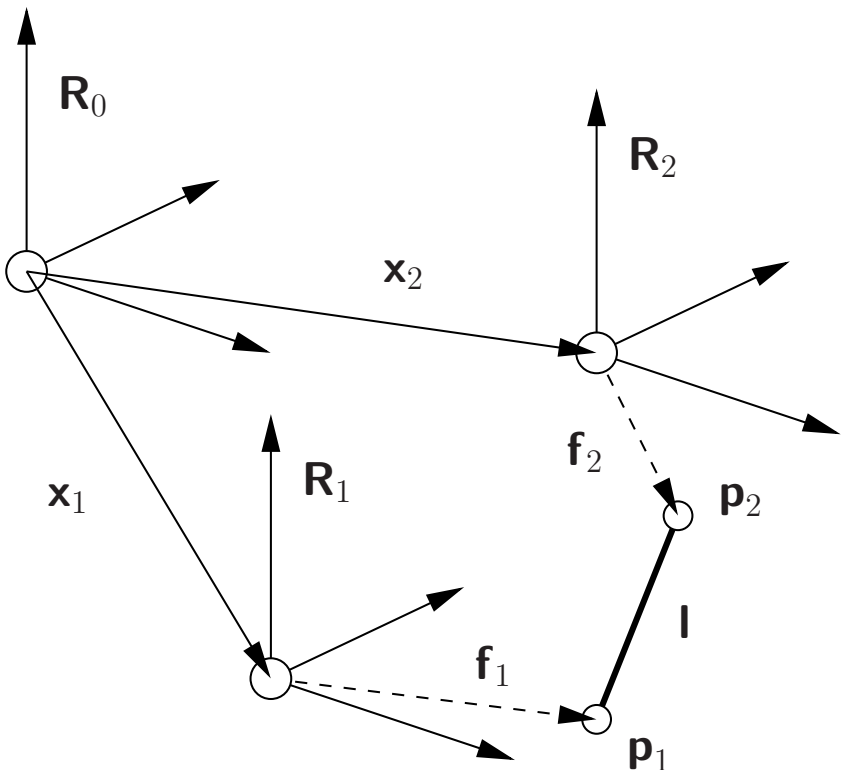
1D Component: Rod



- connects two points $\mathbf{p}_1$, $\mathbf{p}_2$
- optionally offset from the respective nodes:

$$\mathbf{p}_1 = \mathbf{x}_1 + \mathbf{f}_1$$
$$\mathbf{p}_2 = \mathbf{x}_2 + \mathbf{f}_2$$

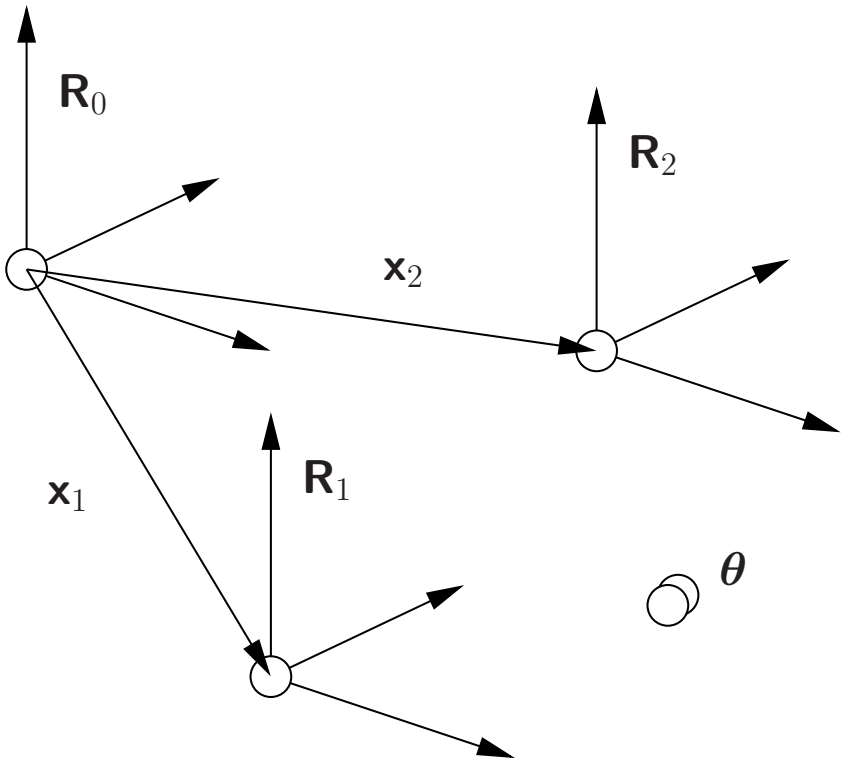
- offsets are rigidly connected to nodes

$$\mathbf{f}_1 = \mathbf{R}_1 \bar{\mathbf{f}}_1$$
$$\mathbf{f}_2 = \mathbf{R}_2 \bar{\mathbf{f}}_2$$

- straining related to distance between points $\mathbf{p}_1$ and $\mathbf{p}_2$:

$$\mathbf{l} = \mathbf{p}_2 - \mathbf{p}_1 \qquad \varepsilon = \frac{\sqrt{\mathbf{l}^T \mathbf{l}}}{l_0} - 1$$

3D Component: Angular Spring



- connects two nodes $\mathbf{x}_1$, $\mathbf{x}_2$
- straining related to perturbation θ of relative orientation $\mathbf{R} = \mathbf{R}_1^T \mathbf{R}_2$
- the joint has no location in space (it is typically paired to a spherical hinge or other relative position constraint)

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$$\mathbf{M}_1 = \mathbf{M}$$
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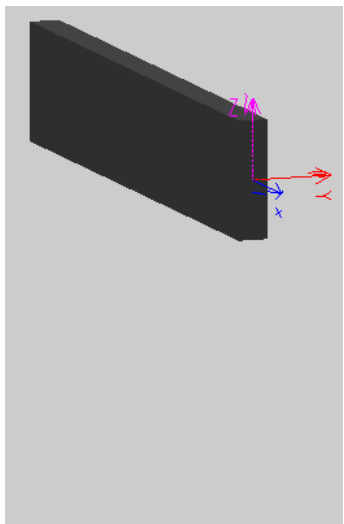

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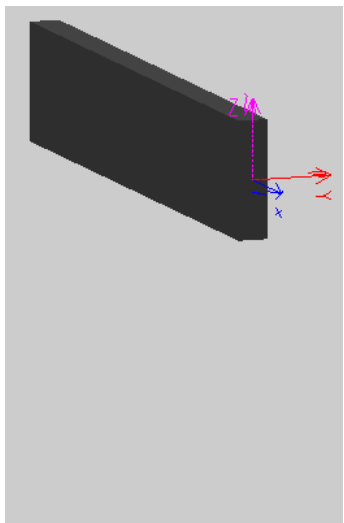

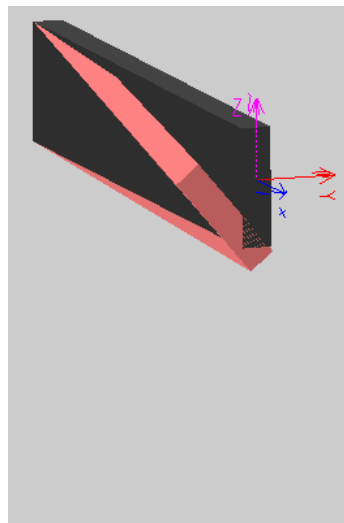

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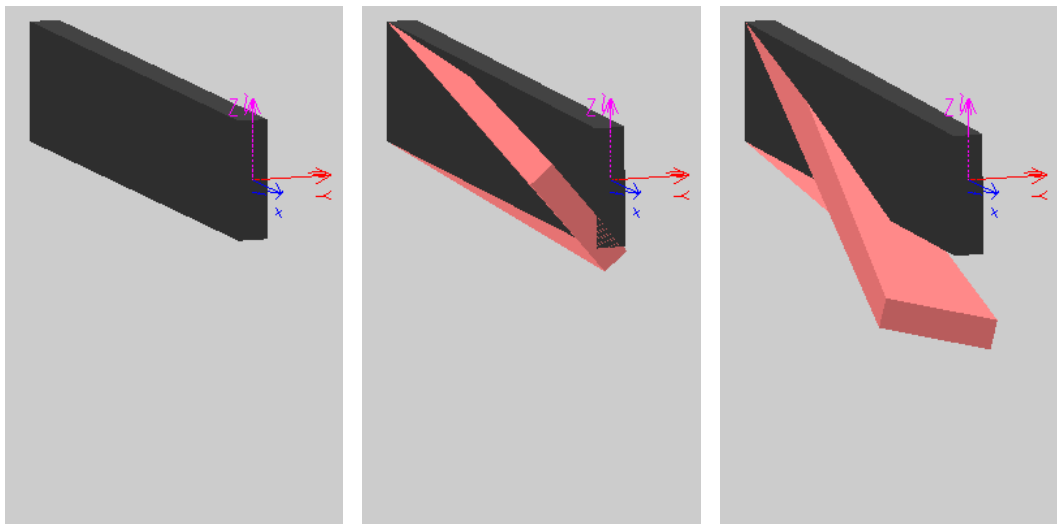

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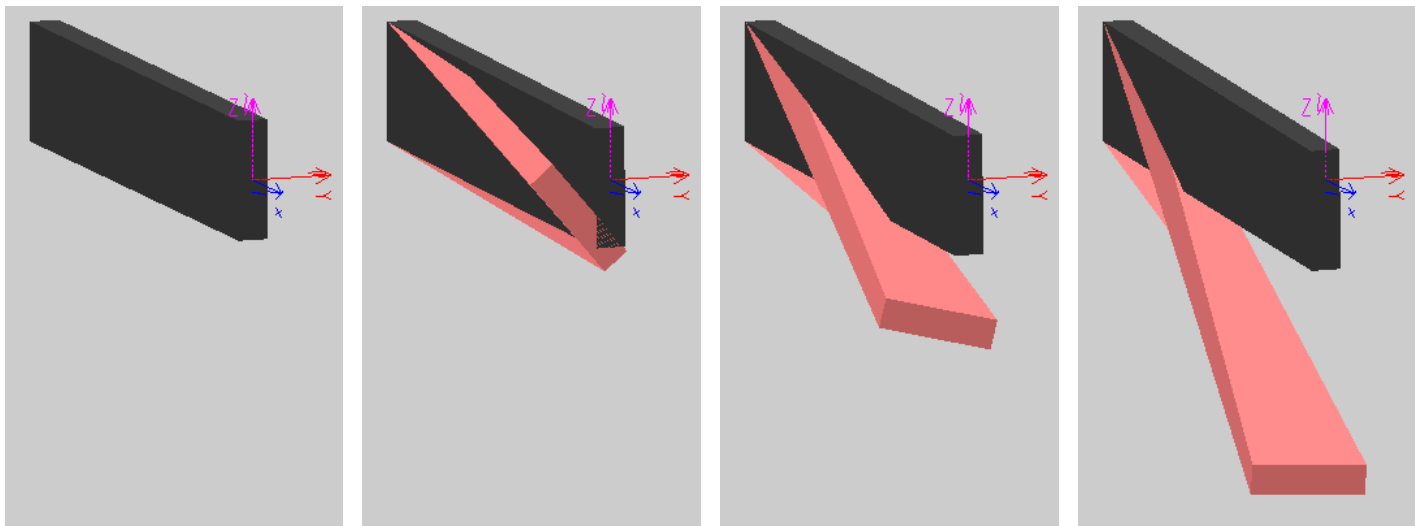

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- otherwise, connectivity must take care of invariance itself

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Cons:

- the formulation is less straightforward
- little bit more computationally expensive

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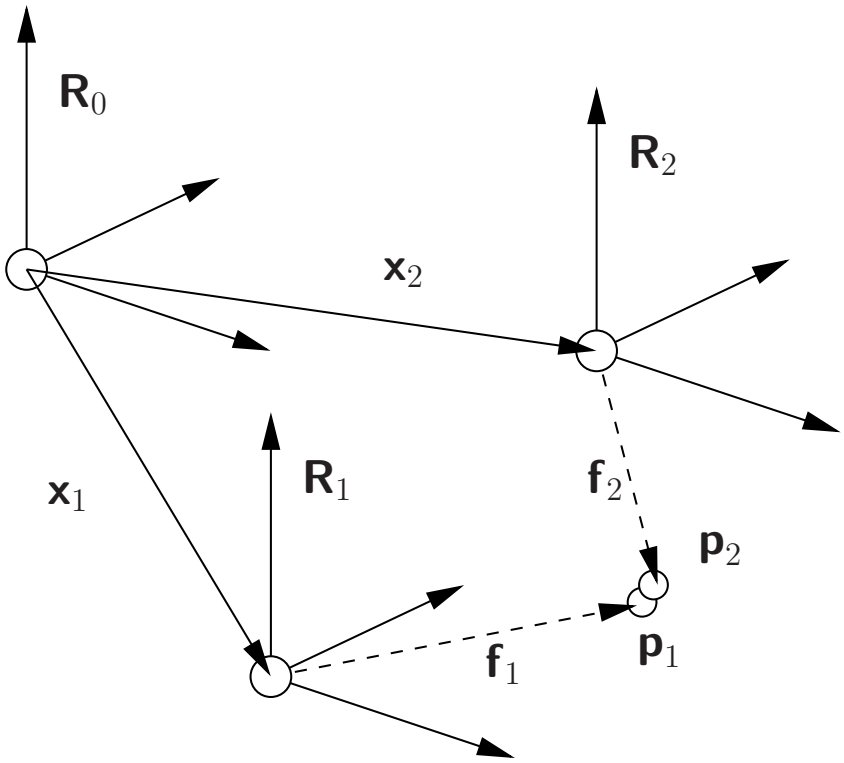

- Same as rod, but...
- straining related to distance between points $\varepsilon = \mathbf{p}_2 - \mathbf{p}_1$
- the joint does not react pure relative rotation (pin; usually paired to relative orientation constraint)

3D Component: Linear Spring

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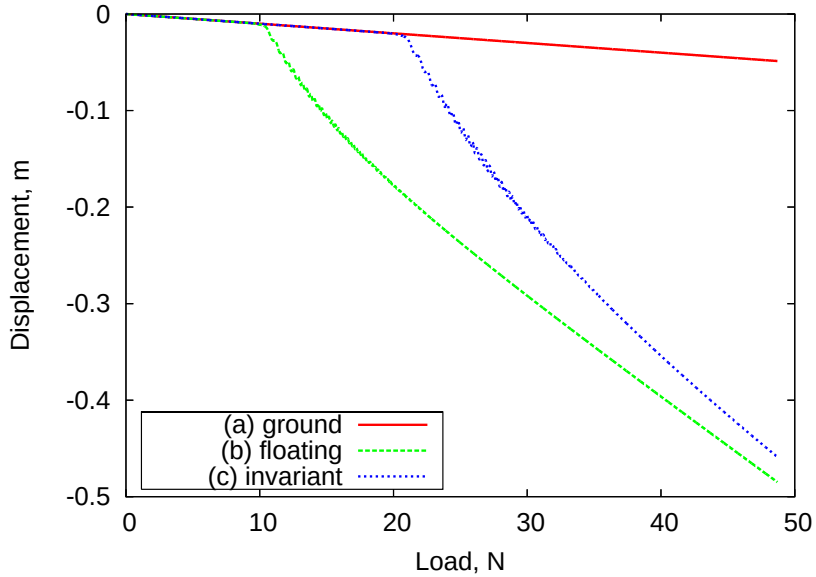
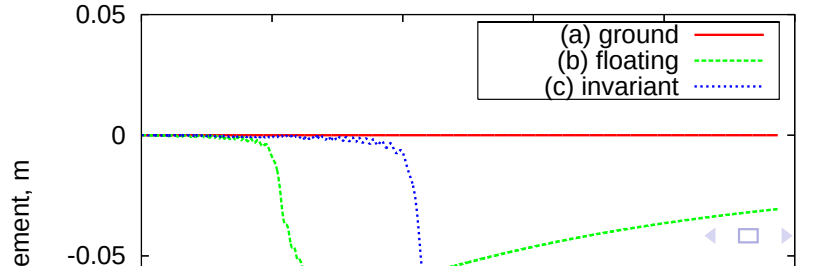
Same linear anisotropic constitutive law, transverse shear case





6D Components

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  - see Ghiringhelli et al., AIAA Journal, 2000

Outline

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- define the input/output relationship required by deformable components

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- this aspect is emphasized in the implementation exploiting C++ *templates* for different dimensionalities (1D, 3D, 6D)

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- or, refer to `mbdyn/base/constltp*` files in the source code

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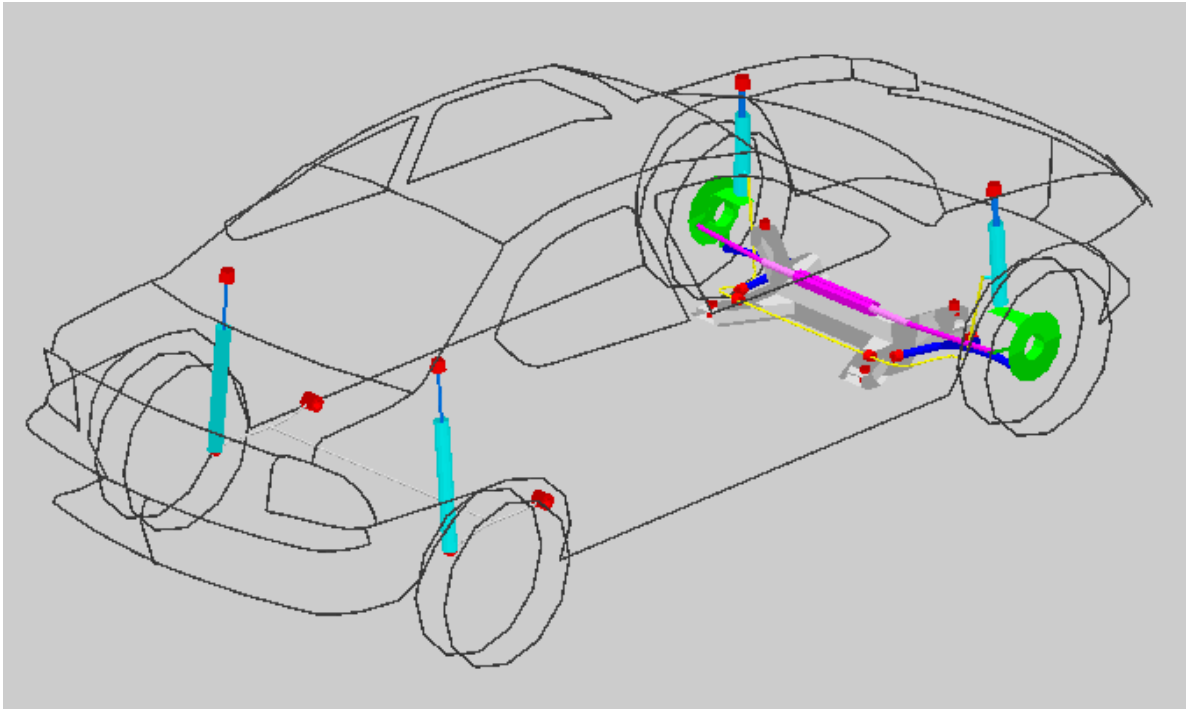

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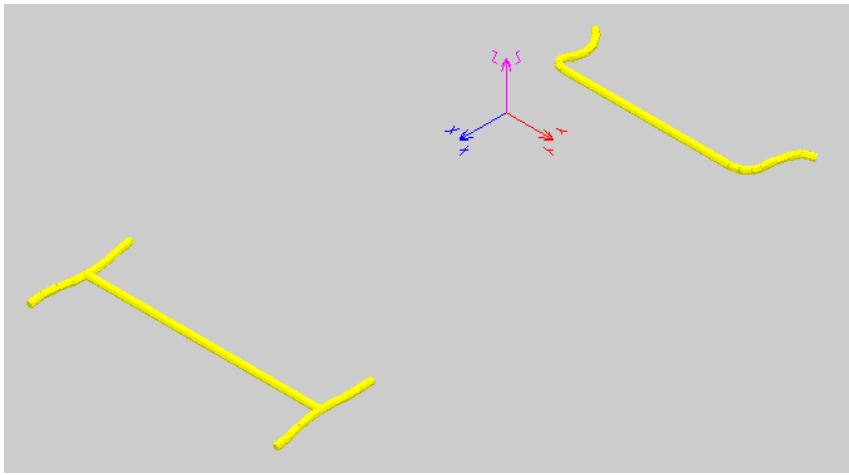

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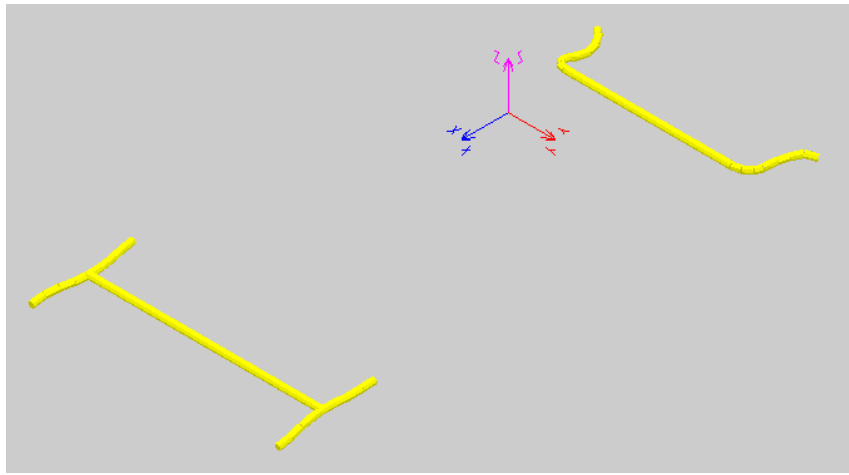

- the model consists in about 1300 equations, related to
  - about 100 structural nodes
  - about 60 nonlinear beam elements
  - more than 80 joints

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- consists in evaluating the loads in rubber components when the model is subjected to test rig excitation

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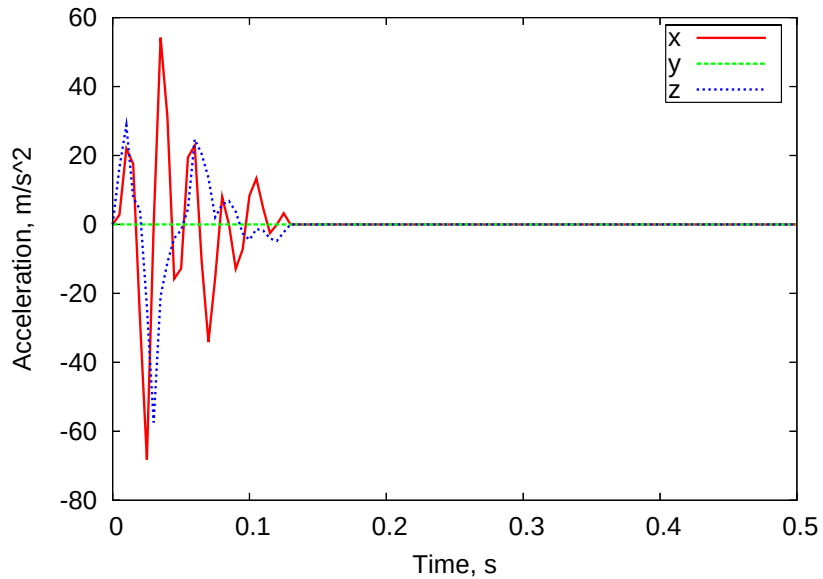
- excitation pattern: acceleration imposed to front right axle



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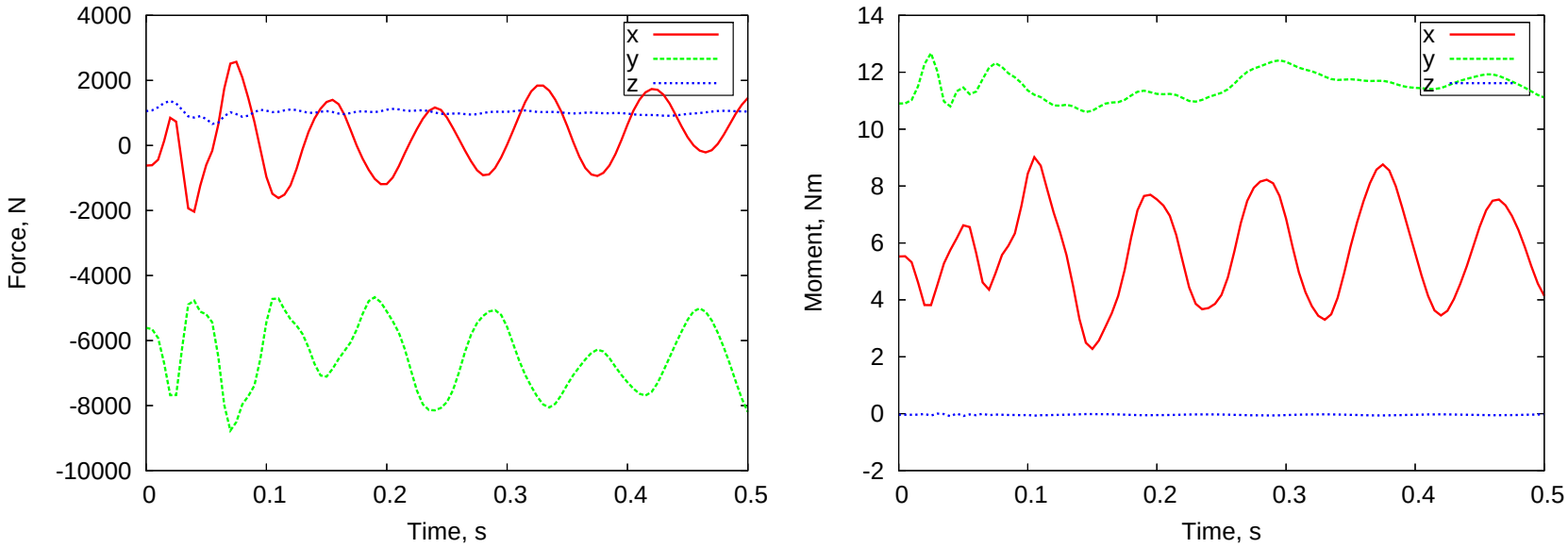

Force and moment in front right shock absorber top bushing

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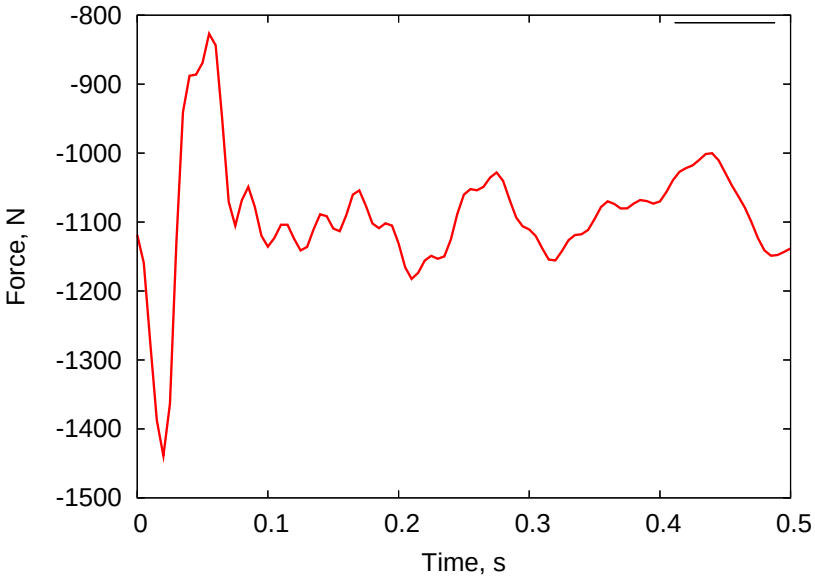

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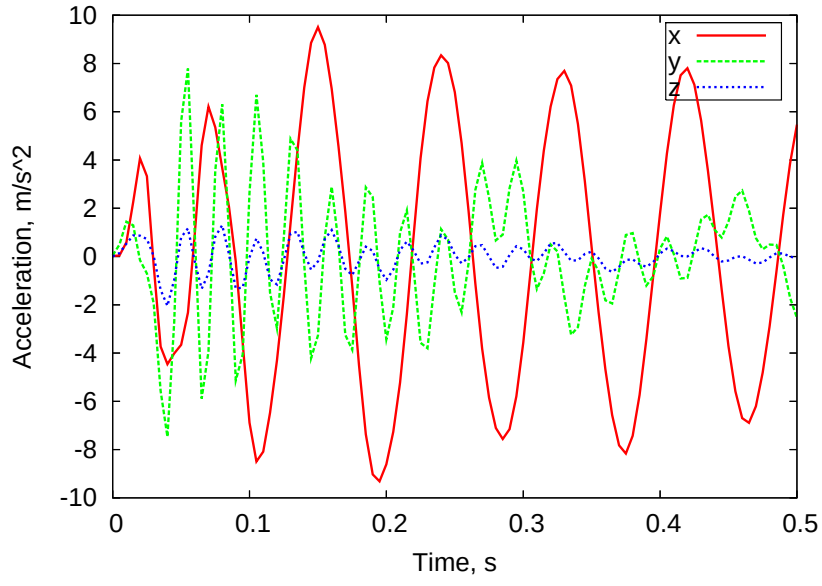

CG acceleration

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ANALYSIS OF LOAD PATTERNS IN RUBBER COMPONENTS FOR VEHICLES

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Questions?